1. **(Challenging)**

a) Solve the pair correlation function P_{12} equation (23) starting from equation (20) using Fourier transformation.

b) Using the definition of $N_\alpha(\vec{x}, \vec{v}, t)$ (25) and the equations of motion (27) and (28), derive the Klimontovich-Dupree equation (33). Start by calculating $\frac{\partial N_\alpha}{\partial t}$.

2. **(Straightforward)**

a) Show that the Vlasov equation conserves particles, i.e. that the time derivative $\frac{\partial}{\partial t} \int nf d\vec{x} d\vec{v} = 0$.

b) Show that the Maxwell-Boltzmann distribution $f = e^{-W/2kT}$, where $W = \frac{1}{2}mv^2 + q\Phi$, is a solution to the Vlasov equation.

3. (**Cool-down**)
Prove the Jean's theorem, i.e., show that any function of integrals of the motion is a solution to the collisionless Boltzman equation.

Hints:
Consider some function, say H, to be a function of integrals of motion α_i (constants of motion), i.e., $H = H(\alpha_i)$. Then apply the collisionless Boltzmann equation to H. Note that the constants of motion are functions of time and phase-space coordinates $\boldsymbol{r}$ and $\boldsymbol{v}$.