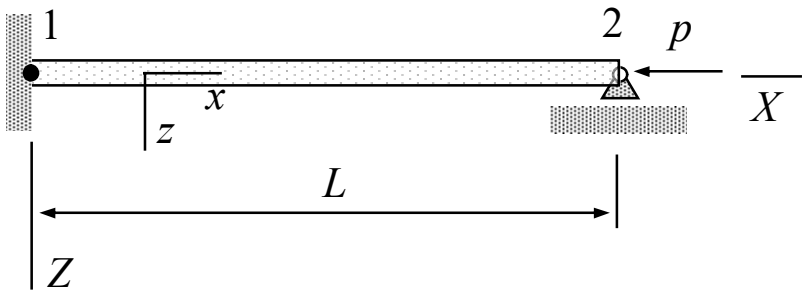
LECTURE ASSIGNMENT 1. Linear and non-linear parts of virtual work expression of internal forces of a beam element (displacements in *xz*-plane) are given by

$$\delta W^{\text{int}} = -\begin{Bmatrix} \delta u_{z1} \\ \delta\theta_{y1} \\ \delta u_{z2} \\ \delta\theta_{y2} \end{Bmatrix}^{\text{T}} \frac{EI_{yy}}{h^3} \begin{bmatrix} 12 & -6h & -12 & -6h \\ -6h & 4h^2 & 6h & 2h^2 \\ -12 & 6h & 12 & 6h \\ -6h & 2h^2 & 6h & 4h^2 \end{bmatrix} \begin{Bmatrix} u_{z1} \\ \theta_{y1} \\ u_{z2} \\ \theta_{y2} \end{Bmatrix},$$

$$\delta W^{\text{sta}} = -\begin{Bmatrix} \delta u_{z1} \\ \delta\theta_{y1} \\ \delta u_{z2} \\ \delta\theta_{y2} \end{Bmatrix}^{\text{T}} \frac{N}{30h} \begin{bmatrix} 36 & -3h & -36 & -3h \\ -3h & 4h^2 & 3h & -h^2 \\ -36 & 3h & 36 & 3h \\ -3h & -h^2 & 3h & 4h^2 \end{bmatrix} \begin{Bmatrix} u_{z1} \\ \theta_{y1} \\ u_{z2} \\ \theta_{y2} \end{Bmatrix}$$

in which I_{yy} is the second moment of area, E is the Young's modulus, and N is the axial force in the beam. Determine the buckling force p_{cr} of the beam shown by using one element. Second moment of area I and Young's modulus E are constants.



Name_____ Student number_____

- The axial force of the beam (in terms of p and negative when compression)

$$N = -p$$

- Linear and non-linear parts of virtual work expression of internal forces of the beam (displacements in xz-plane) are

$$\delta W^{\text{int}} = -\begin{Bmatrix} 0 \\ 0 \\ 0 \\ \delta\theta_{Y2} \end{Bmatrix}^{\text{T}} \frac{EI}{L^3}\begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ 0 \\ \theta_{Y2} \end{Bmatrix} = -\delta\theta_{Y2}\frac{4EI}{L}\theta_{Y2}$$

$$\delta W^{\text{sta}} = -\begin{Bmatrix} 0 \\ 0 \\ 0 \\ \delta\theta_{Y2} \end{Bmatrix}^{\text{T}} \frac{-p}{30L}\begin{bmatrix} 36 & -3L & -36 & -3L \\ -3L & 4L^2 & 3L & -L^2 \\ -36 & 3L & 36 & 3L \\ -3L & -L^2 & 3L & 4L^2 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ 0 \\ \theta_{Y2} \end{Bmatrix} = \delta\theta_{Y2}\frac{4pL}{30}\theta_{Y2}$$

- Principle of virtual work $\delta W = \delta W^{\text{int}} + \delta W^{\text{sta}} = 0 \;\; \forall \delta \text{a}$ implies (assuming that $\theta_{Y2} \neq 0$)

$$(-\frac{4EI}{L} + \frac{4pL}{30})\theta_{Y2} = 0 \quad \Rightarrow \quad p_{\text{cr}} = 30\frac{EI}{L^2}.$$