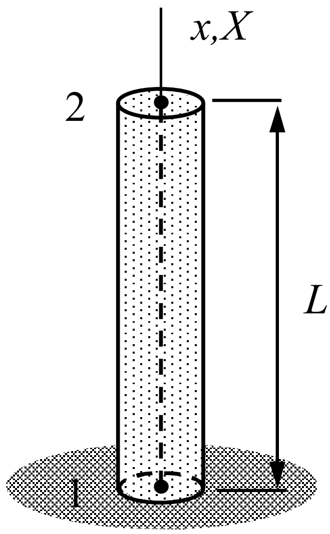
LECTURE ASSIGNMENT 2. Virtual work expression of internal and inertia forces of the bar model is given by

$$\delta W = -\begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^{\mathrm{T}} \left(\frac{EA}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix} + \frac{\rho A h}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_{x1} \\ \ddot{u}_{x2} \end{Bmatrix} \right)$$

in which A is the cross-sectional area, E is the Young's modulus, and ρ is the density of the material. Assuming that node 1 is fixed, derive the expression of the axial displacement $u_{X2}(t)$ at the free end for $t > 0$. The initial conditions at $t = 0$ are $u_{X2}(0) = U$ and $\dot{u}_{X2}(0) = 0$.



Name ______________________ Student number ________________

- In terms of the displacement components of the structural system

$$\delta W = -\begin{Bmatrix} 0 \\ \delta u_{X2} \end{Bmatrix}^{\mathrm{T}} \left(\frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} 0 \\ u_{X2} \end{Bmatrix} + \frac{\rho AL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} 0 \\ \ddot{u}_{X2} \end{Bmatrix}\right).$$

- Initial value problem, consisting of an ordinary differential equation (implied by the virtual work expression) and initial conditions, is given by

$$\frac{EA}{L}u_{X2} + \frac{\rho AL}{3}\ddot{u}_{X2} = 0 \qquad t > 0,$$

$$u_{X2}(0) = U \quad \text{and} \quad \dot{u}_{X2}(0) = 0 \qquad t = 0.$$

- Expression $u_{X2}(t) = A\cos(\omega t)$, which describes a harmonic periodic motion, satisfies all equations of the initial value problem if

$$\omega = \sqrt{3\frac{E}{\rho L^2}} \quad \text{and} \quad A = U.$$