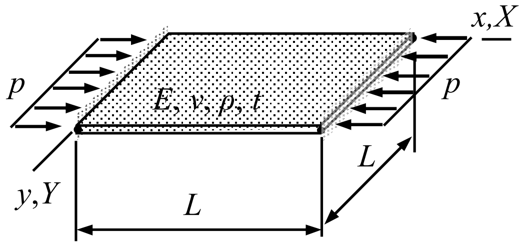
Name____________________________ Student number___________________

Home assignment 3

Determine the critical value of the in-plane loading p_{cr} making the plate of the figure to buckle. The loaded edges are simply supported and the unloaded free. Use the approximation $w(x,y) = a_0(1-x/L)(x/L)$ and assume that $N_{xx} = -p$ and $N_{yy} = N_{xy} = 0$. Problem parameters E, ν, ρ and t are constants.



Solution

Assuming that the material coordinate system is chosen so that the plate bending and thin slab modes decouple in the linear analysis and that the in-plane stress resultants are known (from linear displacement analysis, say), it is enough to consider the virtual work densities of plate bending mode and the coupling of the bending and thin-slab modes

$$\delta w_{\Omega}^{\text{int}} = -\begin{Bmatrix} \partial^2\delta w/\partial x^2 \\ \partial^2\delta w/\partial y^2 \\ 2\partial^2\delta w/\partial x\partial y \end{Bmatrix}^{\mathrm{T}} \frac{t^3}{12}[E]_\sigma \begin{Bmatrix} \partial^2 w/\partial x^2 \\ \partial^2 w/\partial y^2 \\ 2\partial^2 w/\partial x\partial y \end{Bmatrix}, \quad \delta w_{\Omega}^{\text{sta}} = -\begin{Bmatrix} \partial\delta w/\partial x \\ \partial\delta w/\partial y \end{Bmatrix}^{\mathrm{T}} \begin{bmatrix} N_{xx} & N_{xy} \\ N_{yx} & N_{yy} \end{bmatrix} \begin{Bmatrix} \partial w/\partial x \\ \partial w/\partial y \end{Bmatrix}$$

where the elasticity matrix of plane stress

$$[E]_\sigma = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}.$$

Approximation to the transverse displacement and its non-zero derivatives in the density expressions are given by

$$w(x,y) = a_0(1-\frac{x}{L})\frac{x}{L} \quad \Rightarrow \quad \frac{\partial w}{\partial x} = \frac{a_0}{L}(1-2\frac{x}{L}), \quad \frac{\partial^2 w}{\partial x^2} = -2\frac{a_0}{L^2}, \quad \ddot{w} = \ddot{a}_0(1-\frac{x}{L})\frac{x}{L}.$$

When the approximation is substituted there, virtual work density of the internal forces and that of the coupling simplify to (substitute also the known solution $N_{xx} = -p$ and $N_{yy} = N_{xy} = 0$ to the in-plane stress resultants)

$$\delta w_{\Omega}^{\text{int}} = -\delta a_0 \frac{1}{3}\frac{t^3}{L^4}\frac{E}{1-\nu^2}a_0,$$

$$\delta w_{\Omega}^{\text{sta}} = \delta a_0 (1-2\frac{x}{L})^2 \frac{p}{L^2} a_0 .$$

Virtual work expressions are integrals of the densities over the domain occupied by the plate

$$\delta W^{\text{int}} = \int_0^L \int_0^L \delta w_{\Omega}^{\text{int}} dxdy = -\delta a_0 \frac{1}{3} \frac{t^3}{L^2} \frac{E}{1-\nu^2} a_0 ,$$

$$\delta W^{\text{sta}} = \int_0^L \int_0^L \delta w_{\Omega}^{\text{sta}} dxdy = \delta a_0 \frac{1}{3} p a_0 .$$

Virtual work expression

$$\delta W = \delta W^{\text{int}} + \delta W^{\text{sta}} = -\delta a_0 (\frac{1}{3} \frac{t^3}{L^2} \frac{E}{1-\nu^2} - \frac{1}{3} p) a_0 ,$$

principle of virtual work $\delta W = 0 \ \forall \delta a_0$, and the fundamental lemma of variation calculus give

$$(\frac{1}{3} \frac{t^3}{L^2} \frac{E}{1-\nu^2} - \frac{1}{3} p) a_0 = 0 .$$

For a non-trivial solution $a_0 \neq 0$, the loading parameter needs to take the value

$$p_{\text{cr}} = \frac{E}{1-\nu^2} \frac{t^3}{L^2} . \quad \leftarrow$$