LECTURE ASSIGNMENT 1. Strain of a bar can be defined in various ways. The component forms of some of the measures are given by

$$[\varepsilon]=\frac{1}{2}([F]+[F]^{\mathrm{T}}-2[I]) \quad \text{Linear}$$

$$[E]=\frac{1}{2}([F]^{\mathrm{T}}[F]-[I]) \qquad \text{Green-Lagrange}$$

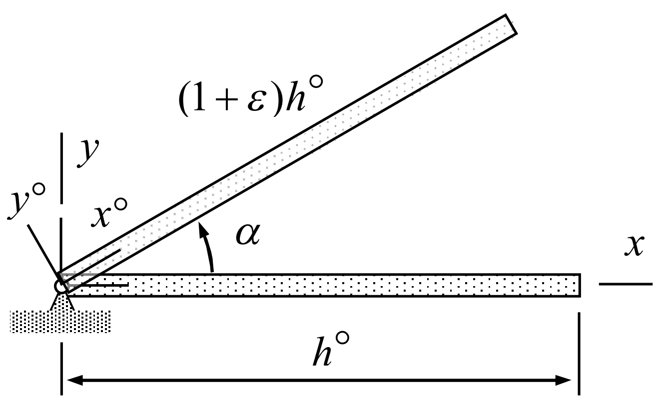
The component forms of the deformation gradient $[F]$, the unit matrix $[I]$, the linear strain, and the Green-Lagrange strain of the planar case are

$$[F]=\begin{bmatrix} \dfrac{\partial x}{\partial x^{\circ}} & \dfrac{\partial x}{\partial y^{\circ}} \\ \dfrac{\partial y}{\partial x^{\circ}} & \dfrac{\partial y}{\partial y^{\circ}} \end{bmatrix},\ [I]=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ [\varepsilon]=\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix},\ [E]=\begin{bmatrix} E_{xx} & E_{xy} \\ E_{xy} & E_{yy} \end{bmatrix}$$

Assume that a typical particle, identified by the material coordinates x° and y°, undergoes displacement so that the position in the structural co-ordinate system is given by

$$\begin{Bmatrix} x \\ y \end{Bmatrix}=\begin{bmatrix} (1+\varepsilon)\cos\alpha & -\sin\alpha \\ (1+\varepsilon)\sin\alpha & \cos\alpha \end{bmatrix}\begin{Bmatrix} x^{\circ} \\ y^{\circ} \end{Bmatrix}$$

in which ε and α are parameters of the mapping (constants). Determine the linear and Green-Lagrange strains of the bar.



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- Strain measures depend on the deformation gradient

$$[F]=\begin{bmatrix} \partial x/\partial x^\circ & \partial x/\partial y^\circ \\ \partial y/\partial x^\circ & \partial y/\partial y^\circ \end{bmatrix}=\begin{bmatrix} (1+\varepsilon)\cos\alpha & -\sin\alpha \\ (1+\varepsilon)\sin\alpha & \cos\alpha \end{bmatrix}$$

- Linear strain easure $[\varepsilon]=\frac{1}{2}([F]+[F]^{\mathrm{T}}-2[I])$

$$[\varepsilon]=\frac{1}{2}(\begin{bmatrix} (1+\varepsilon)\cos\alpha & -\sin\alpha \\ (1+\varepsilon)\sin\alpha & \cos\alpha \end{bmatrix}+\begin{bmatrix} (1+\varepsilon)\cos\alpha & (1+\varepsilon)\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}-\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix})$$

$$[\varepsilon]=\begin{bmatrix} (1+\varepsilon)\cos\alpha-1 & \frac{1}{2}\varepsilon\sin\alpha \\ \frac{1}{2}\varepsilon\sin\alpha & \cos\alpha-1 \end{bmatrix}\approx\begin{bmatrix} \varepsilon & 0 \\ 0 & 0 \end{bmatrix} \quad \text{when } |\alpha|\ll 1$$

- Green-Lagrange strain measure $[E]=\frac{1}{2}([F]^{\mathrm{T}}[F]-[I])$

$$[E]=\frac{1}{2}(\begin{bmatrix} (1+\varepsilon)\cos\alpha & (1+\varepsilon)\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} (1+\varepsilon)\cos\alpha & -\sin\alpha \\ (1+\varepsilon)\sin\alpha & \cos\alpha \end{bmatrix}-\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix})$$

$$[E]=\begin{bmatrix} \frac{1}{2}[(1+\varepsilon)^2-1] & 0 \\ 0 & 0 \end{bmatrix}\approx\begin{bmatrix} \varepsilon & 0 \\ 0 & 0 \end{bmatrix} \quad \text{when } |\varepsilon|\ll 1$$