LECTURE ASSIGNMENT 1. Determine the eigenvalues λ_1, λ_2 and the corresponding eigenvectors $\mathbf{x}_1$, $\mathbf{x}_2$ of the 2×2 matrix $\mathbf{A}$. Write down also the eigenvalue decomposition $\mathbf{A} = \mathbf{X}\boldsymbol{\lambda}\mathbf{X}^{-1}$.

$$\mathbf{A} = \begin{bmatrix} 3 & 0 \\ -2 & 1 \end{bmatrix}$$

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- Eigenvalues given by the characteristic equation $\det(\mathbf{A}-\lambda\mathbf{I})=0$

$$\det\begin{bmatrix} 3-\lambda & 0 \\ -2 & 1-\lambda \end{bmatrix} = (3-\lambda)(1-\lambda)=0 \;\Rightarrow\; \lambda_1 = 3 \quad \text{or} \quad \lambda_2 = 1$$

- Eigenvectors given by equations $(\mathbf{A}-\lambda\mathbf{I})\mathbf{x}=0$

$$\lambda_1 : \begin{bmatrix} 3-3 & 0 \\ -2 & 1-3 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = 0 \;\Rightarrow\; \mathbf{x}_1 = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$$

$$\lambda_2 : \begin{bmatrix} 3-1 & 0 \\ -2 & 1-1 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = 0 \;\Rightarrow\; \mathbf{x}_2 = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

- Matrix of eigenvalues $\boldsymbol{\lambda}$, matrix of eigenvectors $\mathbf{X}$ and its inverse $\mathbf{X}^{-1}$

$$\boldsymbol{\lambda} = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix} \text{ and } \mathbf{X}^{-1} = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}$$

- Eigenvalue decomposition $\mathbf{A} = \mathbf{X}\boldsymbol{\lambda}\mathbf{X}^{-1}$

$$\mathbf{A} = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ -2 & 1 \end{bmatrix}.$$ ←