NBE-E4310 - Biomedical Ultrasonics

EXERCISE 5 (40p)

Independent/group work 21.3.2019 at 12-14; correct solutions 28.3.2019 at 12-14

Submission: Please submit your responses via MyCourses as one zip file containing your responses in pdf and Matlab format.

The deadline for submitting your Exercise 5 responses is at 11:00 AM on Mar 28, 2019.

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1. Cavitation (11p)

Consider the differential equation proposed in the Exercise 4 task 1 and use same parameters.

a) Calculate and plot the contribution of the pressure generated by the pulsating bubble to the total pressure field at a distance of $15\ \mu m$ from the centre of the bubble. Consider the bubble as a pulsating sphere source. (8p)

b) Plot the amplitude spectrum of the total pressure signal. (3p)

The total pressure at a distance r from the centre of the bubble is given by the sum of the driving pressure and the pressure generated by the bubble.

$$p_{\text{tot}} = p_a + p_b$$

where p_b can be approximated as the pressure field generated by a pulsating sphere.

$$p_b = \frac{R}{r} v_r \rho_0 c_0 \frac{-\text{ikR}}{1 - \text{ikR}} e^{\text{ik}(r-R)}$$

where R is the radius of the bubble, r the observation point, v_r the bubble velocity.

```
%Parameters
c= 1500 ;                                   % speed of sound in water, m/s
rho = 998 ;                                 % water density, kg/m^3
```

```
sigma = 0.072 ;                     % surface tension, N/m
mu = 0.001;                         % shear viscosity, Pa*s
v0 = 0;                             % initial bubble boundary velocity, m/s
pinf = 101325;                      % ambient pressure, Pa
amp = 0.9e5;                        % pressure amplitude
gamma = 1.4;                        % ratio between the specific heat capacities
f = 500e3 ;                         % driving frequency, Hz
lambda = c/f;                       % wavelength, m
k = 2*pi/lambda;                    % wave vector, rad/m
omega = 2*pi*f;                     % angular frequency, rad/s
t_max = 65e-6;                      % maximum observation time
dt = 1/(100*f);                     % time step, us
R0 = 6.5e-6;                          % initial bubble radius

%Variables
syms R(t) pa(t)

Rt = diff(R,t);                     % first derivative of bubble radius, dR/dt
Rtt = diff(R,t,2);                  % second derivative of bubble radius, dR/dt
pa = amp*sin(2*pi*f*(t+R/c));       % time delayed driving pressure, Pa
p0 = pinf + 2*sigma/R0;             % internal pressure of the bubble at equilibrium
pg = p0*(R0/R)^(3*gamma);           % gas perssure in the interior of the bubble
pb = pg -  2*sigma/R - 4*mu*(Rt/R); % pressure on the liquid side of the bubble interface
pbt = diff(pb,t);                   %dpb/dt

% Define the differential equation
eqn1 = (1-Rt./c).*R.*Rtt + 3/2.*Rt.^2.*(1-Rt./(3.*c)) == (1+Rt./c).*1./rho.*(pb-pa-pinf) + R./(

%Solve differential equation
[V] = odeToVectorField(eqn1);
M = matlabFunction(V,'vars', {'t','Y'});
sol = ode45(M,[0 t_max],[R0 v0]);

R = deval(sol,[0:dt:t_max],1);      % bubble radius, m
v = deval(sol,[0:dt:t_max],2);      % bubble velocity, m/s
a = [0 diff(v)/dt];                 % bubble acceleration, m/s

% Pressure generated by the bubble
r  = 15e-6;                         % distance from the bubble centre, m
pb = R/r.*max(abs(v)).*rho.*c.*(-i.*k.*R)./(1-i.*k.*R).*exp(i.*k.*(r-R)-i.*omega.*[0:dt:t_max])
pa = amp*sin(2*pi*f*[0:dt:t_max] + r/c); % time delayed driving pressure, Pa
p_tot = real(pb) + pa;                   % total pressure, Pa

% Plot

figure

subplot(211)
plot([0:dt:t_max]*1e6, p_tot*1e-6)
xlabel('Time (\mus)')
ylabel('Pressure (MPa)')
title('Total pressure field')
% xlim([30 50])
```

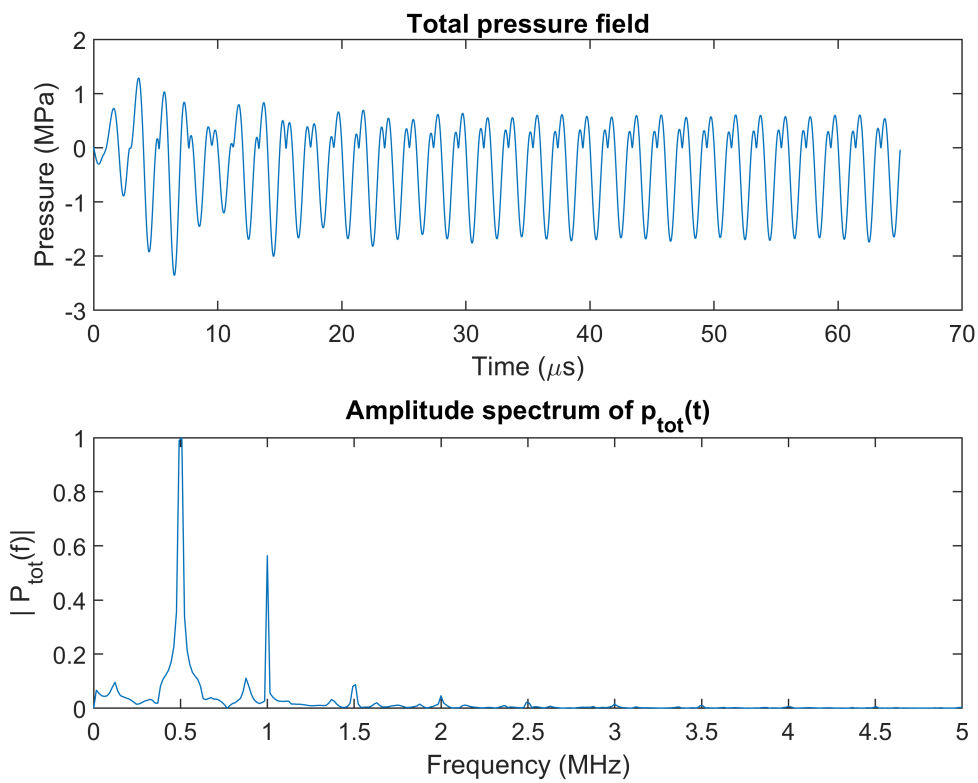
```
% ylim([-0.4 0.4])

subplot(212)
Fs = 1/dt;               % Sampling frequency
T = t_max;                % Sampling period
L = length(R);                % Length of signal
t = (0:L-1)*T;           % Time vector

X = p_tot-mean(p_tot);

Y = fft(X);
P2 = abs(Y/L);
P1 = P2(1:(L/2)+1);
P1(2:end-1) = 2*P1(2:end-1);

f = Fs*(0:(L/2))/L/1e6;
plot(f,P1/max(P1))
title('Amplitude spectrum of p_{tot}(t)')
xlabel('Frequency (MHz)')
ylabel('| P_{tot}(f)|')
xlim([0 5])
```



2. Thermal Dose 1 (9p)

Calculate how long it takes to reach tissue damage ($30 < \text{TD}_{43} < 240$) and tissue necrosis ($\text{TD}_{43} > 240$) in the following cases:

a) $T_1 = 42.5\ °\text{C}$ (3p)

```
T = 42.5;                                              % Costant temperature, °C
tissue_dmg_tsh = 30*60;                              % Equivalent time at 43 °C for tissue damage, s
necrosis_tsh = 240*60;                               % Equivalent time at 43 °C for necrosis, s
R = 0.25;

syms t

eqn1 = tissue_dmg_tsh == int(R^(43-T),t,0,t);
eqn2 = necrosis_tsh == int(R^(43-T),t,0,t);

t_tissue_dmg = double(solve(eqn1,t));       % Time for tissue damage at T, s
t_necrosis = double(solve(eqn2,t));         % Time for necrosis at T, s
```

$t_{\text{tissue damage}} = 1h$

$t_{\text{necrosis}} = 8h$

b) $T_2 = 54\ °\text{C}$ (3p)

```
T = 54;                                     % Costant temperature, °C
R = 0.5;

syms t

eqn1 = tissue_dmg_tsh == int(R^(43-T),t,0,t);
eqn2 = necrosis_tsh == int(R^(43-T),t,0,t);

t_tissue_dmg = double(solve(eqn1,t));       % Time for tissue damage at T, s
t_necrosis = double(solve(eqn2,t));         % Time for necrosis at T, s
```

$t_{\text{tissue damage}} = 0.9s$

$t_{\text{necrosis}} = 7s$

c) $T_3 = 80\ °\text{C}$ (3p)

```
T = 80;                                   % Costant temperature, °C
R = 0.5;

syms t
```

```
eqn1 = tissue_dmg_tsh == int(R^(43-T),t,0,t);
eqn2 = necrosis_tsh == int(R^(43-T),t,0,t);

t_tissue_dmg = double(solve(eqn1,t));       % Time for tissue damage at T, s
t_necrosis = double(solve(eqn2,t));         % Time for necrosis at T, s
```

$t_{\text{tissue damage}} = 13ns$

$t_{\text{necrosis}} = 100\ \mu s$

3. Thermal Dose 2 (10p)

Calculate how long it takes to reach tissue damage ($30 < \text{TD}_{43} < 240$) and tissue necrosis ($\text{TD}_{43} > 240$) in the following cases:

a) $T_1(t) = 70\,(\,1 - e^{-t/10}\,)\ ^\circ\text{C}$ (5p)

```
R = 0.5 ;

syms t
T1(t) = 70*(1-exp(-t/10));

eqn1 = tissue_dmg_tsh == int(R^(43-T1(t)),t,0,t);
eqn2 = necrosis_tsh == int(R^(43-T1(t)),t,0,t);

t_tissue_dmg = double(solve(eqn1,t));       % Time for tissue damage at T, s
t_necrosis = double(solve(eqn2,t));         % Time for necrosis at T, s
```

$t_{\text{tissue damage}} = 14s$

$t_{\text{necrosis}} = 16.7s$

b) $T_2(t) = 55\,(\,1 - e^{-t/15}\,)\ ^\circ\text{C}$ (5p)

```
R = 0.5 ;

syms t
T1(t) = 55*(1-exp(-t/15));

eqn1 = tissue_dmg_tsh == int(R^(43-T1(t)),t,0,t);
eqn2 = necrosis_tsh == int(R^(43-T1(t)),t,0,t);

t_tissue_dmg = double(solve(eqn1,t));       % Time for tissue damage at T, s
t_necrosis = double(solve(eqn2,t));         % Time for necrosis at T, s
```

$t_{\text{tissue damage}} = 41s$

$t_{\text{necrosis}} = 55s$

4. Biomedical applications (10p)

a) Define the parameters that shoud be used in order to achieve histotripsy at 1 MHz without inducing thermal damage in tissue. (5p)

In order to obtain tissue homogenisation, the mechanical index should be high enough to produce cavitation. If we consider a $\text{MI} = 3$ then:

$$\text{PNP} = \text{MI}\ \sqrt{f} = 3\,\text{MPa}$$

Assuming that the signal is sinusoidal, with $\text{PNP} = \text{PPP} = p_0$, the I_{spta} is given by:

$$I_{\text{spta}} = \text{DC}\, I_{\text{sppa}} = \frac{I_0}{2}\,\text{DC}$$

where $I_0 = \dfrac{\text{PNP}^2}{\rho\, c}$ ans PRP has to be defined.

Thermal damage should be also avoided, which means:

$$\text{TD}_{43} = \int_0^t R^{43-T(t)}\text{dt} < 30\text{ min}$$

From the Pennes' Bioheat Transfer Equation one can obtain the relation between the rate of temperature change and the the ultrasound power

deposition per unit volume:

$$\rho_t\, c_t \frac{\text{dT}}{\text{dt}} = 2\alpha\, I_{\text{spta}}$$

where ρ_t is the tissue density, c_p is the tissue specific heat capacity and α is he absorption coefficient. The time variation of the temperature is given by:

$$T_2(t) = T_1 + 2\alpha \frac{I_{\text{spta}}}{\rho_t c_t}\, t$$

By including the temperature epression into the termal dose expression and integrating we obtain:

$$\text{TD}_{43} = -\frac{\rho_t c_t}{\alpha\, I_0 \text{DC}}\ \frac{R^{43-T_1-\alpha\frac{I_0\text{DC}}{\rho_t c_t}t}}{\ln(R)} < 30\text{ min}$$

which gives the general formula to calculate what the DC should for a specific signal duration in order not to cause thermal damage in tissue.

If we consider a singal duration of $1h$, then $\mathrm{DC} < 0.007\%$.

```
alpha = 0.1e-2 ;                % Absorption coefficient in liver, 1/m (Ultrasonic absorption i
rho = 1079 ;                    % Liver density, kg/m^3
c_p = 3540 ;                    % heat capacity of liver
c = 1600 ;                      % speed of sound in liver, m/s
f = 1e6 ;                       % US frequency, Hz
TD_43 = 30*60;                  % thermal dose threshold, s
R = 0.25 ;
T1 = 37 ;                       % initial temperature, °C
PNP = 3e6 ;                     % Peak negative pressure, Pa
I0 = PNP^2/rho/c;               % acoustic intensitity peak, W/m^2
t = 3600;                       % signal duration, s

syms  DC

eqn = -rho*c_p/(alpha * I0 *DC) * 1/log(R)*R^(43 - T1 - alpha * I0 * DC/(rho*c_p)*t) == TD_43;

DC = double(solve(eqn, DC));
```

b) Define the parameters that shoud be used in order to achieve thermal ablation at $1\,\mathrm{MHz}$ without inducing cavitation. (5p)

In order to obtain tissue thermal ablation, the thermal dose has to be greater than $240\,\mathrm{CEM}_{43}$. The mechanical index instead has to be lower than 0.5 to avoid cavitation. The peak negative pressure should be:

$$\mathrm{PNP} = \sqrt{f} * \mathrm{MI} \; = 500\,\mathrm{kPa}$$

In this case $\mathrm{DC} > 4\%$.

```
alpha = 0.1e-2 ;                % Absorption coefficient in liver, 1/m (Ultrasonic absorption i
rho = 1079 ;                    % Liver density, kg/m^3
c_p = 3540 ;                    % heat capacity of liver
c = 1600 ;                      % speed of sound in liver, m/s
f = 1e6 ;                       % US frequency, Hz
TD_43 = 240*60;                 % thermal dose threshold, s
R = 0.5 ;
T1 = 37 ;                       % initial temperature, °C
PNP = 500e3 ;                   % Peak negative pressure, Pa
I0 = PNP^2/rho/c;               % acoustic intensitity peak, W/m^2
t = 3600;                       % signal duration, s

syms  DC

eqn = -rho*c_p/(alpha * I0 *DC) * 1/log(R)*R^(43 - T1 - alpha * I0 * DC/(rho*c_p)*t) == TD_43;
```

```
DC = double(solve(eqn, DC));
```