LECTURE ASSIGNMENT 2. Virtual work densities of the bar model according to the large displacement theory are given by

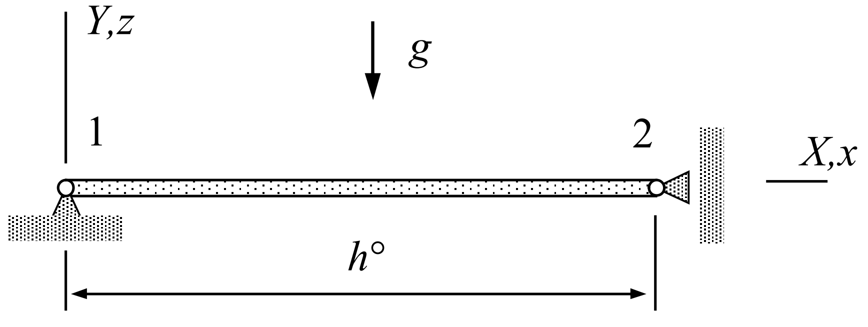
$$\delta w_{\Omega^\circ}^{\text{int}} = -\delta E_{xx} C A^\circ E_{xx} \quad \text{and} \quad \delta w_{\Omega^\circ}^{\text{ext}} = \rho^\circ A^\circ (\delta u g_x + \delta v g_y + \delta w g_z)$$

in which the Green-Lagrange strain measure and its variation

$$E_{xx} = \frac{du}{dx} + \frac{1}{2}(\frac{du}{dx})^2 + \frac{1}{2}(\frac{dv}{dx})^2 + \frac{1}{2}(\frac{dw}{dx})^2,$$

$$\delta E_{xx} = \frac{d\delta u}{dx} + \frac{d\delta u}{dx}\frac{du}{dx} + \frac{d\delta v}{dx}\frac{dv}{dx} + \frac{d\delta w}{dx}\frac{dw}{dx}.$$

Derive the virtual work densities for the element shown in terms of nodal displacement u_{Y2}. Use linear approximations to the displacement components. Cross-sectional area and density of the initial geometry are A° and ρ°, respectively, and elasticity parameter C.



Name________________________ Student number_________________

- Linear approximations to displacement components in the element (material) coordinate system in terms of u_{Y2}

$$u = 0 \qquad v = 0 \qquad w = \frac{x}{h^\circ} u_{Y2}$$

$$\frac{du}{dx} = 0 \qquad \frac{dv}{dx} = 0 \qquad \frac{dw}{dx} = \frac{u_{Y2}}{h^\circ}$$

- Green-Lagrange strain measure and its variation in terms of u_{Y2}

$$E_{xx} = \frac{1}{2}\left(\frac{u_{Y2}}{h^\circ}\right)^2 \quad \Rightarrow \quad \delta E_{xx} = \frac{u_{Y2}}{h^\circ}\frac{\delta u_{Y2}}{h^\circ}.$$

- Virtual work densities of internal and external distributed forces in terms of u_{Y2}

$$\delta w_{\Omega^\circ}^{\text{int}} = -\frac{u_{Y2}}{h^\circ}\frac{\delta u_{Y2}}{h^\circ} CA^\circ \frac{1}{2}\left(\frac{u_{Y2}}{h^\circ}\right)^2$$

$$\delta w_{\Omega^\circ}^{\text{ext}} = -\frac{x}{h^\circ}\delta u_{Y2}\rho^\circ A^\circ g$$

- Virtual work expressions are integrals over the initial domain

$$\delta W^{\text{int}} = \int_0^{h^\circ} \delta w_{\Omega^\circ}^{\text{int}} dx = -\delta u_{Y2} CA^\circ \frac{1}{2}\left(\frac{u_{Y2}}{h^\circ}\right)^3$$

$$\delta W^{\text{ext}} = \int_0^{h^\circ} \delta w_{\Omega^\circ}^{\text{ext}} dx = -\delta u_{Y2}\frac{\rho^\circ A^\circ h^\circ}{2} g$$