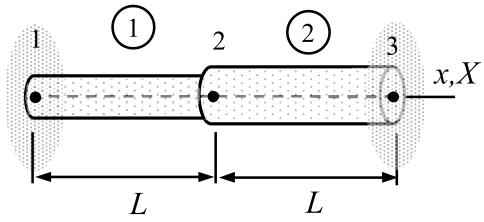
Name________________________ Student number________________

Home assignment 2

The bar shown consists of two elements having different cross-sectional areas $A_1 = A$, $A_2 = 2A$. Material properties E, k, and α are the same. Determine the stationary displacement u_{X2} and temperature ϑ_2 at node 2, when the temperature at the left wall (node 1) is ϑ° and that of the right wall is $2\vartheta^\circ$ (node 3). Stress vanishes, when the temperature in the wall and bar is ϑ°.



Solution template

Element contribution of a bar needed in this case are

$$\delta W^{\text{int}} = -\begin{Bmatrix}\delta u_{x1}\\ \delta u_{x2}\end{Bmatrix}^{\mathrm{T}} \frac{EA}{h}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_{x1}\\ u_{x2}\end{Bmatrix}, \quad \delta W^{\text{cpl}} = \begin{Bmatrix}\delta u_{x1}\\ \delta u_{x2}\end{Bmatrix}^{\mathrm{T}} \frac{\alpha EA}{2}\begin{bmatrix}-1 & -1\\ 1 & 1\end{bmatrix}\begin{Bmatrix}\Delta\vartheta_1\\ \Delta\vartheta_2\end{Bmatrix},$$

$$\delta P^{\text{int}} = -\begin{Bmatrix}\delta\vartheta_1\\ \delta\vartheta_2\end{Bmatrix}^{\mathrm{T}} \frac{kA}{h}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}\vartheta_1\\ \vartheta_2\end{Bmatrix}.$$

The expressions assume linear approximations and constant material properties. The temperature relative to the initial temperature without stress is denoted by $\Delta\vartheta = \vartheta - \vartheta^\circ$. The unknown nodal displacement and temperature are u_{X2} and ϑ_2.

When the nodal displacements and temperatures are substituted there, the element contributions of bar 1 take the forms

$\delta W^1 =$ ____,

$\delta P^1 =$ ____.

When the displacements and temperatures are substituted there, the element contributions of bar 2 take the forms

$\delta W^2 =$ ____,

$\delta P^2 =$ ____.

Virtual work expression of a structure is the sum over the expressions of the elements

$$\delta W = -\delta u_{X2}(\underline{\hspace{8cm}}),$$

$$\delta P = -\delta \vartheta_2(\underline{\hspace{8cm}}).$$

Variational principle $\delta P = 0$ and $\delta W = 0 \quad \forall \mathbf{a}$ gives the linear equation system

$$\begin{bmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{bmatrix} \begin{Bmatrix} u_{X2} \\ \vartheta_2 \end{Bmatrix} - \begin{Bmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{Bmatrix} = 0 \quad \Leftrightarrow$$

$\vartheta_2 = \underline{\hspace{3cm}}$ and $u_{X2} = \underline{\hspace{3cm}}$.