Exercise 10

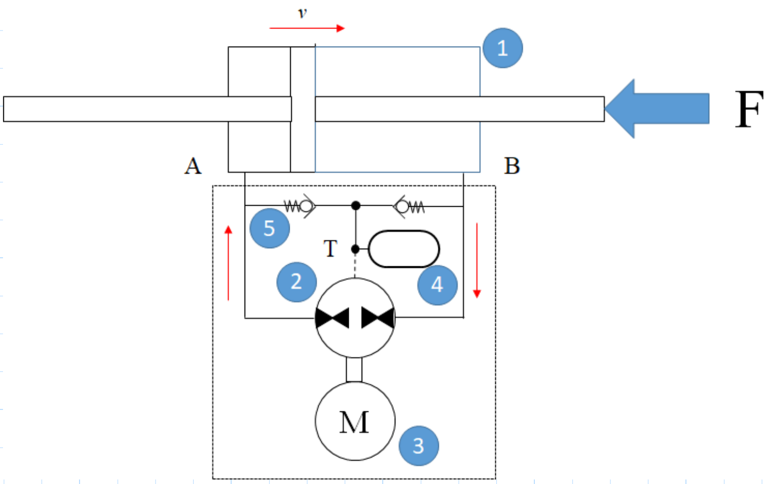


Figure 1 DDH cylinder system

Inputs

V_p	pump's swept volume [m³/r]
F	load force [N]
v	piston velocity [m/s]
D	piston diameter [m]
d	piston rod diameter [m]
R	laminar resistance of leakage throttles [Pa/(m³/s)]
Δp_{crack}	cracking (open) pressure difference of check valve [Pa]
K	laminar resistance of check valves [Pa/(m³/s)]
p_{ACCU}	accumulator's pressure [gauge pressure - Pa]

$V_p := 21 \cdot 10^{-6} \cdot m^3$

$F := 30000 \cdot N$ load force is group dependent

$v := 0.1 \cdot \frac{m}{s}$

$D := 0.1 \cdot m$

$d := 0.060 \cdot m$

$A := \frac{\pi}{4} \left(D^2 - d^2\right)$

$A = 0.00503\ m^2$

Pump

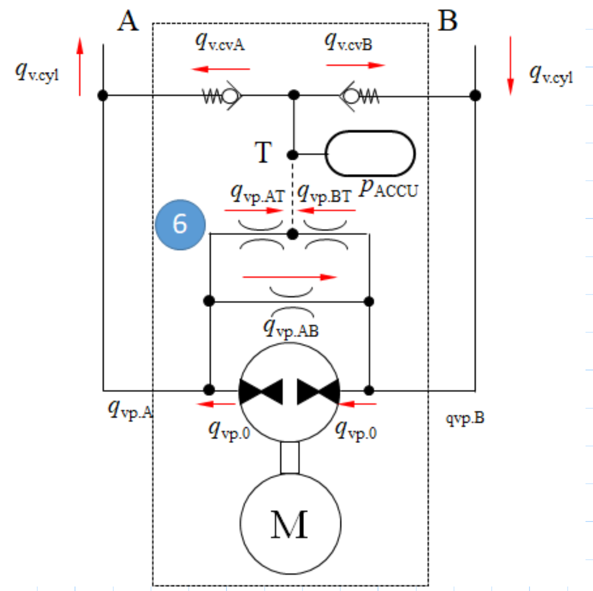


Figure 2 DDH pump system

$R := \frac{60000 \cdot 50 \cdot 10^5}{0.5} \cdot \frac{Pa \cdot s}{m^3}$ (laminar) leakage flor resistance

$R = (6 \cdot 10^{11}) \frac{Pa \cdot s}{m^3}$

$K := \frac{5 \cdot 10^5}{0.001} \cdot \frac{Pa \cdot s}{m^3}$

$\Delta p_{cv} = p_{crack} + K \cdot q_{v.cv}$

$K = (5 \cdot 10^8) \frac{kg}{m^4 \cdot s}$

$p_{crack} := 0.2 \cdot 10^5 \cdot Pa$

check valve's laminar resistance and
cracking pressure (difference)

$p_{ACCU} := 0.5 \cdot 10^5 \cdot Pa$

accumulator's constant pressure (gauge), gas pressure should be calculated by using absolute pressure values (not needed in this case)

$\Delta p := \frac{F}{A}$ cylinder's pressure difference

$\Delta p = 59.683 \ bar$

$q_{v.cv} = \frac{\Delta p_{cv} - p_{crack}}{K}$

check valve's flow rate, if pressure difference is higher than cracking pressure, otherwise flow rate is 0

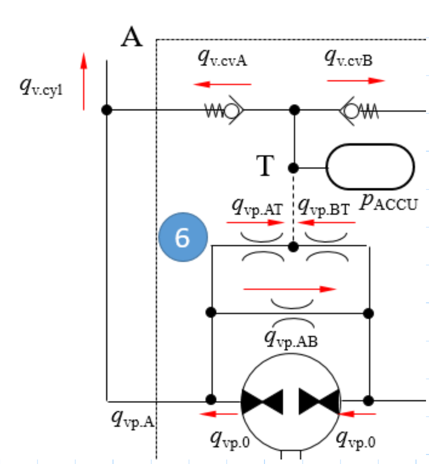
System equations

All the flow rate equations for valves must be formulated.
Accumulator pressure gives the pressure reference for the system and defines all of the absolute pressure levels in the system.

Check valve B flow (T->B)

$$q_{v.cvB} = \frac{p_T - p_B - p_{crack}}{K}$$

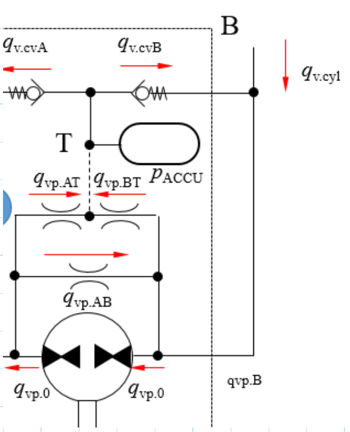
flow rate through check valve B from tank (accumulator) to B chamber pipe, pressure difference over check valve is $p_T - p_B$



A chamber flow

$$v \cdot A = q_{vp.0} - q_{vp.AB} - q_{vp.AT}$$

vA represents cylinder flow which is "ideal pump" flow - internal leakage - case drain to accumulator



B chamber flow

$$v \cdot A + q_{v.cvB} + q_{vp.AB} - q_{vp.BT} = q_{vp.0}$$

$$v \cdot A = q_{vp.0} - q_{v.cvB} - q_{vp.AB} + q_{vp.BT}$$

vA represents (again) cylinder flow "ideal pump" flow is flow from cylinder + check valve (TB) flow + internal leakage - case drain from B to accumulator
it is probable that pB < pT, so case drain will be negative in this case

Subtract cylinder flow equations

Now some of the flows (flow equations) are common for the A and B cylinder chamber equations. You can get rid of them by subtracting equations.

$$0 = -q_{vp.AT} - q_{vp.BT} + q_{v.cvB}$$

The remaining flow rates are external drain (AT), external drain (BT) and check valve (TB).

$$q_{v.cvB} = q_{vp.AT} + q_{vp.BT}$$

These flow rates are all unknown so we have to utilize pressure information. We know accumulator pressure and pressure difference between cylinder chamber pressures A and B.

We substitute flow rates equations with the corresponding valve equations based on pressure information.

$$\frac{p_{ACCU} - p_B - p_{crack}}{K} = \frac{p_A - p_{ACCU}}{R} + \frac{p_B - p_{ACCU}}{R}$$

There are still too many unknown pressures (2) but we know the relation between cylinder chamber pressures.

$$(p_A - p_B) \cdot A = F \qquad p_A = \frac{F}{A} + p_B$$

$$\frac{p_{ACCU} - p_B - p_{crack}}{K} = \frac{\frac{F}{A} + p_B - p_{ACCU}}{R} + \frac{p_B - p_{ACCU}}{R}$$

Now we have only one unknown (pressure) and we can solve it and also all the other variables.

$$p_B \cdot \left(\frac{2}{R} + \frac{1}{K}\right) = \frac{p_{ACCU} - p_{crack}}{K} - \frac{F}{A \cdot R} + \frac{2 \cdot p_{ACCU}}{R}$$

$$p_B = \frac{\frac{p_{ACCU} - p_{crack}}{K} - \frac{F}{A \cdot R} + \frac{2 \cdot p_{ACCU}}{R}}{\frac{2}{R} + \frac{1}{K}}$$

$$p_B := \frac{p_{ACCU} \cdot \left(\frac{1}{K} + \frac{2}{R}\right) - \frac{p_{crack}}{K} - \frac{F}{A \cdot R}}{\frac{2}{R} + \frac{1}{K}}$$

$p_B = 0.251$ **bar**

$p_A := \frac{F}{A} + p_B$

$p_B = 0.251\ \text{bar}$

$p_A = 59.934\ \text{bar}$

$q_{v.cyl} := v \cdot A$

$q_{v.cyl} = 30.159\ \frac{l}{min}$

"Ideal pump" flow rate from B side flow equation

$v \cdot A = q_{vp.0} - q_{v.cvB} - q_{vp.AB} + q_{vp.BT}$

$q_{vp.0} = v \cdot A + q_{v.cvB} + q_{vp.AB} - q_{vp.BT}$

$q_{vp.0} := v \cdot A + \frac{p_{ACCU} - p_B - p_{crack}}{K} + \frac{\Delta p}{R} - \frac{p_B - p_{ACCU}}{R}$

$q_{vp.0} = 31.35\ \frac{l}{min}$

$q_{vp.0} = \left(5.225 \cdot 10^{-4}\right) \frac{m^3}{s}$

"Ideal pump" flow rate from A side flow equation

$q_{vp.0.A} := v \cdot A + \frac{\Delta p}{R} + \frac{p_A - p_{ACCU}}{R}$

$q_{vp.0.A} = 31.35\ \frac{l}{min}$

Pump's rotational speed

$n := \frac{q_{vp.0}}{V_p}$

$n = \left(1.493 \cdot 10^3\right) \frac{1}{min}$

$n = 24.881\ \frac{1}{s}$

Pump's internal leakage

$q_{vp.AB} := \frac{\Delta p}{R}$

$q_{vp.AB} = 0.59683\ \frac{L}{min}$

$q_{vp.AB} = \left(9.947 \cdot 10^{-6}\right) \frac{m^3}{s}$

Pump's case drain from A to tank (accumulator)

$q_{vp.AT} := \frac{p_A - p_{ACCU}}{R}$

$q_{vp.AT} = 0.59434\ \frac{L}{min}$

$q_{vp.AT} = \left(9.906 \cdot 10^{-6}\right) \frac{m^3}{s}$

$$q_{vp.BT} := \frac{p_B - p_{ACCU}}{R}$$

$$q_{vp.BT} = -0.00249 \ \frac{L}{min}$$

$$q_{vp.BT} = -4.155 \cdot 10^{-8} \ \frac{m^3}{s}$$

Note: leakage flow rate is from accumulator to B side (negative sign)

Check valve (TB) flow rate

$$q_{v.cvB} := \frac{p_{ACCU} - p_B - p_{crack}}{K}$$

$$q_{v.cvB} = 0.59184 \ \frac{L}{min}$$

$$q_{v.cvB} = \left(9.864 \cdot 10^{-6}\right) \frac{m^3}{s}$$

Pump-motor torque

$$T := \frac{\Delta p \cdot V_p}{2 \cdot \pi}$$

$$T = 19.948 \ N \cdot m$$

$$P_{out} := F \cdot v$$

$$P_{out} = 3 \ kW$$

$$P_{in} := T \cdot 2 \cdot \pi \cdot n$$

$$P_{in} = 3.118 \ kW$$

Power losses

AB

$$P_{AB} := q_{vp.AB} \cdot \left(p_A - p_B\right)$$

$$P_{AB} = 59.368 \ W$$

BT

$$P_{BT} := q_{vp.BT} \cdot \left(p_B - p_{ACCU}\right)$$

$$P_{BT} = 0.001 \ W$$

AT

$$P_{AT} := q_{vp.AT} \cdot \left(p_A - p_{ACCU}\right)$$

$$P_{AT} = 58.873 \ W$$

Check Valve

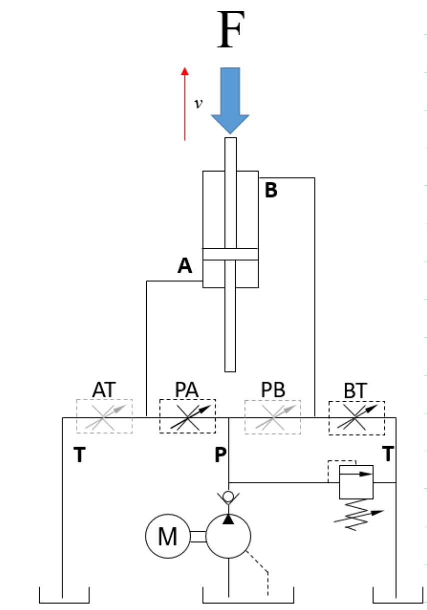
$P_{CV} := q_{v.cvB} \cdot (p_{ACCU} - p_B)$

$P_{CV} = 0.246\ W$

$P_{AB} + P_{BT} + P_{AT} + P_{CV} = 118.488\ W$

$P_{in} - P_{out} = 118.488\ W$

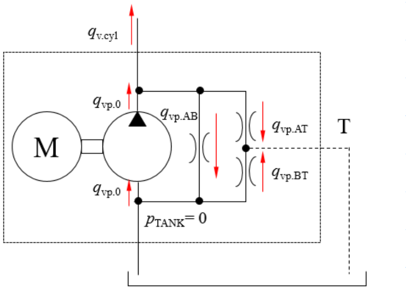
Proportional Control Valve operated system



Pressure differerence in (symmetric) cylinder

$\Delta p := \frac{F}{A}$

$\Delta p = 59.683\ bar$



$q_{v.nom} := \frac{40}{60000} \cdot \frac{m^3}{s}$

$\Delta p_{nom} := 35 \cdot 10^5 \cdot Pa$

$U := 10 \cdot V$

$q_v = K_{valve} \cdot \frac{U}{10 \cdot V} \cdot \sqrt[2]{\Delta p}$

Proportional control valve capacity parameter

$K_{valve} := \frac{q_{v.nom}}{\sqrt[2]{\Delta p_{nom}}}$

$p_T := 0 \cdot bar$

$\Delta p = 59.683\ bar$

$$q_{vp.AB} := \frac{p_A - p_T}{R}$$

$$q_{vp.AB} = \left(9.989 \cdot 10^{-6}\right) \frac{m^3}{s}$$

$$q_{vp.AT} := \frac{p_A - p_T}{R}$$

$$q_{vp.AT} = \left(9.989 \cdot 10^{-6}\right) \frac{m^3}{s}$$

$$q_{vp.BT} := \frac{p_T - p_T}{R}$$

$$q_{vp.BT} = 0 \frac{m^3}{s}$$

Cylinder flow rate(s)

$$q_{A.propo} := v \cdot A$$

$$q_{B.propo} := v \cdot A$$

Pressure losses in proportional control valve's control edges

$$\Delta p_{propo} := \frac{q_{B.propo}^{\ 2}}{K_{valve}^{\ 2}} \cdot \frac{10^2 \cdot V^2}{U^2}$$

$$\Delta p_{propo} = 19.897 \text{ bar}$$

Cylinder's B chamber pressure is the pressure loss in control edge BT (assuming tak pressure 0)

$$p_{B.propo} := \Delta p_{propo}$$

$$p_{B.propo} = 19.897 \text{ bar}$$

Cylinder's A chamber pressure

$$p_{A.propo} := \frac{F + p_{B.propo} \cdot A}{A}$$

$$p_{A.propo} = 79.58 \text{ bar}$$

PUMP pressure

$$p_{p.propo} := p_{A.propo} + \Delta p_{propo}$$

$$p_{p.propo} = 99.477 \text{ bar}$$

Orifice AT

$$q_{v.AT.propo} := \frac{p_{p.propo} - p_T}{R}$$

$$q_{v.AT.propo} = 0.995 \frac{l}{min}$$

$$q_{v.AT.propo} = \left(1.658 \cdot 10^{-5}\right) \frac{m^3}{s}$$

Orifice AB

$$q_{v.AB.propo} := \frac{p_{p.propo} - p_T}{R}$$

$$q_{v.AB.propo} = 0.995 \, \frac{l}{min}$$

$$q_{v.AB.propo} = \left(1.658 \cdot 10^{-5}\right) \frac{m^3}{s}$$

$$q_{v.BT.propo} := \frac{p_T - p_T}{R}$$

$$q_{v.BT.propo} = 0 \, \frac{m^3}{s}$$

"Ideal pump" flow rate is divided into cylinder flow rate and leakage flow rates (case drain and internal leakage)

$$q_{v0.propo} := v \cdot A + q_{v.AT.propo} + q_{v.AB.propo}$$

$$q_{v0.propo} = 32.149 \, \frac{l}{min}$$

$$q_{v0.propo} = \left(5.358 \cdot 10^{-4}\right) \frac{m^3}{s}$$

Volumetric efficiency

$$\eta_v := \frac{v \cdot A}{v \cdot A + q_{v.AT.propo} + q_{v.AB.propo}}$$

$$\eta_v = 0.938$$

Pump's rotational speed depends on "ideal pump" flow rate

$$n_{p.propo} := \frac{q_{v0.propo}}{V_p}$$

$$n_{p.propo} = 25.515 \, \frac{1}{s}$$

$$n_{p.propo} = \left(1.531 \cdot 10^{3}\right) \frac{1}{min}$$

or, by using net flow rate (actuator flow rate) AND volumetric efficiency.
this is the "normal way".

$$n_{pump} := \frac{v \cdot A}{\eta_v \cdot V_p}$$

$$n_{pump} = 25.515 \, \frac{1}{s}$$

Pump's torque (hydromechanical efficiency is 1, no friction)

$$T_{p.propo} := \frac{p_{p.propo} \cdot V_p}{2 \cdot \pi}$$

$$T_{p.propo} = 33.248 \, J$$

Pump's input power (with hydraulic variables)

$$P_{in.propo} := q_{v0.propo} \cdot p_{p.propo}$$

$$P_{in.propo} = 5.33\ kW$$

Pump's input power (with mechanical variables)

$$P_{in.mech} := 2 \cdot \pi \cdot n_{pump} \cdot T_{p.propo}$$

$$P_{in.mech} = 5.33\ kW$$

Actuator's (cylinder) output power

$$P_{out.propo} := F \cdot v$$

$$P_{out.propo} = 3\ kW$$

Power loss in control edge PA

$$P_{PA} := \Delta p_{propo} \cdot v \cdot A$$

$$P_{PA} = 1.00014\ kW$$

Power loss in control edge BT (symmetric cylinder)

$$P_{BT} := \Delta p_{propo} \cdot v \cdot A$$

$$P_{BT} = 1.00014\ kW$$

Power loss in pump's orifice AB (internal leakage)

$$P_{ORI.AB} := q_{v.AB.propo} \cdot p_{p.propo}$$

$$P_{ORI.AB} = 164.929\ W$$

Power loss in pump's orifice AT (case drain)

$$P_{ORI.AT} := q_{v.AT.propo} \cdot p_{p.propo}$$

$$P_{ORI.AT} = 164.929\ W$$

Power loss in pump's orifice BT (case drain)

$$P_{ORI.BT} := q_{v.BT.propo} \cdot p_{p.propo}$$

$$P_{ORI.BT} = 0\ W$$

Power usage together

$$P_{out.propo} + P_{PA} + P_{BT} + P_{ORI.AB} + P_{ORI.AT} + P_{ORI.BT} = 5.33\ kW$$

Pump's input power (for comparison)

$$P_{in.propo} = 5.33\ kW$$

System's mechanical efficiency

$$\frac{P_{out.propo}}{P_{in.propo}} = 0.563$$

The major power losses take place in proportional control valve's orifices (control edges). Typically there is also some leakage in orifices PB and AT.