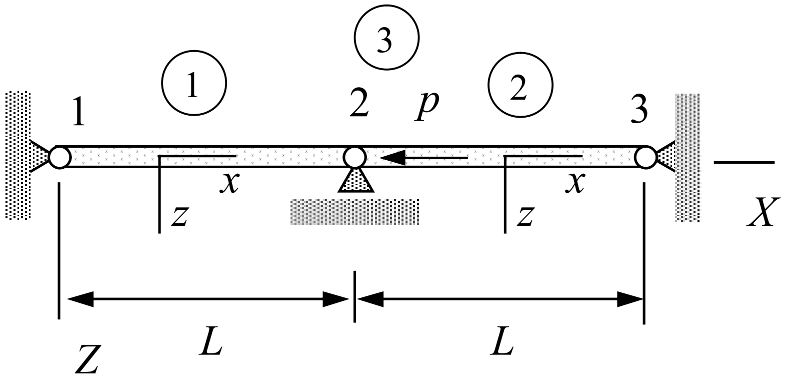
LECTURE ASSIGNMENT 2. Stability analysis of a truss (beams connected by joints) requires the axial forces $N(p)$ of the beams as functions of the loading parameter $p \geq 0$ and the buckling criterion

$$N(p) = \begin{cases} -\pi^2 \dfrac{EI}{h^2} & \text{when } N < 0 \\ \infty & \text{when } N > 0 \end{cases}$$

to identify the first beam (under compression) to buckle when the loading parameter p is increased from zero. Determine the critical load p_{cr} of the truss shown. First, determine the axial forces by using the principle of virtual work and the relationship between the axial displacements and axial forces

$$\delta W^{\text{int}} = -\begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^{\text{T}} \frac{EA}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix} \text{ and } N = EA \frac{u_{x2} - u_{x1}}{h}.$$

Thereafter, determine the critical load p_{cr} by using the buckling criterion.



Name______________________ Student number________________

- Element contributions are given by

$$\delta W^1 = -\begin{Bmatrix} 0 \\ \delta u_{X2} \end{Bmatrix}^{\mathrm{T}} \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} 0 \\ u_{X2} \end{Bmatrix} = -\delta u_{X2}\frac{EA}{L}u_{X2}$$

$$\delta W^2 = -\begin{Bmatrix} \delta u_{X2} \\ 0 \end{Bmatrix}^{\mathrm{T}} \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_{X2} \\ 0 \end{Bmatrix} = -\delta u_{X2}\frac{EA}{L}u_{X2}$$

$$\delta W^3 = -\delta u_{X2}p$$

- Principle of virtual work gives the axial displacement

$$-\delta u_{X2}(2\frac{EA}{L}u_{X2} + p) = 0 \quad \Rightarrow \quad u_{X2} = -\frac{1}{2}\frac{pL}{EA} .$$

- Axial forces of the beams and the corresponding buckling loads are given by (superscript denotes the element)

$$N^1 = EA\frac{u_{x2} - u_{x1}}{h} = -\frac{1}{2}p \quad \Rightarrow \quad p = 2\pi^2\frac{EI}{L^2}$$

$$N^2 = EA\frac{u_{x3} - u_{x2}}{h} = \frac{1}{2}p \quad \Rightarrow \quad p = \infty$$

- The critical load is the smallest of the values

$$p_{\mathrm{cr}} = 2\pi^2\frac{EI}{L^2} . \quad \leftarrow$$