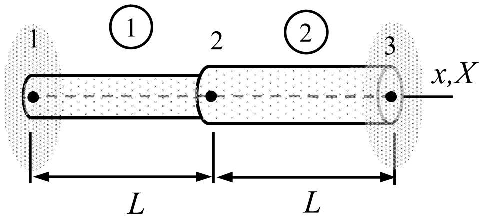
Name______________________________ Student number____________________

# Home assignment 2

The bar shown consists of two elements having different cross-sectional areas $A_1 = A$, $A_2 = 2A$. Material properties $E$, $k$, and $\alpha$ are the same. Determine the stationary displacement $u_{X2}$ and temperature $\vartheta_2$ at node 2, when the temperature at the left wall (node 1) is $\vartheta^\circ$ and that of the right wall is $2\vartheta^\circ$ (node 3). Stress vanishes, when the temperature in the wall and bar is $\vartheta^\circ$.




**Solution template**

Element contribution of a bar needed in this case are

$$\delta W^{\text{int}} = -\begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^{\text{T}} \frac{EA}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_{x1} \\ u_{x2} \end{Bmatrix}, \quad \delta W^{\text{cpl}} = \begin{Bmatrix} \delta u_{x1} \\ \delta u_{x2} \end{Bmatrix}^{\text{T}} \frac{\alpha EA}{2} \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \begin{Bmatrix} \Delta\vartheta_1 \\ \Delta\vartheta_2 \end{Bmatrix},$$

$$\delta P^{\text{int}} = -\begin{Bmatrix} \delta\vartheta_1 \\ \delta\vartheta_2 \end{Bmatrix}^{\text{T}} \frac{kA}{h} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} \vartheta_1 \\ \vartheta_2 \end{Bmatrix}.$$

The expressions assume linear approximations and constant material properties. The temperature relative to the initial temperature without stress is denoted by $\Delta\vartheta = \vartheta - \vartheta^\circ$. The unknown nodal displacement and temperature are $u_{X2}$ and $\vartheta_2$.

When the nodal displacements and temperatures are substituted there, the element contributions of bar 1 take the forms

$$\delta W^1 = -\begin{Bmatrix} 0 \\ \delta u_{X2} \end{Bmatrix}^{\text{T}} \left( \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ u_{X2} \end{Bmatrix} - \frac{\alpha EA}{2} \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ \vartheta_2 - \vartheta^\circ \end{Bmatrix} \right),$$

$$\delta P^1 = -\begin{Bmatrix} 0 \\ \delta\vartheta_2 \end{Bmatrix}^{\text{T}} \frac{kA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} \vartheta^\circ \\ \vartheta_2 \end{Bmatrix}.$$

When the displacements and temperatures are substituted there, the element contributions of bar 2 take the forms

$$\delta W^2 = -\begin{Bmatrix} \delta u_{X2} \\ 0 \end{Bmatrix}^{\text{T}} \left( \frac{2EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_{X2} \\ 0 \end{Bmatrix} - \alpha EA \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \begin{Bmatrix} \vartheta_2 - \vartheta^\circ \\ \vartheta^\circ \end{Bmatrix} \right),$$

$$\delta P^2 = -\begin{Bmatrix} \delta\vartheta_2 \\ 0 \end{Bmatrix}^{\text{T}} \frac{2kA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} \vartheta_2 \\ 2\vartheta^\circ \end{Bmatrix}.$$

Virtual work expression of a structure is the sum over the expressions of the elements

$$\delta W = -\delta u_{X2}(3\frac{EA}{L}u_{X2} + \frac{\alpha EA}{2}(\vartheta_2 + \vartheta^\circ)),$$

$$\delta P = -\delta\vartheta_2(\frac{kA}{L}(3\vartheta_2 - 5\vartheta^\circ)).$$

Variational principle $\delta P = 0$ and $\delta W = 0$ $\forall \mathbf{a}$ gives the linear equation system

$$\begin{bmatrix} 3\frac{EA}{L} & \frac{EA}{2}\alpha \\ 0 & 3\frac{kA}{L} \end{bmatrix} \begin{Bmatrix} u_{X2} \\ \vartheta_2 \end{Bmatrix} - \begin{Bmatrix} -\frac{1}{2}\alpha EA\vartheta^\circ \\ 5\frac{kA}{L}\vartheta^\circ \end{Bmatrix} = 0 \quad \Leftrightarrow$$

$\vartheta_2 = \frac{5}{3}\vartheta^\circ$ and $u_{X2} = -\frac{4}{9}\alpha L\vartheta^\circ$. ←