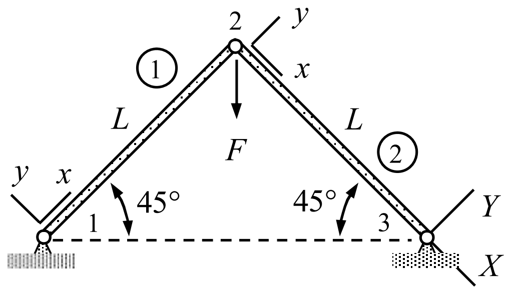
Name______________________________ Student number_____________________

Home assignment 1

Determine the displacement components u_{X2} and u_{Y2} of the bar structure of the figure. Use the principle of virtual work and the virtual work expressions for the linear bar element. Young's modulus of the material E and the cross-sectional area of the bars A are constants.



Solution template

Virtual work expression for the displacement analysis consists of parts coming from internal and external forces δW^{int} and δW^{ext}. For a bar and linear interpolation of the nodal values, the element contributions are

$$\delta W^{\text{int}} = -\begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^{\mathrm{T}} \frac{EA}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix}, \quad \delta W^{\text{ext}} = \begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^{\mathrm{T}} \frac{f_x h}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}.$$

The element contribution of the point force follows from the definition or work and is given by

$$\delta W^{\text{ext}} = \delta u_{X1} F_X + \delta u_{Y1} F_Y + \delta u_{Z1} F_Z .$$

According to the figure, the axial displacements of the nodes of bar 1 are given by $u_{x1} = 0$ and $u_{x2} = u_{Y2}$

$$\delta W^1 = -\begin{Bmatrix} 0 \\ \delta u_{Y2} \end{Bmatrix}^{\mathrm{T}} \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ u_{Y2} \end{Bmatrix} = -\delta u_{Y2} \frac{EA}{L} u_{Y2} .$$

The axial displacements of the nodes of bar 2 are given by $u_{x2} = u_{X2}$ and $u_{x3} = 0$

$$\delta W^2 = -\begin{Bmatrix} \delta u_{X2} \\ 0 \end{Bmatrix}^{\mathrm{T}} \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_{X2} \\ 0 \end{Bmatrix} = -\delta u_{X2} \frac{EA}{L} u_{X2} .$$

The point force is considered as element 3. By dividing the force into component in X- and Y-directions

$$\delta W^3 = \delta u_{X2} (\frac{F}{\sqrt{2}}) + \delta u_{Y2} (-\frac{F}{\sqrt{2}}) = \begin{Bmatrix} \delta u_{X2} \\ \delta u_{Y2} \end{Bmatrix}^{\mathrm{T}} \begin{Bmatrix} F/\sqrt{2} \\ -F/\sqrt{2} \end{Bmatrix}.$$

Virtual work expression of the structure is the sum of element contributions. When written in the standard form

$$\delta W^1 + \delta W^2 + \delta W^3 = -\begin{Bmatrix} \delta u_{X2} \\ \delta u_{Y2} \end{Bmatrix}^{\mathrm{T}} \left(\begin{bmatrix} EA/L & 0 \\ 0 & EA/L \end{bmatrix} \begin{Bmatrix} u_{X2} \\ u_{Y2} \end{Bmatrix} - \begin{Bmatrix} F/\sqrt{2} \\ -F/\sqrt{2} \end{Bmatrix} \right).$$

Principle of virtual work and fundamental lemma of variation calculus imply the system of equations

$$\begin{bmatrix} EA/L & 0 \\ 0 & EA/L \end{bmatrix} \begin{Bmatrix} u_{X2} \\ u_{Y2} \end{Bmatrix} - \begin{Bmatrix} F/\sqrt{2} \\ -F/\sqrt{2} \end{Bmatrix} = 0 .$$

Solution to the displacement components

$$u_{X2} = \frac{1}{\sqrt{2}} \frac{FL}{EA} \quad \text{and} \quad u_{Y2} = -\frac{1}{\sqrt{2}} \frac{FL}{EA} .$$