SOLUTION 1

A) Piston velocities

NOTE: Unit handling is easier when calculating in SI-units (even though initial values and results are in many cases more describing in non SI-units such as l/min or bar)

Volume flow to SI-units (m^3/s)

$$q_{Vs11} = \frac{q_{Vs11}}{60 \cdot 1000} = 1.67 \cdot 10^{-4} \text{ m}^3/\text{s}$$

Piston area of cylinder 1 (actuating cylinder)

$$A_{s11} = \frac{\pi \cdot d_{s11}{}^2}{4} = 3.12 \cdot 10^{-3} \text{ m}^2$$

Piston velocity of cylinder 1

$$v_{s1} = \frac{q_{Vs11}}{A_{s11}} = 0.0535 \text{ m/s}$$

In order to calculate the exiting flow of cylinder 1, the ring surface area A_{s13} (rod side) must be calculated

$$A_{s13} = \frac{\pi \cdot d_{s11}{}^2}{4} - \frac{\pi \cdot d_{s12}{}^2}{4} = 2.41 \cdot 10^{-3} \text{ m}^2$$

The exiting flow from cylinder 1

$$q_{Vs13} = v_{s1} \cdot A_{s13} = 1.29 \cdot 10^{4} \text{ m}^3/\text{s}$$

Changing the unit to l/min

$$q_{Vs13} \cdot 60 \cdot 1000 = 7.73 \quad \text{l/min}$$

This is the volume flow entering the cylinder 2 (lift cylinder) generating its piston movement. In order to calculate the velocity, area A_{s21} must be calculated

$$A_{s21} = \frac{\pi \cdot d_{s21}{}^2}{4} = 1.26 \cdot 10^{-3} \text{ m}^2$$

The piston velocity of cylinder 2 is

$$v_{s2} = \frac{q_{Vs13}}{A_{s21}} = 0.1026 \text{ m/s}$$

B) Velocities equal?

In order for the piston velocities to be equal, the piston area A_{s21} of cylinder 2 must be the same as the ring surface area A_{s13} of cylinder 1.

C) Lift height

The total oil volume, exiting cylinder 1 during full stroke, is

$$V_{s13} = l_{s1} \cdot A_{s13} = 6.03 \cdot 10^{-4} \text{ m}^3$$

This volume induces in cylinder 2 a piston motion of

$$x_{s2} = \frac{V_{s13}}{A_{s21}} = 0.48 \text{ m}$$

D) Lift height, would the stroke of cylinder 1 be doubled

The total oil volume, exiting cylinder 1 during full stroke, would be

$$V_{s13} = 2 \cdot l_{s1} \cdot A_{s13} = 1.21 \cdot 10^{-3} \text{ m}^3$$

This volume would result in a piston motion of (in cylinder 2)

$$x_{s2} = \frac{V_{s13}}{A_{s21}} = 0.96 \text{ m}$$

This, however, exceeds the stroke of cylinder 2 (500 mm) and is hence is not achievable. The maximum lift height in this case is limited by stroke to 500 mm (cylinder 2 would stop to position of maximum extension, also stopping the actuating cylinder).

E) The pressure p_{s11} required for lifting the load

The load m exerts force F_{s2} in cylinder 2

$$F_{s2} = mg = 4.91 \cdot 10^4 \text{ N}$$

The pressure need p_{s21} in cylinder 2 to counter the force is

$$p_{s21} = \frac{F_{s2}}{A_{s21}} = 3.907 \cdot 10^7 \text{ Pa}$$

, which is in bars

$$p_{s21} \cdot 10^5 = 390.7 \text{ bar}$$

This pressure acting on the rod side of cylinder 1 creates force F_{s13} which acts to retract the cylinder (move piston to left, inwards).

$$F_{s13} = p_{s21} \cdot A_{s13} = 9.42 \cdot 10^4 \text{ N}$$

In order to extend the cylinder 1, a equal but opposing force is required (in an ideal system). This is achieved by creating a pressure p_{s11} on the piston side

$$p_{s11} = \frac{F_{s13}}{A_{s11}} = 3.021 \cdot 10^7 \text{ Pa}$$

Alternatively, the piston side pressure can be calculated directly utilizing known rod side pressure and area ratio (piston area vs. rod side annular surface area).

$$p_{s11} = \frac{A_{s13}}{A_{s11}} \cdot p_{s21} = 3.021 \cdot 10^7 \text{ Pa}$$

, which is in bars

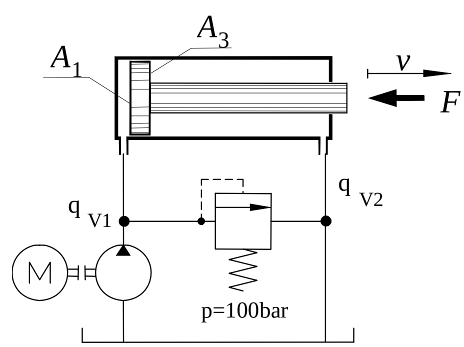
$$p_{s11} \cdot 10^{-5} = 302.1 \text{ bar}$$

SOLUTION 2

Case 1:

$$v = \frac{q_{V1}}{A_1} = \frac{300 \frac{\text{cm}^3}{\text{s}}}{100 \text{ cm}^2} = 3 \frac{\text{cm}}{\text{s}} = 0.03 \frac{\text{m}}{\text{s}}$$

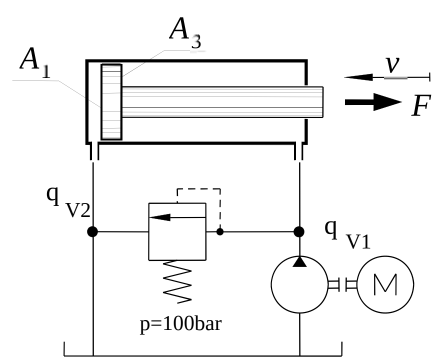
$$F = p \cdot A_1 = 100 \cdot 10^5 \frac{\text{N}}{\text{m}^2} \cdot 100 \cdot 10^{-4} \text{m}^2 = 100 \text{kN}$$



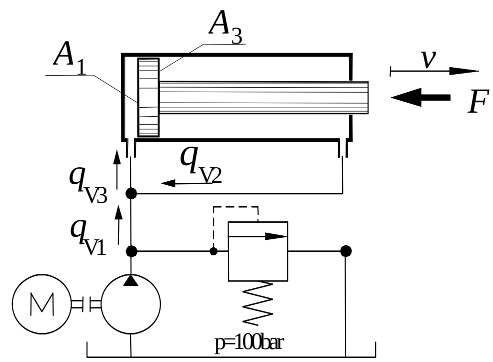
Case 2:

$$v = \frac{q_{V1}}{A_3} = \frac{300 \frac{\text{cm}^3}{\text{s}}}{50 \text{ cm}^2} = 6 \frac{\text{cm}}{\text{s}} = 0.06 \frac{\text{m}}{\text{s}}$$

$$F = p \cdot A_3 = 100 \cdot 10^5 \frac{\text{N}}{\text{m}^2} \cdot 50 \cdot 10^{-4} \text{m}^2 = 50 \text{kN}$$



Case 3:



$$q_{V3} = v \cdot A_1 \ , \quad q_{V2} = v \cdot A_3 \ , \quad q_{V3} = q_{V1} + q_{V2}$$

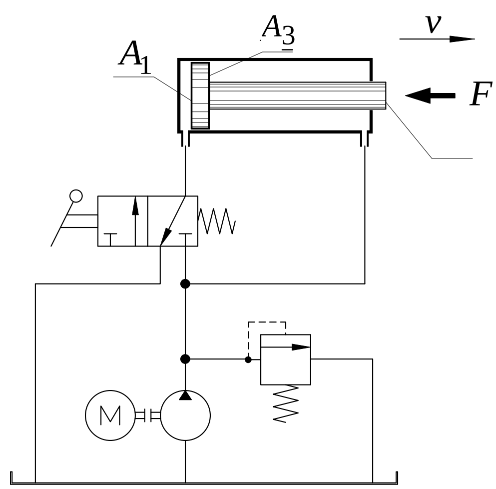
$$v \cdot A_1 = v \cdot A_3 + q_{V1} \quad \Rightarrow \quad v = \frac{q_{V1}}{(A_1 - A_3)}$$

$$v = \frac{300\,\frac{\text{cm}^3}{\text{s}}}{(100-50)\text{cm}^2} = 6\,\frac{\text{cm}}{\text{s}} = 0.06\,\frac{\text{m}}{\text{s}}$$

$$-F + p \cdot A_1 - p \cdot A_3 = 0$$

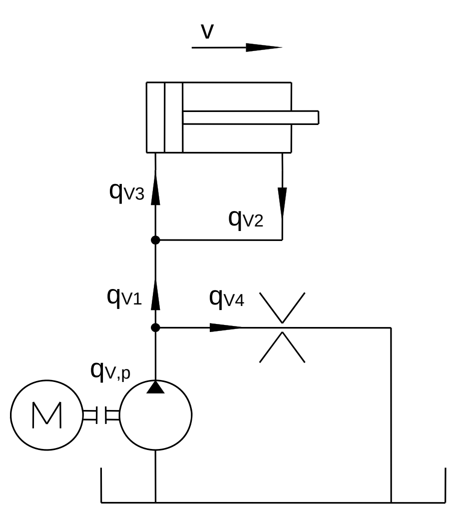
$$F = p(A_1 - A_3) = 100 \cdot 10^5\,\frac{\text{N}}{\text{m}^2} \cdot 50 \cdot 10^{-4}\,\text{m}^2 = 50\,\text{kN}$$

Selecting the piston areas to fulfill criteria $A_1 = 2 \cdot A_3$ will result in equal piston velocities in both directions when connections for outward stroke are as in case 3 and for inward stroke as in case 2. One possible implementation of such system adjacent.



SOLUTION 3

Applying equation of continuity,



$$q_{\text{V,p}} = q_{V1} + q_{V4}$$

$$q_{V3} = q_{V1} + q_{V2}$$

Piston velocity $v = 0.5\,\frac{\text{m}}{\text{s}}$

$$q_{V2} = (A_1 - A_2) \cdot v = (0.003 - 0.001)\text{m}^2 \cdot 0.5\,\frac{\text{m}}{\text{s}}$$

$$= 0.001\,\frac{\text{m}^3}{\text{s}}$$

$$q_{V3} = A_1 \cdot v = 0.003\,\text{m}^2 \cdot 0.5\,\frac{\text{m}}{\text{s}} = 0.0015\,\frac{\text{m}^3}{\text{s}}$$

$$q_{V1} = q_{V3} - q_{V2} = (0.0015 - 0.001)\frac{\text{m}^3}{\text{s}} = 0.0005\frac{\text{m}^3}{\text{s}} = 30\frac{\text{l}}{\text{min}}$$

$$q_{V4} = q_{\text{V,p}} - q_{V1} = (50 - 30)\frac{\text{l}}{\text{min}} = 20\frac{\text{l}}{\text{min}}$$

Case A: one restrictor

Calculating the pressure loss in the restrictor

$$q_{V4} = C_q \cdot A_\text{k} \cdot \sqrt{\frac{2 \cdot \Delta p}{\rho}} \quad \Rightarrow \Delta p = \left(\frac{q_{V4}}{C_q \cdot A_\text{k}}\right)^2 \cdot \frac{\rho}{2} = \left(\frac{\frac{20}{60000}\frac{\text{m}^3}{\text{s}}}{0.7 \cdot \pi \cdot \frac{(0.002\,\text{m})^2}{4}}\right)^2 \cdot \frac{860\frac{\text{kg}}{\text{m}^3}}{2}$$

$$= 98.79\,\text{bar}$$

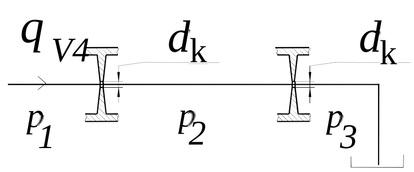
Forming force balance of piston and resolving the force

$$\Delta p \cdot A_1 = \Delta p \cdot (A_1 - A_2) + F \quad \Rightarrow \quad F = \Delta p \cdot A_2$$

$$\Rightarrow F = 98.79 \cdot 10^5 \cdot 0.001 = 9879\,\text{N} = 9.9\,\text{kN}$$

Case B: two restrictos in series

The same volume flow flows through both restrictors (q_{V4}).



The total pressure difference is the sum of pressure differences across restrictors.

$$\Delta p_{tot} = (p_1 - p_2) + (p_2 - p_3) \quad \Rightarrow \quad \Delta p_{kok} = p_1 - p_3$$

$$p_1 - p_2 = \left(\frac{q_{V4}}{C_q \cdot A_k}\right)^2 \cdot \frac{\rho}{2}$$

$$p_2 - p_3 = \left(\frac{q_{V4}}{C_q \cdot A_k}\right)^2 \cdot \frac{\rho}{2}$$

$$\Delta p_{tot} = \left(\frac{q_{V4}}{C_q \cdot A_k}\right)^2 \cdot \frac{\rho}{2} + \left(\frac{q_{V4}}{C_q \cdot A_k}\right)^2 \cdot \frac{\rho}{2} = \left(\frac{q_{V4}}{C_q}\right)^2 \cdot \frac{\rho}{2}\left(\frac{1}{A_k^2} + \frac{1}{A_k^2}\right)$$

$$= \left(\frac{q_{V4}}{C_q}\right)^2 \cdot \rho \cdot \frac{1}{A_k^2} = \left(\frac{\frac{20}{60000}\frac{m^3}{s}}{0.7}\right)^2 \cdot 860\frac{kg}{m^3} \cdot \frac{1}{\left(\pi \cdot \frac{(0.002m)^2}{4}\right)^2}$$

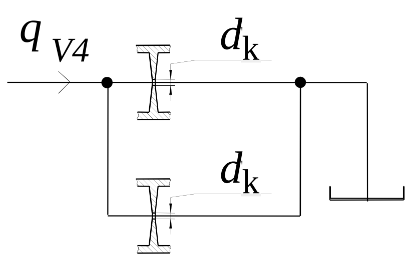
$$= 19.7MPa$$

Forming force balance of piston and resolving the force

$$F = \Delta p \cdot A_2 = 19.7 \cdot 10^6\, Pa \cdot 0.001 m^2 = 19.7kN$$

Case C: two paraller restrictors

There pressure difference is common across both restrictors. The volume flow q_{V4} is in this case the sum of flows through bot restrictors.



Solving the pressure difference.

$$q_{V4} = C_q \cdot A_k \cdot \sqrt{\frac{2 \cdot \Delta p}{\rho}} + C_q \cdot A_k \cdot \sqrt{\frac{2 \cdot \Delta p}{\rho}} = C_q \cdot 2 \cdot A_k \cdot \sqrt{\frac{2 \cdot \Delta p}{\rho}}$$

$$\Rightarrow \Delta p = \frac{q_{V4}{}^2 \cdot \rho}{2 \cdot C_q{}^2 \cdot (2 \cdot A_k)^2} = \frac{\left(\frac{20}{60000} \cdot \frac{m^3}{s}\right)^2 \cdot 860\frac{kg}{m^3}}{2 \cdot 0.7^2 \cdot \left(2 \cdot \pi \cdot \frac{(0.002m)^2}{4}\right)^2}$$

$$= 2.47MPa$$

Forming force balance of piston and resolving the force

$$F = \Delta p \cdot A_2 = 2.47 \cdot 10^6 Pa \cdot 0.001 m^2 = 2.47 kN$$