GIS-E3010 Least-Squares Methods in Geoscience

Lecture 5, activation 3

Relative orientation

The coplanarity equation is $G = \begin{vmatrix} b_X & b_Y & b_Z \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix} = b_X(Y_1Z_2 - Y_2Z_1) - b_Y(X_1Z_2 - X_2Z_1) + b_Z(X_1Y_2 - X_2Y_1) = 0$

And after linearization it looks like

$$\begin{bmatrix} \frac{\partial G}{\partial x_1} & \frac{\partial G}{\partial y_1} & \frac{\partial G}{\partial x_2} & \frac{\partial G}{\partial y_2} \end{bmatrix} \begin{bmatrix} dx_1 \\ dy_1 \\ dx_2 \\ dy_2 \end{bmatrix} + \begin{bmatrix} \frac{\partial G}{\partial b_Y} & \frac{\partial G}{\partial b_Z} & \frac{\partial G}{\partial \omega_2} & \frac{\partial G}{\partial \phi_2} & \frac{\partial G}{\partial \kappa_2} \end{bmatrix} \begin{bmatrix} db_Y \\ db_Z \\ d\omega_2 \\ d\phi_2 \\ d\kappa_2 \end{bmatrix} + G^0 = 0$$

$$C^T dl + Ddp + G^0 = 0$$

dp is the solution vector including the corrections of unknown relative orientation parameters (the size of this vector doesn't change). G^0 is the value of the coplanarity equation with current approximate parameters of relative orientation (e.g. initial values).
What is the interpretation of dx_1, dy_1, dx_2 and dy_2?

Let's assume that we have two tie points (corresponding points) measured between images (this is not enough to make solution). However, write (symbolically) the contents of matrices C^T and D and vector dl in such case. Add a sub-index to x, y and G indicating the number of corresponding observation), e.g. for the first corresponding point, for *dl* could be (number 1 added as an index)

$$\begin{bmatrix} dx_{11} \\ dy_{11} \\ dx_{21} \\ dy_{21} \end{bmatrix}$$

Solution:

The interpretation is that this vector contains corrections to image observations. x_1 and y_1 refer to image observations from (e.g.) the left image and x_2 and y_2 to image observations from the right image.

$$C^T = \begin{bmatrix} \frac{\partial G_1}{\partial x_{11}} & \frac{\partial G_1}{\partial y_{11}} & \frac{\partial G_1}{\partial x_{21}} & \frac{\partial G_1}{\partial y_{21}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial G_2}{\partial x_{12}} & \frac{\partial G_2}{\partial y_{12}} & \frac{\partial G_2}{\partial x_{22}} & \frac{\partial G_2}{\partial y_{22}} \end{bmatrix}$$

$$dl = \begin{bmatrix} dx_{11} \\ dy_{11} \\ dx_{21} \\ dy_{21} \\ dx_{12} \\ dy_{12} \\ dx_{22} \\ dy_{22} \end{bmatrix}$$

$$D = \begin{bmatrix} \frac{\partial G_1}{\partial b_Y} & \frac{\partial G_1}{\partial b_Z} & \frac{\partial G_1}{\partial \omega_2} & \frac{\partial G_1}{\partial \varphi_2} & \frac{\partial G_1}{\partial \kappa_2} \\ \frac{\partial G_2}{\partial b_Y} & \frac{\partial G_2}{\partial b_Z} & \frac{\partial G_2}{\partial \omega_2} & \frac{\partial G_2}{\partial \varphi_2} & \frac{\partial G_2}{\partial \kappa_2} \end{bmatrix}$$