Theory exercise 7.1

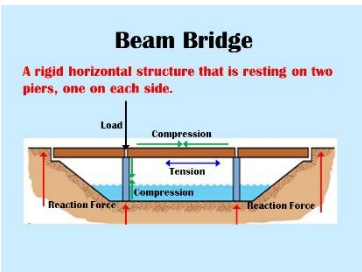
(i) Spend one to two hours for carefully reading lecture slides, sections 7.1, 7.2 and 7.3 (not necessarily the extra material), and making notes.

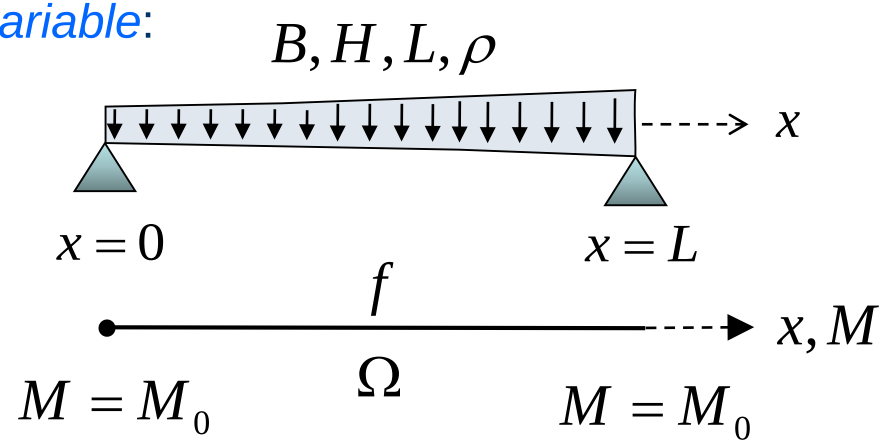
(ii) List three main points from each section by a few sentences and/or formulae.

(iii) List the mathematical notations not familiar to you (for recalling them from your earlier study material or Wikipedia, Wolfram MathWorld, …).

Theory exercise 7.2

Let us consider a vertically gravity-loaded (statically determined) *beam* with *bending moment* M (not *deflection*) as the *primary variable*:





Formulate the *weak form* of the problem (serving as a basis for FE formulations).

Hint: First, find out the strong form, i.e., the differential equation and boundary conditions, of the problem in terms of bending moment by recalling your previous studies (or Wikipedia).

Since the strong form of the problem is analogous to the one of the bar (or the heat diffusion) problem, you can simply imitate the weak form of the bar problem (or the heat diffusion problem).

Computer exercise 7.1

(i) In the program below, change the number of points in *x* to 20 and 200 and plot the result in the same figure as the original one. What do you learn from this?

(ii) Raise the noise factor from 0.3 to 1 (with 200 points) and plot the result in the same figure as the original one.

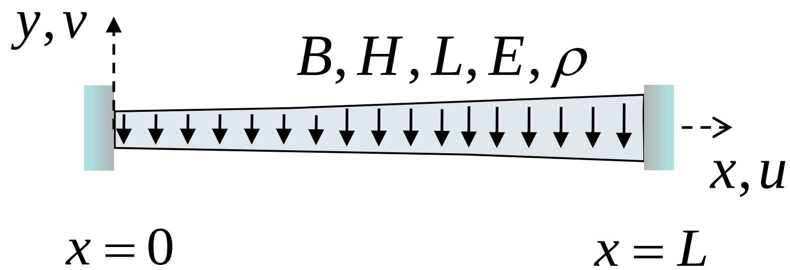
```
% This is a program that generates a clean and noisy signal
x=linspace(0,10*pi,200);
% Compute and plot the clear signal in Fig. 1
y=sin(x);
figure(1); plot(x,y); hold on;
% Compute the noise
z=0.3*rand(1,200);
% Add the noise to the signal
y=y+z
% Plot the noisy signal in Fig.1
plot(x,y); grid
% Finalize the figure
title(' Clean And Noisy Sinusoid')
xlabel('x')
ylabel('y')
hold off;
```

Computer exercise 7.2

Let us consider a *beam bending* problem as a *plane stress elasticity* problem with displacements *u* and *v* in the *x*- and *y*-directions, correspondingly, as primary variables and self weight as a body load:

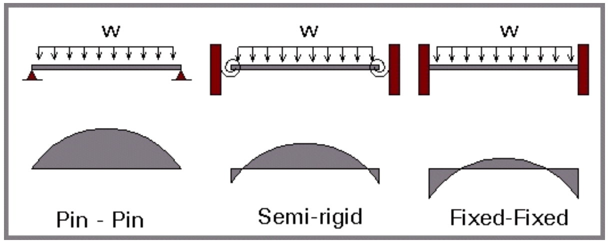
$$B = 40 \text{ cm}, H = 20 \text{ cm}, L = 1 \text{ m}$$

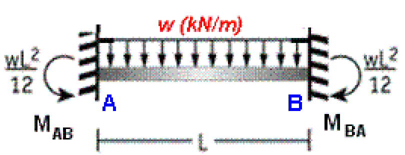
$$E = 200 \text{ GPa}, \rho = 8 \text{ kg/m}^3$$

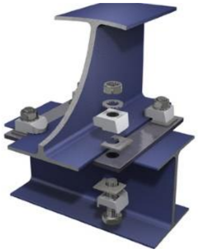


Solve the 2D deformation state of the beam by utilizing Matlab *PDEToolbox* relying on the finite element method (of triangular elements with linear basis functions).

Hint: Take part in the guided tour in the exercise session.







Computer exercise 7.3

(i) Find out how to numerically integrate function *f = f(x)* in Matlab over a line segment of the *x*-axis.

(ii) Integrate numerically function *f = sin(x)* over the line segment *(-pi/2, pi/2)*.

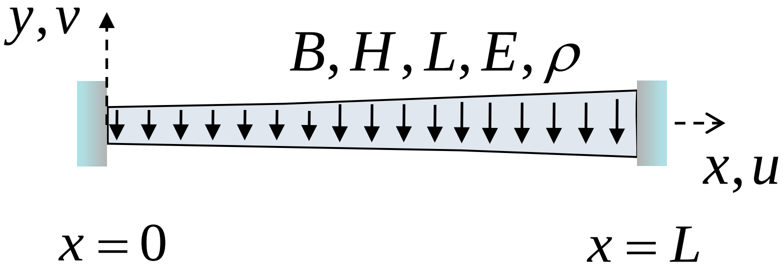
(iii) Check the result by a hand calculation.

Computer exercise 7.4

Let us consider a *beam bending* problem as a *plane stress elasticity* problem with displacements *u* and *v* in the *x*- and *y*-directions, correspondingly, as primary variables and self weight as a body load:

$$B = 40\ \text{cm}, H = 20\ \text{cm}, L = 1\ \text{m}$$

$$E = 200\ \text{GPa},\ \rho = 8\ \text{kg/m}^3$$

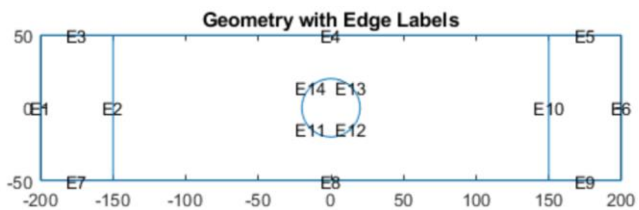


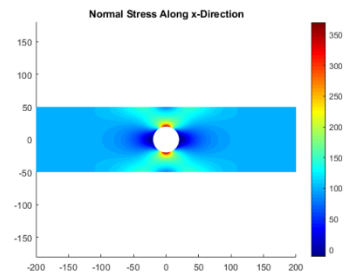
After solving the 2D deformation state of the beam in one of the previous exercises, compute the *bending moment* of the beam (engineering beam model) in the middle of the beam (at $x = L/2$), $M(L/2)$, from the axial stress distribution.

Hint: Recall the definition of bending moment of the engineering beam model – involving the *x*-component of the stress, the *y*-coordinate and an integral across the cross section of the beam – from your previous studies (or Wikipedia). Utilize the assumption that the distribution of the axial stress across the thickness is linear.

Computer exercise 7.5

Let us consider a thin rectangular plate strip under a uniaxial tensional loading implying a uniform stress distribution. Introducing a circular hole in the plate disturbs the uniform stress distribution near the hole, resulting in significantly higher stress values. A thin plate subject to in-plane loading can be analyzed as a 2D *plane stress elasticity* problem. In theory, if the plate is infinite, then the stress near the hole is three times higher than the average stress.





Find out the stress distribution of this problem in case of a finite strip length by utilizing Matlab *PDEToolbox* relying on the finite element method (of triangular elements with linear basis functions).

Hint: Follow the Matlab tutorial Stress Concentration in Plate with Circular Hole:
https://se.mathworks.com/help/pde/examples/stress-concentration-in-plate-with-circular-hole.html