NBE-E4310 - Biomedical Ultrasonics

EXERCISE 2 (30p)

Independent/group work 31.1.2019 at 12-14; correct solutions 7.2.2019 at 12-14

Submission: Please submit your responses via MyCourses as one zip file containing your responses in pdf and Matlab format.

The deadline for submitting your Exercise 2 responses is at 11:00 AM on Feb 7, 2019.

Please, note that not all details needed for the exercises have been necessarily presented during the lectures or the demo. If missing information, please refer to open sources or course book. Students are expected to have basic knowledge of signal processing and Matlab skills.

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1. Common measures of intensity, (5 p)

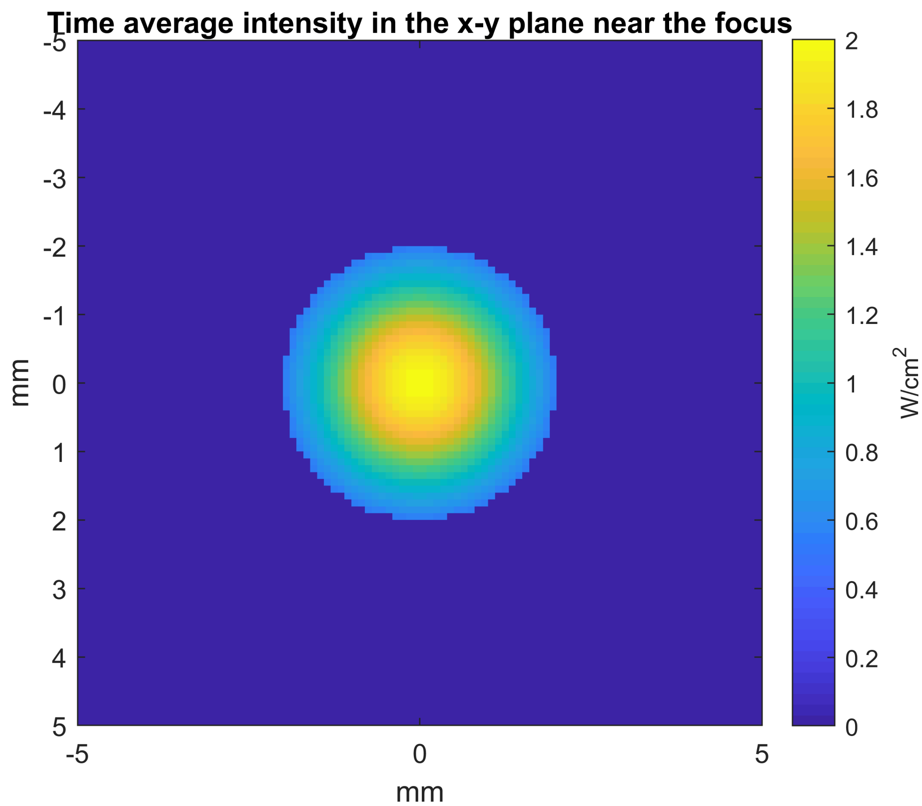
A focused continuous wave is traveling along the *z-axis*. The file (I_ta.txt) contains a *z-slice* of intensity data in the *x-y* plane near the focus of the beam. Each element corresponds to the time average intensity (in W/cm^2) in a 100 μm by 100 μm area.

```
% Data loading
I_ta = load('I_ta.txt');
n_pxls = length(I_ta);  % number of pixels of I_ta
pxl_size = 100e-6;  % length of a single pixel, m

%create a x-y grid of 100um x 100 um with the origo at the center of the
%matrix
axes = [-(n_pxls-1)/2 : (n_pxls-1)/2]*pxl_size;
[x, y] = meshgrid(axes, axes);
```

a) What is the total power of the beam within a 2 mm distance from the center of the beam in the *x-y* plane? (3p)

```
d_r = 2e-3;                        % distance from the center of the beam, m
```

```
mask_d_r = x.^2+y.^2 <= d_r.^2; % boolean map that defines the location of pixels
                                % within a 2 mm distance from the center of the beam
%Plot
figure
imagesc(axes*1e3,axes*1e3,I_ta.*mask_d_r )
c = colorbar ;
daspect([1 1 1])
xlabel('mm')
ylabel('mm')
ylabel(c,'W/cm^2')
title('Time average intensity in the x-y plane near the focus')
```




```
W = sum(sum(I_ta.*mask_d_r/1e-4*pxl_size.^2)); % total power, W
```

$W = 140\ \mathrm{mW}$

b) Calculate the I_{sata} in the region defined in a). (2p)

```
A = pi*d_r.^2;
I_sata = W/A*1e-4; % W/cm2
```

$I_{\text{sata}} = 1.1 \, \frac{\text{W}}{\text{cm}^2}$

2. Radiation Force (5p)

You apply the field presented in Problem 1 to a perfect absorber. The entire power defined by I_ta.txt is absorbed. (10p)

a) What is the radiation force exerted on the absorber? (3p)

$$F = \frac{W}{c}$$

```
c=1.5e3;    %speed of sound in water

W_tot = sum(sum(I_ta/1e-4*pxl_size.^2)); % total power, W
F = W_tot/c*1e6; % uN
```

$F = 130 \, \mu N$

b) What is the maximum Langevin pressure applied to the interface, given that the interface is at the focus of the beam? (2p)

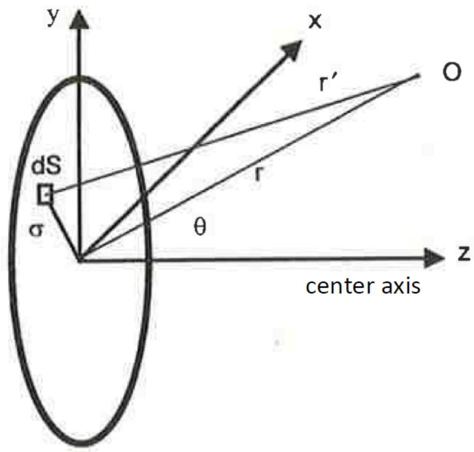
$$P_{\text{Lan}} = \frac{I}{c}$$

```
P_Lan_max = max(max(I_ta*1e4))/c; % Pa
```

$\{P_{\text{Lan}}\}_{\text{max}} = 13.3 \, \text{Pa}$

3. Transducers (20p)

Consider a circular ultrasound transducer of a radius $a = 6 \, \text{mm}$ and operating frequency of $f_0 = 1 \, \text{MHz}$.

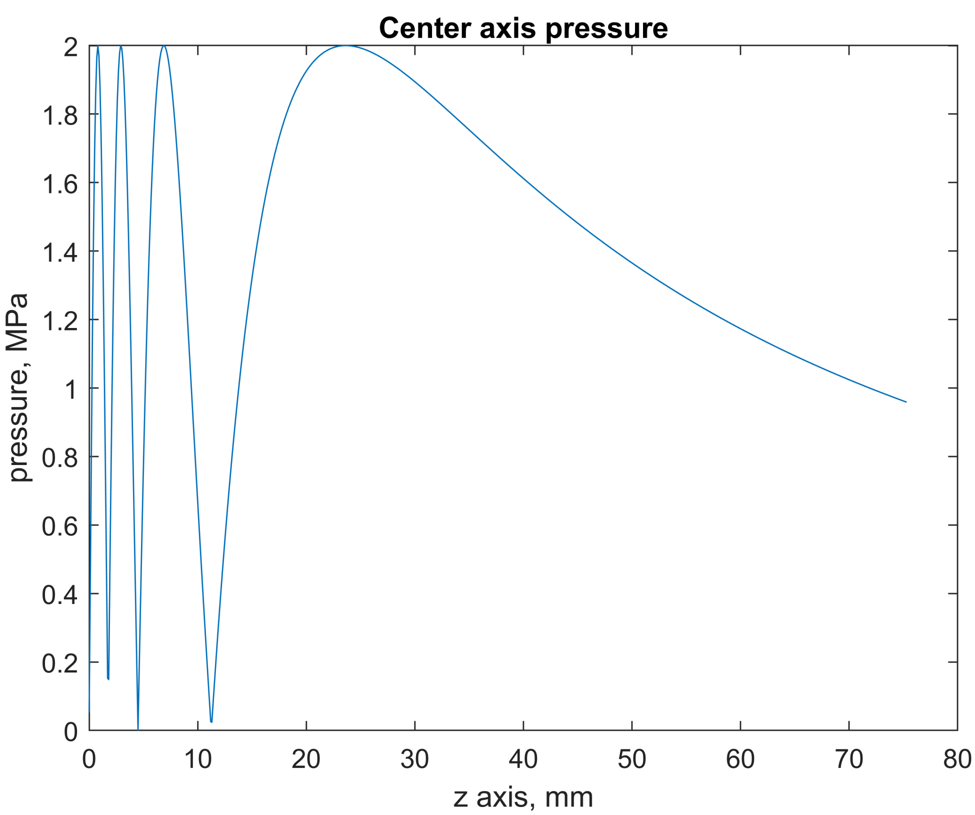


a) Calculate numerically the pressure field at the center axis z as a complex presentation. Plot then envelope of the pressure at the center axis at $t \sim 0\,s$ in the near field. (5p)

$$p_z(r) = i\frac{p_0 k}{2\pi}\int\int\frac{e^{-ikr}}{r}\mathrm{dS}$$

```
%General parameters
rho = 1e3; % kg/m3
f0 = 1e6; % MHz
c = 1500; %m/s
k = 2*pi*f0/c; % wave vector, 1/m
w = 2*pi*f0;
p0 = 1e6; % initial pressure at the transducer, Pa
a = 6e-3; % transducer radius, m
element_size = 50e-6; % transducer element size, m
Z_R = k*a^2/2; % Raleigh distance, m

%create a x-y grid of 1.5cm x 1.5cm with the origo at the center of the
%matrix. This grid defines the x-y plane where the transducer is laying.

L = 1.5e-2;   %size of the grid, m
n_pxls = L/element_size;
axes = [-(n_pxls-1)/2 : (n_pxls-1)/2]*element_size;
[x, y] = meshgrid(axes, axes);

% Define the z-axis vector
z_max = 1*Z_R; % maximum value of the z axis, m
z = 0:element_size*2:z_max;

%Define the transudcer region
mask_transuder = x.^2+y.^2 <= a.^2;

%Define the pressure field at the center axis

p_z = zeros(1,length(z));
dS = element_size.^2;

for j =1: length(p_z)
```

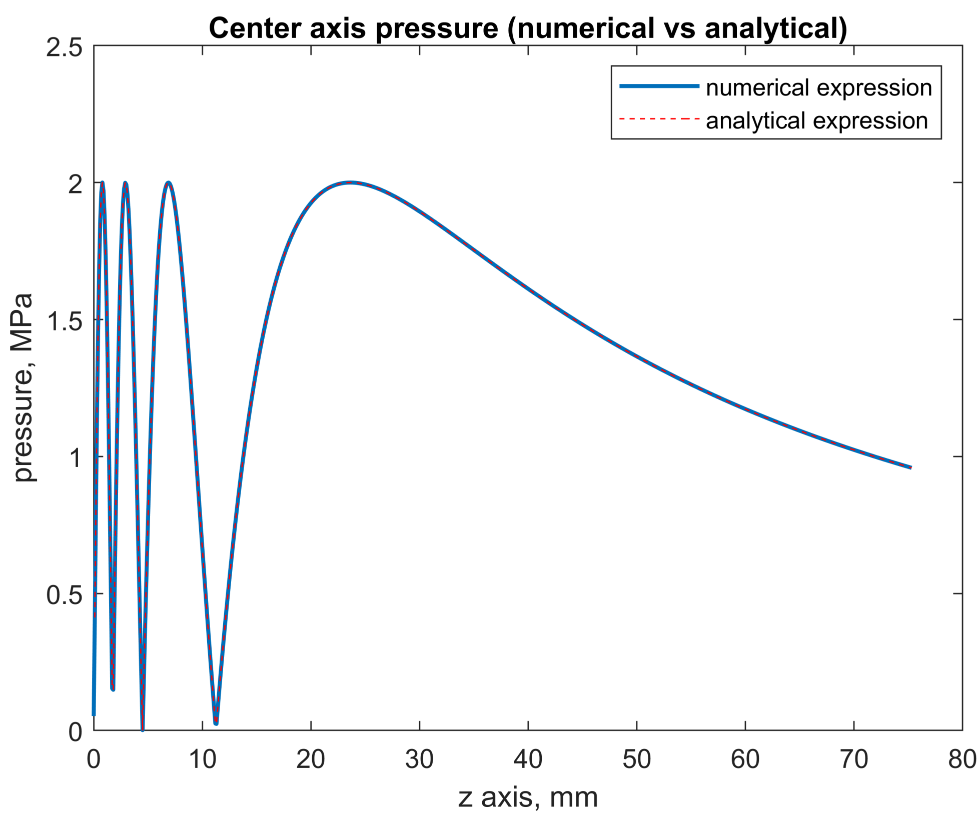
```
    transd_coordinates = horzcat(x(mask_transuder),y(mask_transuder),...
                         zeros(length(y(mask_transuder)),1));

    z_axis_coordinate = [0, 0, z(j)];

    z_axis_coordinate = repmat(z_axis_coordinate, length(y(mask_transuder)),1);

    % Euclidean distance map betwen the z-axis point z(i) and the
    % transducer elements
    r = sqrt((z_axis_coordinate(:,1)-transd_coordinates(:,1)).^2 + ...
             (z_axis_coordinate(:,2)-transd_coordinates(:,2)).^2 + ...
             (z_axis_coordinate(:,3)-transd_coordinates(:,3)).^2);

    integral = sum(exp(i*(-k*r))./r*dS);
    p_z(j) =    i*p0*k/(2*pi)*integral;

end

figure
plot(z/1e-3, abs(p_z)/1e6)
xlabel('z axis, mm')
ylabel('pressure, MPa')
title('Center axis pressure')
```



b) Plot the numerical expression and analytical expression of the pressure at the centre axis in i the same graph. (5p)

$$p_z(z) = 2p_0\left|\sin\left\{\frac{kz}{2}\left[\sqrt{1+\left(\frac{a}{z}\right)^2}-1\right]\right\}\right|$$

```
p_z_analytical = 2*p0*abs(sin(k*z/2.*(sqrt(1+(a./z).^2)-1)));

figure
plot(z/1e-3, abs(p_z)/1e6, 'LineWidth',1.5)
hold on
plot(z/1e-3, p_z_analytical/1e6,'r--')
xlabel('z axis, mm')
ylabel('pressure, MPa')
title('Center axis pressure (numerical vs analytical)')
legend('numerical expression', 'analytical expression')
```



c) Consider the complex vector $p_z(z)$ representig the pressure at center axis. Plot as a video 8 cycles of the pressure signal in time dependent expression $p_z(z)\, e^{i\omega t}$. (5p)

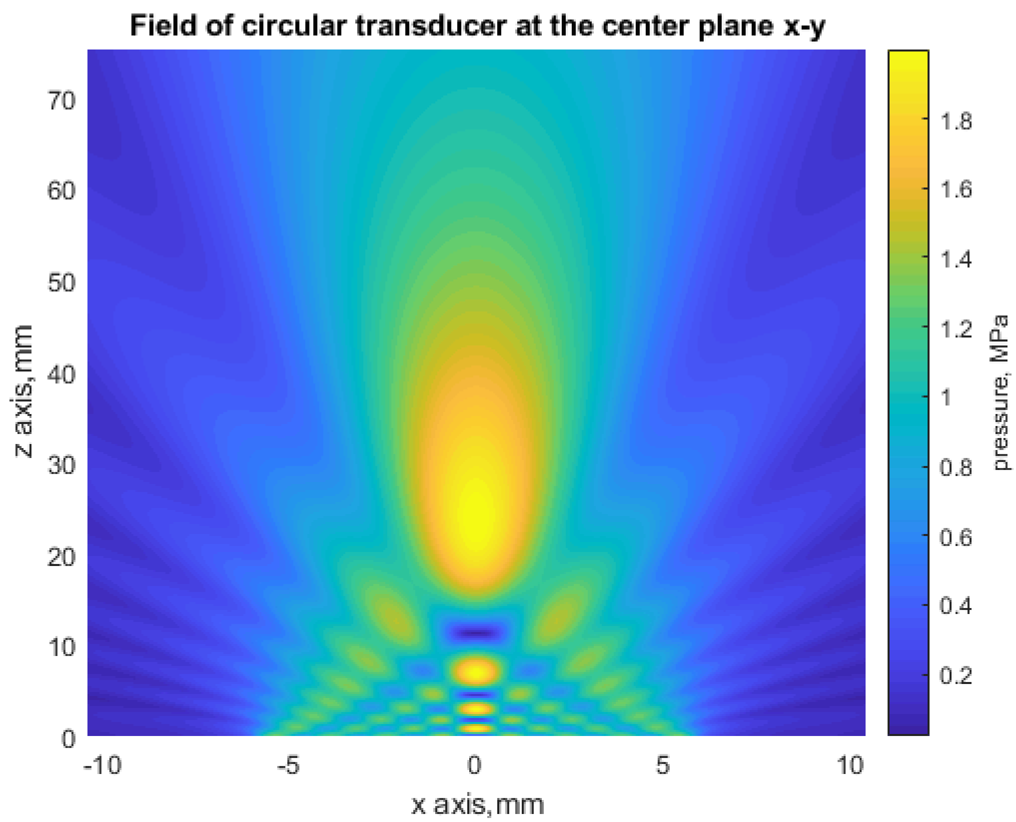
```
n_cyc = 10;
t = 0:1/(100*f0):n_cyc/f0-1/(100*f0);
```

```
figure
for j = 1:length(t)

    p_z_t = p_z.*exp(i*w*t(j));

    plot(z/1e-3, real(p_z_t)/1e6,'b')
    hold on
    plot(z/1e-3, abs(p_z)/1e6,'r')
    xlabel('z axis, mm')
    ylabel('pressure, MPa')
    title('Center axis pressure, p(t,z)')

    ylim=get(gca,'ylim');
    xlim=get(gca,'xlim');
    text(60,1.5,['t = ' num2str(t(j)) ' s'])
        set(gcf,'Visible','on')
    drawnow
    clf
end
```

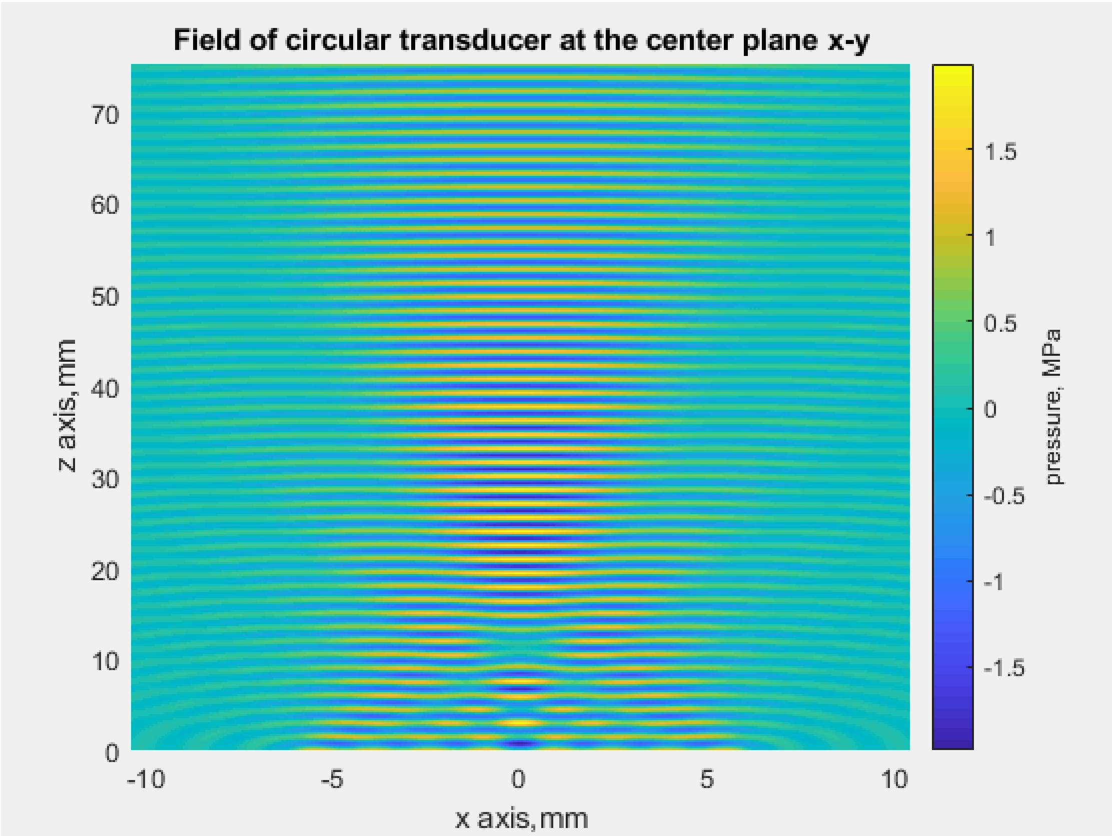
d) Calculate numerically the pressure field at the center plane xy as a complex presentation. Plot then envelope of the pressure at the center plane xy at $t \sim 0\,s$. (5p)

```
rho = 1e3; % kg/m3
f0 = 1e6; % MHz
c = 1500; %m/s
k = 2*pi*f0/c; % wave vector, 1/m
w = 2*pi*f0;
p0 = 1e6; % initial pressure at the transducer, Pa

a = 6e-3; % transducer radius, m
element_size = 100e-6; % transducer element size, m
Z_R = k*a^2/2; % Raleigh distance, m

%create a x-y grid of 5cm x 5cm with the origo at the center of the
%matrix. This grid defines the x-y plane where the transducer is laying.

L = 1.5e-2;   %size of the grid, m
n_pxls = L/element_size;
axes = [-(n_pxls-1)/2 : (n_pxls-1)/2]*element_size;
[x, y] = meshgrid(axes, axes);

% Define the x-z plane grid
z_max = 1*Z_R; % maximum value of the z axis, m
z = 0:element_size*2:z_max;
axes = [-(n_pxls+n_pxls*0.4-1)/2 : (n_pxls+n_pxls*0.4-1)/2]*element_size;
[x2, z] = meshgrid(axes, z);

%Define the transudcer region
mask_transuder = x.^2+y.^2 <= a.^2;
```

```
%Define the pressure field at the center axis

p_xz = zeros(size(x2));
dS = element_size.^2;

for j =1: size(p_xz,2)  % loope over x2
   for m =1: size(p_xz,1)  % loope over z

    transd_coordinates = horzcat(x(mask_transuder),y(mask_transuder),zeros(length(y(mask_transu

    xz_plane_coordinate = [x2(m,j), 0, z(m,j)];

    xz_plane_coordinate = repmat(xz_plane_coordinate, length(y(mask_transuder)),1);

    % Euclidean distance map betwen the z-axis point z(i) and the
    % transducer elements

    r = sqrt((xz_plane_coordinate(:,1)-transd_coordinates(:,1)).^2 + ...
             (xz_plane_coordinate(:,2)-transd_coordinates(:,2)).^2 + ...
             (xz_plane_coordinate(:,3)-transd_coordinates(:,3)).^2);

    integral = sum(exp(i*(-k*r))./r*dS);
    p_xz(m,j) =    i*p0*k/(2*pi)*integral;

   end
end

figure
surf(x2(1,:)*1e3,z(:,1)*1e3,abs(p_xz)/1e6)
view(2)
axis tight
shading interp
xlabel('x axis,mm')
ylabel('z axis,mm')
c = colorbar;
ylabel(c,'pressure, MPa')
title('Field of circular transducer at the center plane x-y')
```




```
%time dependent pressure field
n_cyc = 10;
t = 0:1/(100*f0):n_cyc/f0-1/(100*f0);

figure
for j = 1:length(t)

    p_xz_t =  p_xz.*exp(i*w*t(j));
    surf(x2(1,:)*1e3,z(:,1)*1e3,real(p_xz_t)/1e6)
    view(2)
    axis tight
    shading interp
    xlabel('x axis,mm')
    ylabel('z axis,mm')
    c = colorbar;
    ylabel(c,'pressure, MPa')
    title('Field of circular transducer at the center plane x-y')
    set(gcf,'Visible','on')
    drawnow
end
```



Field of circular transducer at the center plane x-y
z axis, mm
x axis, mm
pressure, MPa
70
60
50
40
30
20
10
0
-10
-5
0
5
10
1.5
1
0.5
0
-0.5
-1
-1.5