GIS-E3010 Least-Squares Methods in Geoscience

Lecture 5 activation 1

Make linearization to collinearity equations with respect to the parameter omega. In order to make linearization easier, we introduce new parameters *U*, *V* and *W.*

Collinearity equations:

$$\begin{cases} x = -c\dfrac{r_{11}(X-X_0)+r_{12}(Y-Y_0)+r_{13}(Z-Z_0)}{r_{31}(X-X_0)+r_{32}(Y-Y_0)+r_{33}(Z-Z_0)} = f_x \\ y = -c\dfrac{r_{21}(X-X_0)+r_{22}(Y-Y_0)+r_{23}(Z-Z_0)}{r_{31}(X-X_0)+r_{32}(Y-Y_0)+r_{33}(Z-Z_0)} = f_y \end{cases}$$

Inner part of the Collinearity equations using parameters *U*, *V* and *W*

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = R \begin{bmatrix} X-X_0 \\ Y-Y_0 \\ Z-Z_0 \end{bmatrix} \qquad x = f_x = -c\frac{U}{W} \quad \text{ja} \quad y = f_y = -c\frac{V}{W}$$

Use rule of partial derivation of a quotient equation (p=any parameter with respect to you make partial derivation; in this case=omega) and a pre-given solution, how to calculate $\dfrac{\partial R}{\partial \omega}$

$$\begin{cases} \dfrac{\partial f_x}{\partial p} = \dfrac{-c}{W^2}\left(\dfrac{\partial U}{\partial p}W - U\dfrac{\partial W}{\partial p}\right) = \dfrac{-c}{W}\left(\dfrac{\partial U}{\partial p} - \dfrac{U}{W}\dfrac{\partial W}{\partial p}\right) \\ \dfrac{\partial f_y}{\partial p} = \dfrac{-c}{W^2}\left(\dfrac{\partial V}{\partial p}W - V\dfrac{\partial W}{\partial p}\right) = \dfrac{-c}{W}\left(\dfrac{\partial V}{\partial p} - \dfrac{V}{W}\dfrac{\partial W}{\partial p}\right) \end{cases}$$

$$\frac{\partial R}{\partial \omega} = R\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

As the result, we get two first row elements of the first column of design matrix A $\dfrac{\partial f_x}{\partial \omega}\ and\ \dfrac{\partial f_y}{\partial \omega}$

(One point observation establishes two rows and the total amount of rows is, therefore, 2*number of observations)

Solution:

At first, we make partial derivation of functions U, V and W, because we need this information when applying the *rule of partial derivation of a quotient equation*

$$\frac{\partial}{\partial\omega}\begin{bmatrix}U\\V\\W\end{bmatrix}=\begin{bmatrix}\partial U/\partial\omega\\\partial V/\partial\omega\\\partial W/\partial\omega\end{bmatrix}=\frac{\partial R}{\partial\omega}\begin{bmatrix}X-X_0\\Y-Y_0\\Z-Z_0\end{bmatrix}$$

$$=R\begin{bmatrix}0&0&0\\0&0&1\\0&-1&0\end{bmatrix}\begin{bmatrix}X-X_0\\Y-Y_0\\Z-Z_0\end{bmatrix}$$

We use the pre-known partial derivation of R with respect to omega

$$\frac{\partial R}{\partial\omega}=R\begin{bmatrix}0&0&0\\0&0&1\\0&-1&0\end{bmatrix}$$

$$=R\begin{bmatrix}0\\Z-Z_0\\-(Y-Y_0)\end{bmatrix}=\begin{bmatrix}r_{11}&r_{12}&r_{13}\\r_{21}&r_{22}&r_{23}\\r_{31}&r_{32}&r_{33}\end{bmatrix}\begin{bmatrix}0\\Z-Z_0\\-(Y-Y_0)\end{bmatrix}=\begin{bmatrix}r_{12}(Z-Z_0)-r_{13}(Y-Y_0)\\r_{22}(Z-Z_0)-r_{23}(Y-Y_0)\\r_{32}(Z-Z_0)-r_{33}(Y-Y_0)\end{bmatrix}$$

We place the result to equations

$$\begin{cases}\dfrac{\partial f_x}{\partial\omega}=\dfrac{-c}{W^2}\left(\dfrac{\partial U}{\partial\omega}W-U\dfrac{\partial W}{\partial\omega}\right)=\dfrac{-c}{W}\left(\dfrac{\partial U}{\partial\omega}-\dfrac{U}{W}\dfrac{\partial W}{\partial\omega}\right)\\[2ex]\dfrac{\partial f_y}{\partial\omega}=\dfrac{-c}{W^2}\left(\dfrac{\partial V}{\partial\omega}W-V\dfrac{\partial W}{\partial\omega}\right)=\dfrac{-c}{W}\left(\dfrac{\partial V}{\partial\omega}-\dfrac{V}{W}\dfrac{\partial W}{\partial\omega}\right)\end{cases}$$

The result is (Notice that we should also write out temporary parameters U, V and W)

$$\begin{cases}\dfrac{\partial f_x}{\partial\omega}=\dfrac{-c}{W}\left(\dfrac{\partial U}{\partial\omega}-\dfrac{U}{W}\dfrac{\partial W}{\partial\omega}\right)=\dfrac{-c}{W}\left[\{r_{12}(Z-Z_0)-r_{13}(Y-Y_0)\}-\dfrac{U}{W}\{r_{32}(Z-Z_0)-r_{33}(Y-Y_0)\}\right]\\[2ex]\dfrac{\partial f_y}{\partial\omega}=\dfrac{-c}{W}\left(\dfrac{\partial V}{\partial\omega}-\dfrac{V}{W}\dfrac{\partial W}{\partial\omega}\right)=\dfrac{-c}{W}\left[\{r_{22}(Z-Z_0)-r_{23}(Y-Y_0)\}-\dfrac{V}{W}\{r_{32}(Z-Z_0)-r_{33}(Y-Y_0)\}\right]\end{cases}$$