NBE-E4310 - Biomedical Ultrasonics

EXERCISE 3 (20p)

Independent/group work 14.2.2019 at 12-14; correct solutions 28.2.2019 at 12-14

Submission: Please submit your responses via MyCourses as one zip file containing your responses in pdf and Matlab format.

The deadline for submitting your Exercise 3 responses is at 11:00 AM on Feb 28, 2019.

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1. Acoustic mercury “fountain” (2p)

An ultrasonic beam is focused on the surface of liquid mercury directly from below. As a result a $100\ \mu m$ tall fountain is formed. What is the I_{spta} ? How about I_{sppa}, when the duty cycle of the ultrasound exposure is 40% ?

The acoustic impedance of mercury is much greater than the acoustic impedance of the air, so the incident wave is almost totally reflected at the liquid-air interface ($R \approx 1$). The radiation force must balance the hydrostatic pressure:

$$d_r \frac{W}{c} = \Delta p\ A$$

$$2\frac{I_{\mathrm{spta}}}{c} = \rho_0 \mathrm{gh}$$

$$I_{\mathrm{spta}} = \rho_0 \mathrm{ghc}/2$$

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rho = 13593; % density, kg/m3
c = 1450;   % speed of sound, m/s
h = 100e-6;  % fountain height, m
DC = 0.4;    % duty cycle
g = 9.8;     % gravity, m/s^2

I_spta = rho*g*h*c/2*1e-4; %W/cm^2
I_sppa = I_spta/DC;
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$$I_{\mathrm{spta}} = 0.96\ \frac{W}{\mathrm{cm}^2}$$

The I_{sppa} is given by :

$$I_{\text{sppa}} = \frac{I_{\text{spta}}}{\text{DC}} = 2.41 \ \frac{W}{\text{cm}^2}$$

2. Geometric attenuation (6p)

Derive the expression of the geometric attenuation when the sound is produced by a:

a) point source.

A point source has spherical wavefronts, which means that the sound propagates as spherical waves. Assuming lossless medium, the total power carried by the waves remains constant, but the intensity decreases moving away from the source since the same power is distributing over bigger areas.

Let's call W_1 and W_2 the powers calculated respectevely at r_1 and r_2.

$$W_1 = W_2 \rightarrow I_1 \, 4\pi r_1^2 = I_2 \, 4\pi r_2^2 \rightarrow I_2 = I_1 \ \frac{r_1^2}{r_2^2}$$

One can notice that the intensity decreases as a function of $\frac{1}{r^2}$. Given the relationship between the pressure and the intensity one obtains:

$$p_2 = \ p_1 \frac{r_1}{r_2}$$

The pressure decreases as a function of $\frac{1}{r}$.

b) line source.

A line source has cylindrical wavefronts, which means that the sound propagates as cylindrical waves. Assuming lossless medium, the total power carried by the waves remains constant, but the intensity decreases moving away from the source since the same power is distributing over bigger areas.

Let's call W_1 and W_2 the powers calculated respectevely at r_1 and r_2.

$$W_1 = W_2 \rightarrow I_1 \, \text{H} 2\pi r_1 = I_2 \, \text{H} 2\pi r_2 \rightarrow I_2 = I_1 \ \frac{r_1}{r_2}$$

One can notice that the intensity decreases as a function of $\frac{1}{r}$. Given the relationship between the pressure and the intensity one obtains:

$$p_2 = \ p_1 \frac{\sqrt{r_1}}{\sqrt{r_2}}$$

The pressure decreases as a function of $\frac{1}{\sqrt{r}}$.

3. Cavitation (6p)

Calculate the resonance size for a bubble excited by 1 MHz Ultrasound in water,

a) near the water-air interface at 1 atm

The resonance bubble size is given by:

$$R_{\text{res}} = \frac{1}{2\pi f}\sqrt{3\frac{p_0}{\rho}}$$

where p_0 can be considered as the atmospheric pressure.

```
f = 1e6;         % frequency, Hz
p0 = 101325;     % atmospheric pressure, Pa
rho = 1e3;       % water density, kg/m^3
g = 9.8;         % gravity, m/s^2

h1 = 0;          %distance1 from the surface, m
h2 = 10e-2;      %distance2 from the surface, m
h3 = 1;          %distance3 from the surface, m

p = p0 + rho*g*h1; %pressure at distance1 from the surface, Pa

R_res = 1/(2*pi*f)*sqrt(3*p/rho);
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$R_{\text{res}} = 2.77\ \mu m$

b) at 10 cm depth

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p = p0 + rho*g*h2; %pressure at distance1 from the surface, Pa

R_res = 1/(2*pi*f)*sqrt(3*p/rho);
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$R_{\text{res}} = 2.79\ \mu m$

c) at 1 m depth

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p = p0 + rho*g*h3; %pressure at distance1 from the surface, Pa
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R_res = 1/(2*pi*f)*sqrt(3*p/rho);
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$R_{\mathrm{res}} = 2.91\ \mu m$

4. Ultrasound safety (6p)

The instantaneous intensity at the PNP is:

a) $1\ \frac{W}{\mathrm{cm}^2}$

b) $100\ \frac{W}{\mathrm{cm}^2}$

c) $1000\ \frac{W}{\mathrm{cm}^2}$

Which of these are likely to cause biological damage? Why?

Assume that the ultrasound is delivered in short pulses with no macroscopic thermal effects.

Assuming that $p = p_0 \sin(2\pi\ f\ t\ )$ and that PNP = PPP, the intensity can be expressed as $I(t) = \frac{p(t)^2}{\rho c} = \frac{p_0^2}{\rho c}\ \sin(2\pi\ f\ t)^2 = I_{\mathrm{PNP}} \sin(2\pi\ f\ t)^2$.

Since the ultrasound is delivered in short pulses we can assume for exaple that that only one cycle of sinusoid is delivered. We can now measure the I_{sppa} by averaging $I(t)$ on a time window of duration PD = T.

$$I_{\mathrm{sppa}} = \frac{1}{T}\int_0^T I(t)\mathrm{dt} = \ \frac{1}{T}\int_0^T I_{\mathrm{PNP}} \sin(2\pi\ f\ t)^2 \mathrm{dt} = \frac{I_{\mathrm{PNP}}}{T}\int_0^T \sin(2\pi\ f\ t)^2 \mathrm{dt} = I_{\frac{\mathrm{PNP}}{2}}$$

Let's now consider the following table:

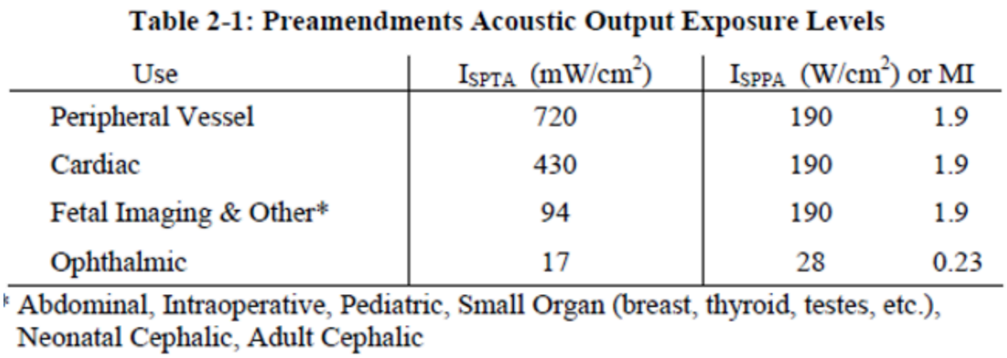

Table 2-1: Preamendments Acoustic Output Exposure Levels

Use	I_{SPTA} (mW/cm^2)	I_{SPPA} (W/cm^2)	or MI
Peripheral Vessel	720	190	1.9
Cardiac	430	190	1.9
Fetal Imaging & Other*	94	190	1.9
Ophthalmic	17	28	0.23

* Abdominal, Intraoperative, Pediatric, Small Organ (breast, thyroid, testes, etc.), Neonatal Cephalic, Adult Cephalic

a) $I_{\mathrm{sppa}} = I_{\frac{\mathrm{PNP}}{2}} = 0.5\ \frac{W}{\mathrm{cm}^2}$. According to the table this intensity is not likely to cause any damage in any case.

b) $I_{\mathrm{sppa}} = 50\ \frac{W}{\mathrm{cm}^2}$. This intensity is not likely to cause any damage in any case, except from ophtalmic use.

c) $I_{\mathrm{sppa}} = 500\ \frac{W}{\mathrm{cm}^2}$. This intensity is likely to cause damage in all cases.