NBE-E4310 - Biomedical Ultrasonics

EXERCISE 4 (30p)

Independent/group work 7.3.2019 at 12-14; correct solutions 14.3.2019 at 12-14

Submission: Please submit your responses via MyCourses as one zip file containing your responses in pdf and Matlab format.

The deadline for submitting your Exercise 4 responses is at 11:00 AM on Mar 14, 2019.

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1. Cavitation (20p)

Based on the article https://doi.org/10.1121/1.402855:

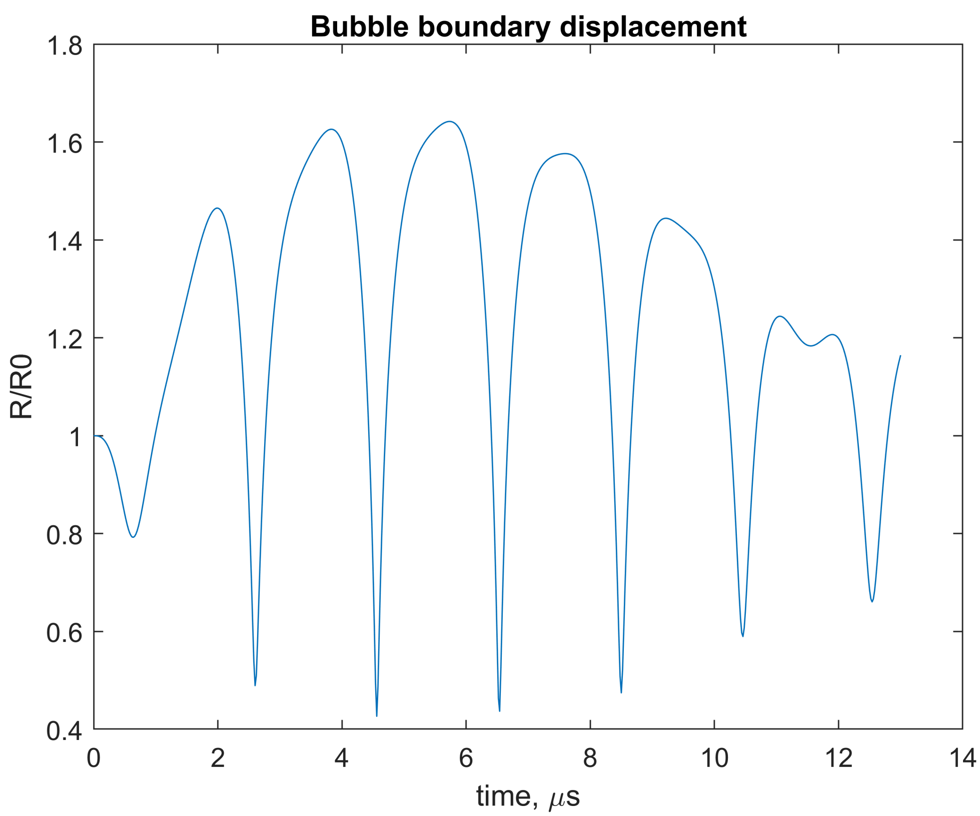
a) implement in matlab (you can find a template script on the course page) the following equation describing the motion of a single bubble in a spatially uniform acoustic field.

$$\left(1-\frac{\mathrm{dR}}{\mathrm{dt}}\frac{1}{c}\right)R\frac{\mathrm{d}^2}{\mathrm{d}t^2}R+\frac{3}{2}\frac{\mathrm{dR}^2}{\mathrm{dt}}\left(1-\frac{\mathrm{dR}}{\mathrm{dt}}\frac{1}{3c}\right)=\left(1+\frac{\mathrm{dR}}{\mathrm{dt}}\frac{1}{c}\right)\frac{1}{\rho_l}\left[p_B(R,t)-p_A\left(t+\frac{R}{c}\right)-p_\infty\right]+\frac{R}{\rho_l c}\frac{\mathrm{dp}_B(R,t)}{\mathrm{dt}}$$

Then plot the relative bubble boundary displacement $R(t)/R_0$. (5p)

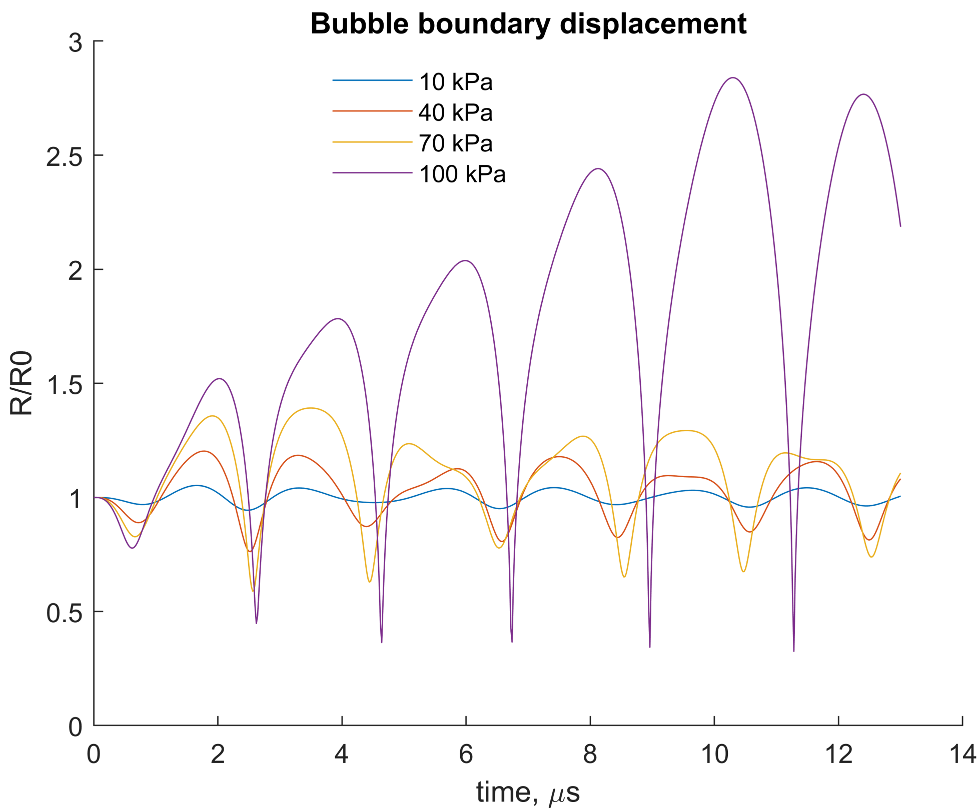
```
%Parameters
c= 1500 ;                           % speed of sound in water, m/s
rho = 998 ;                         % water density, kg/m^3
sigma = 0.072 ;                     % surface tension, N/m
mu = 0.001;                         % shear viscosity, Pa*s
v0 = 0;                             % initial bubble boundary velocity, m/s
pinf = 101325;                      % ambient pressure, Pa
amp = 0.9e5;                        % pressure amplitude
gamma = 1.4;                        % ratio between the specific heat capacities
f = 500e3 ;                         % driving frequency, Hz
t_max = 13e-6;                      % maximum observation time
dt = 1/(100*f);                     % time step, us
R0 = 5e-6;                          % initial bubble radius

%Variables
syms R(t) pa(t)
```

```
Rt = diff(R,t);                       % first derivative of bubble radius, dR/dt
Rtt = diff(R,t,2);                    % second derivative of bubble radius, dR/dt
pa = amp*sin(2*pi*f*(t+R/c));         % time delayed driving pressure, Pa
p0 = pinf + 2*sigma/R0;               % internal pressure of the bubble at equilibrium
pg = p0*(R0/R)^(3*gamma);             % gas perssure in the interior of the bubble
pb = pg -  2*sigma/R - 4*mu*(Rt/R);  % pressure on the liquid side of the bubble interface
pbt = diff(pb,t);                     %dpb/dt

% Define the differential equation
eqn1 = (1-Rt./c).*R.*Rtt + 3/2.*Rt.^2.*(1-Rt./(3.*c)) == (1+Rt./c).*1./rho.*(pb-pa-pinf) + R./

%Solve differential equation
[V] = odeToVectorField(eqn1);
M = matlabFunction(V,'vars', {'t','Y'});
sol = ode45(M,[0 t_max],[R0 v0]);

%Plots
R = deval(sol,[0:dt:t_max],1);       % bubble radius, m
v = deval(sol,[0:dt:t_max],2);       % bubble velocity, m/s
a = [0 diff(v)/dt];                   % bubble acceleration, m/s^2

figure
plot([0:dt:t_max]*1e6,abs(R)/(R0))
title('Bubble boundary displacement')
xlabel('time, \mus')
ylabel('R/R0')
```



b) Determine the pressure threshold that enables inertial cavitation, that can be considered as when the $R(t) > 2\,R_0$ where R_0 is the initial bubble radius. (5p)

```
i = 1;

figure

for amp = 0.1e5 : 0.3e5 :1e5

    %Variables
    syms R(t) pa(t)

    %Variables
    pa = amp*sin(2*pi*f*(t+R/c));       % time delayed driving pressure, Pa

    % Define the differential equation
    eqn1 = (1-Rt./c).*R.*Rtt + 3/2.*Rt.^2.*(1-Rt./(3.*c)) == (1+Rt./c).*1./rho.*(pb-pa-pinf) +

    %Solve differential equation
    [V] = odeToVectorField(eqn1);
    M = matlabFunction(V,'vars', {'t','Y'});
    sol = ode45(M,[0 t_max],[R0 v0]);

    %Plots
    R = deval(sol,[0:dt:t_max],1);      % bubble radius, m

    hold on
    plot([0:dt:t_max]*1e6,abs(R)/(R0))
    title('Bubble boundary displacement')
    xlabel('time, \mus')
    ylabel('R/R0')

    legendInfo{i} = ['' num2str(amp/1000) ' kPa'];
    i = i + 1 ;

end

legend(legendInfo,'Location','best','box','off')
```



From the graph above can be noticed that, in this case, a pressure amplitude > $100\,\mathrm{kPa}$ enables inertial cavitation, for the bubble radius becomes 2 times grater than the initial radius.

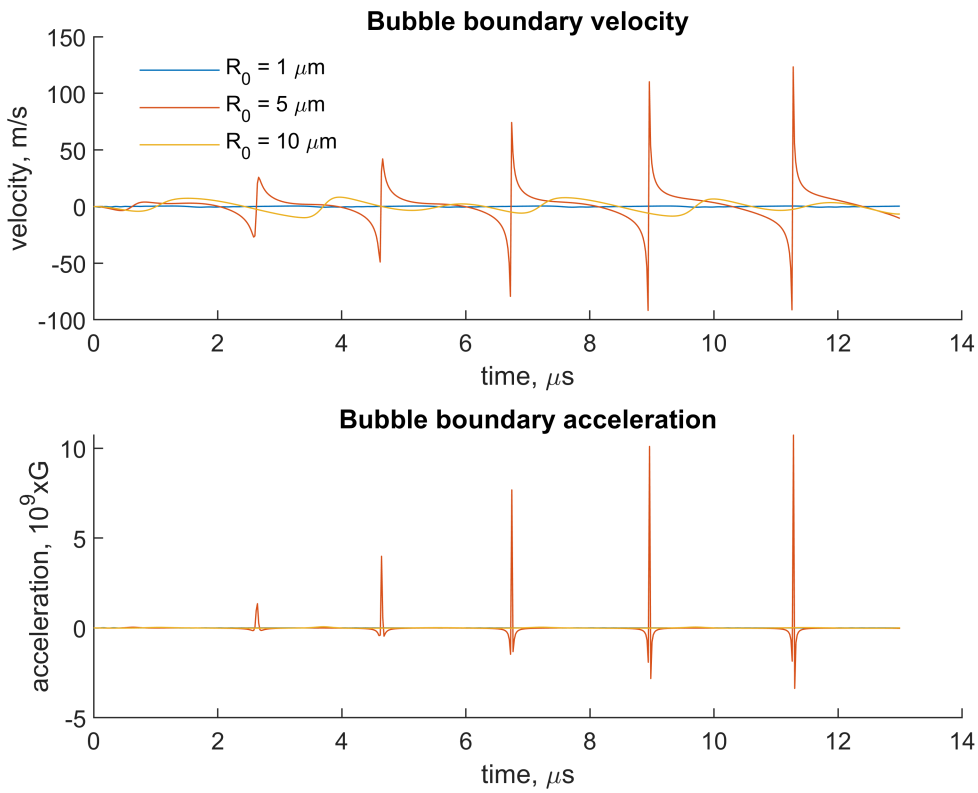
c) Plot the bubble boundary velocity for the following cases: (5p)

1) $R_{0,1} = 1\mu m$
2) $R_{0,2} = 5\mu m$
3) $R_{0,3} = 10\mu m$

d) Plot the bubble boundary acceleration for the same cases as in the previous point. What differences do you observe? Why? (5p)

```
i = 1;

figure

for R0 = [1e-6 5e-6 10e-6]

    %Variables
    syms R(t)

    p0 = pinf + 2*sigma/R0;             % internal pressure of the bubble at equilibrium
    pg = p0*(R0/R)^(3*gamma);           % gas perssure in the interior of the bubble
```

```
    pb = pg -  2*sigma/R - 4*mu*(Rt/R); % pressure on the liquid side of the bubble interface
    pbt = diff(pb,t);                   %dpb/dt

    % Define the differential equation
    eqn1 = (1-Rt./c).*R.*Rtt + 3/2.*Rt.^2.*(1-Rt./(3.*c)) == (1+Rt./c).*1./rho.*(pb-pa-pinf) +

    %Solve differential equation
    [V] = odeToVectorField(eqn1);
    M = matlabFunction(V,'vars', {'t','Y'});
    sol = ode45(M,[0 t_max],[R0 v0]);

    %Plots
    R = deval(sol,[0:dt:t_max],1);      % bubble radius, m
    v = deval(sol,[0:dt:t_max],2);      % bubble velocity, m/s
    a = [0 diff(v)/dt];                 % bubble acceleration, m/s^2


    legendInfo{i} = ['R_0 = ' num2str(R0 * 1e6) ' \mum'];

    subplot(211)
    hold on
    plot([0:dt:t_max]*1e6,v)
    title('Bubble boundary velocity')
    xlabel('time, \mus')
    ylabel('velocity, m/s')
    legend(legendInfo,'Location','best','Box','off')

    subplot(212)
    hold on
    plot([0:dt:t_max]*1e6,a/1e9)
    title('Bubble boundary acceleration')
    xlabel('time, \mus')
    ylabel('acceleration, 10^9xG')

    i = i + 1 ;

end
hold off
```



At the excitation frequency of $500\,\mathrm{kHz}$ the radius at which the bubble starts resonating is $\sim 5.5\ \mu m$, as calculated with the Minnaert formula. The highest velocities and acceleration are observed when $R_0 = 5\ \mu m$ because is close to the bubble resonance size, while in the other cases the bubble is off resonance.

2. Radiation force in absorbing medium (2p)

What is the radiation pressure gradient in muscle tissue at $1\,\mathrm{MHz}$, when $I_{\mathrm{spta}} = 5\ \frac{W}{\mathrm{cm}^2}$?

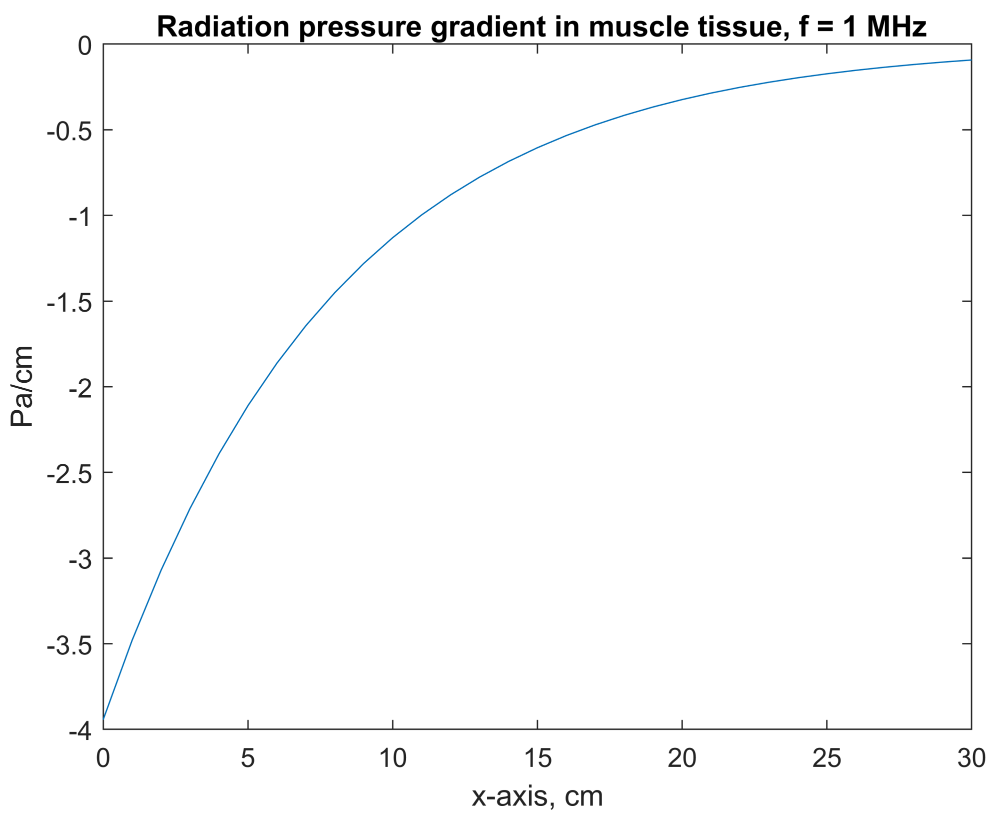
The Langevin pressure is defined as:

$$P_{\mathrm{Lan}} = \frac{I}{c}$$

Since the muscle is an absorbing medium its attenuation coefficient α is taken into account. The gradient of the radiation pressure is :

$$\nabla P_{\text{Lan}} = \nabla \frac{I}{c} = \nabla \frac{I_0 e^{-\alpha x}}{c} = -\alpha \frac{I_0 e^{-\alpha x}}{c}$$

```
%Parameters
c = 1585 ;  % speed of sound in tissue, cm/s
alpha = 12.5; % attenuation coefficient in muscle, Np/cm
I = 5e4 ; % acoustic intensity, W/m^2

x = [0:30]*1e-2; % x-axis, m

P_lan = I/c;
P_grad = -alpha * P_lan * exp(-alpha*x);

figure
plot(x*1e2, P_grad*1e-2)
xlabel('x-axis, cm')
ylabel('Pa/cm')
title(' Radiation pressure gradient in muscle tissue, f = 1 MHz')
```



3. Acoustic levitation (5p)

You have a polystyrene ball with a radius of 1 mm, and a λ / 2 levitator operating at 20 kHz. What is the minimum PPP in the standing wave that can levitate the sphere in air? Convert this peak pressure to SPL.

The acoustic radiation force on a sphere within a standing wave is given by :

$$F = -\left(\frac{\pi \hat{p}^2 V_s \beta_m}{2\lambda}\right) \phi \sin(2kx) = -\frac{4\pi}{3} R^3 \frac{k}{2} \frac{\hat{p}^2}{2\rho_m {c_m}^2} \phi \sin(2kx)$$

In order to make levitate the sphere in air, the acoustic radiation force must win the gravity force exerted on the particle

$$\max(F_{\text{rad}}) > F_{\text{gravity}}$$

```
%Parameters
R = 30e-6; % sphere radius, m
Vp = 4/3*pi*R^3; % sphere volume, m^3
cm = 343; % speed of sound in air, m/s
cs = 2400; % speed of sound in polystyrene, m/s
rhos = 1.04e3; % density of polystyrene, kg/m^3
rhom = 1.225; % density of air, kg/m^3
f = 20e3; % driving frequency, Hz
lambda = cm/f; % wavelength, m
k = 2*pi/lambda; % wave vector
P0 = 2e-5; % reference pressure
Lambda = rhos/rhom;
sigma = cs/cm;
Phi = (5*Lambda-2)/(2*Lambda+1)-1/Lambda/sigma^2; % contrast factor;
g = 9.8;
Fg = rhos*Vp*g;
x = -lambda/8;

p =  sqrt(Fg*1/(Vp*k*Phi/(2*2*rhom*cm^2))); %Pressure, Pa
SPL = 20*log10(p/P0);
```

$P = 2.5\,\text{kPa}$

$\text{SPL} = 162\,\text{dB}$

4. Acoustic streaming (3p)

You are using a HIFU setup at $1\,\text{MHz}$, where the $I_{\text{spta}} = 1\,\frac{W}{\text{cm}^2}$. The geometric factor G is 2.

a) What is the streaming velocity in water at the focus?

b) What is the streaming velocity in blood at the focus?

The acoustic streaming velocity at the focus is given by:

$$v_s = \frac{2\,\alpha\,I_{\text{ta}}}{c\,\mu}\,d^2\,\text{G}$$

```
% Parameters
alpha_w = 100 ; % 1/m, in water
c_w = 1.5e3; % m/s, in water
mu_w = 1e-3; % Ns/m^2, dynamic viscosity, in water

alpha_b = 102 ; % 1/m, in blood
c_b = 1.575e3; % m/s, in blood
mu_b = 3e-3; % Ns/m^2, dynamic viscosity, in blood

I = 1e4; % W/m^2
G = 2;
d = 10e-2; % m

v_w = 2*alpha_w * I *G/(c_w*mu_w)*d^2; % velocity in water, m/s

v_b = 2*alpha_b * I *G/(c_b*mu_b)*d^2; % velocity in water, m/s
```

$v_{\text{water}} = 26.7\ \frac{\text{km}}{s}$

$v_{\text{blood}} = 8.6\,\frac{\text{km}}{s}$