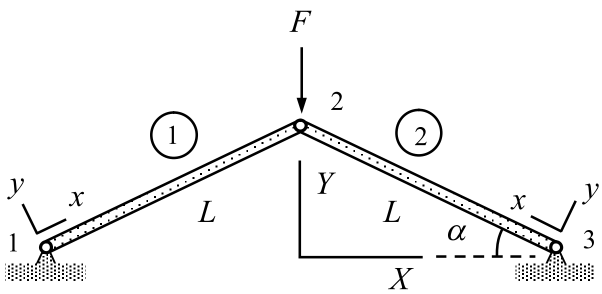
Name________________________ Student number________________

Home assignment 2

Consider the bar structure subjected to large displacements in the figure. Determine the relationship between the displacement of node 2 and force F. Start with the virtual work density $\delta w_{\Omega^\circ}^{\text{int}}$ of the non-linear bar model, linear approximations to displacement components, and assume that $u_{X2}=0$ (due to symmetry). Cross-sectional area of the initial geometry is A° and Young's modulus of the material is C.



Solution template

Virtual work density of the non-linear bar model

$$\delta w_{\Omega^\circ}^{\text{int}} = -(\frac{d\delta u}{dx}+\frac{du}{dx}\frac{d\delta u}{dx}+\frac{dv}{dx}\frac{d\delta v}{dx}+\frac{dw}{dx}\frac{d\delta w}{dx})CA^\circ[\frac{du}{dx}+\frac{1}{2}(\frac{du}{dx})^2+\frac{1}{2}(\frac{dv}{dx})^2+\frac{1}{2}(\frac{dw}{dx})^2]$$

is based on the Green-Lagrange strain definition which is physically correct also when rotations/displacements are large. The expression depends on all displacement components, material property is denoted by C (constitutive equation $S_{xx}=CE_{xx}$), and the superscript in the cross-sectional area A° (and in other quantities) refers to the initial geometry (strain and stress vanishes). Otherwise, equilibrium equations follow in the same manner as in the linear case.

For element 1, the non-zero displacement components are $u_{y2}=u_{Y2}\cos\alpha$ and $u_{x2}=u_{Y2}\sin\alpha$. As the initial length of the element $h^\circ = L$, linear approximations to the displacement components are

$$u=\frac{x}{L}u_{Y2}\sin\alpha\,,\quad v=\frac{x}{L}u_{Y2}\cos\alpha\,,\quad \text{and}\quad w=0 \quad \Rightarrow$$

$$\frac{du}{dx}=\frac{u_{Y2}}{L}\sin\alpha\,,\quad \frac{dv}{dx}=\frac{u_{Y2}}{L}\cos\alpha\,,\ \text{and}\quad \frac{dw}{dx}=0\,.$$

When the approximations are substituted there, virtual work density of the internal forces and thereby the virtual work expression (density is constant) in terms of the dimensionless displacement $\text{a}=u_{Y2}/L$ simplify to

$$\delta w_{\Omega^\circ}^{\text{int}} = -(\delta\text{a}\sin\alpha+\text{a}\delta\text{a})CA^\circ(\text{a}\sin\alpha+\frac{1}{2}\text{a}^2) \quad \Rightarrow$$

$$\delta W^1 = -(\delta \mathrm{a} \sin\alpha + \mathrm{a}\delta \mathrm{a}) CA^\circ L(\mathrm{a}\sin\alpha + \frac{1}{2}\mathrm{a}^2) .$$

For element 2, the non-zero displacement components are $u_{y2} = u_{Y2}\cos\alpha$ and $u_{x2} = u_{Y2}\sin\alpha$. As the initial length of the element $h^\circ = L$, linear approximations to the displacement components are

$$u = \frac{x}{L} u_{Y2} \sin\alpha , \quad v = \frac{x}{L} u_{Y2}\cos\alpha , \text{ and } w = 0 \quad \Rightarrow$$

$$\frac{du}{dx} = \frac{u_{Y2}}{L}\sin\alpha , \quad \frac{dv}{dx} = \frac{u_{Y2}}{L}\cos\alpha , \text{ and } \frac{dw}{dx} = 0 .$$

When the approximations are substituted there, virtual work density of the internal forces and thereby the virtual work expression (density is constant) in terms of the dimensionless displacement $\mathrm{a} = u_{Y2} / L$ simplify to

$$\delta w_{\Omega^\circ}^{\mathrm{int}} = -(\delta \mathrm{a}\sin\alpha + \mathrm{a}\delta \mathrm{a}) CA^\circ (\mathrm{a}\sin\alpha + \frac{1}{2}\mathrm{a}^2) \quad \Rightarrow$$

$$\delta W^2 = -(\delta \mathrm{a}\sin\alpha + \mathrm{a}\delta \mathrm{a}) CA^\circ L(\mathrm{a}\sin\alpha + \frac{1}{2}\mathrm{a}^2) .$$

Virtual work expression of the point force, in terms of the dimensionless displacement $\mathrm{a} = u_{Y2} / L$, is

$$\delta W^3 = -F\delta u_{Y2} = -FL\delta \mathrm{a} .$$

Virtual work expression of the structure is obtained as the sum of the element contributions

$$\delta W = -\delta \mathrm{a}((\sin\alpha + \mathrm{a}) 2CA^\circ L(\mathrm{a}\sin\alpha + \frac{1}{2}\mathrm{a}^2) + FL) .$$

Principle of virtual work and the fundamental lemma of variation calculus imply that

$$CA^\circ \mathrm{a}(\sin\alpha + \mathrm{a})(2\sin\alpha + \mathrm{a}) + F = 0 \quad \text{in which} \quad \mathrm{a} = \frac{u_{Y2}}{L} . \; \leftarrow$$