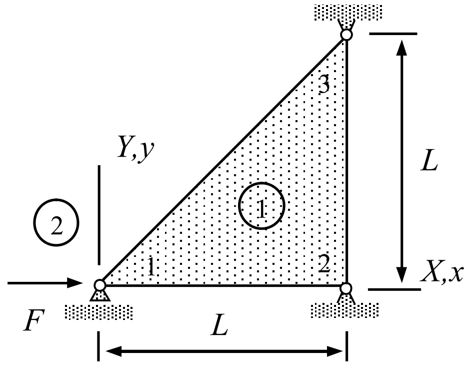
Name________________________ Student number________________

Home assignment 3

A thin triangular slab (assume plane stress conditions) loaded by a horizontal force is allowed to move horizontally at node 1 and nodes 2 and 3 are fixed. At the constant initial temperature ϑ° and loading $F=0$, stress vanishes. If the slab is heated to the constant temperature $2\vartheta^\circ$, what is the required force F to have $u_{X1}=0$? Material properties E, ν, α and thickness t of the slab are constants.



Solution

As temperature is known and the external distributed force vanishes, virtual work densities needed are (formulae collection)

$$\delta w_\Omega^{\text{int}} = -\begin{Bmatrix}\delta\varepsilon_{xx}\\ \delta\varepsilon_{yy}\\ \delta\gamma_{xy}\end{Bmatrix}^{\mathrm{T}} t[E]_\sigma \begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix} \text{ where } \begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\partial u/\partial x\\ \partial v/\partial y\\ \partial u/\partial y + \partial v/\partial x\end{Bmatrix},$$

$$\delta w_\Omega^{\text{cpl}} = \begin{Bmatrix}\partial\delta u/\partial x\\ \partial\delta v/\partial x\end{Bmatrix}^{\mathrm{T}} \frac{E\alpha t}{1-\nu}\Delta\vartheta\begin{Bmatrix}1\\1\end{Bmatrix}$$

in which $\Delta\vartheta = \vartheta - \vartheta^\circ$ is the difference between temperature at the deformed and initial geometries.

Approximation is the first thing to be considered. As the origin of the material $xy-$coordinate system is placed at node 1 and the axes are aligned with the axes of the structural $XY-$coordinate system

$$u = (1-\frac{x}{L})u_{X1}, \quad v = 0, \text{ and } \Delta\vartheta = \vartheta^\circ \text{ (constant).}$$

When the approximations are substituted there, virtual work density (composed of the internal and coupling parts) simplifies to

$$\delta w_\Omega = -\begin{Bmatrix}-\delta u_{X1}/L\\ 0\\ 0\end{Bmatrix}^{\mathrm{T}} \frac{Et}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & (1-\nu)/2\end{bmatrix}\begin{Bmatrix}-u_{X1}/L\\ 0\\ 0\end{Bmatrix} + \begin{Bmatrix}-\delta u_{X1}/L\\ 0\end{Bmatrix}^{\mathrm{T}} \frac{Et}{1-\nu}\alpha\vartheta^\circ\begin{Bmatrix}1\\1\end{Bmatrix} \quad \Leftrightarrow$$

$$\delta w_\Omega = -\frac{\delta u_{X1}}{L}\frac{Et}{1-\nu^2}\frac{u_{X1}}{L} - \frac{\delta u_{X1}}{L}\frac{Et}{1-\nu}\alpha\vartheta^\circ.$$

Virtual work expression is integral of the density expression over the domain occupied by the element. Here, virtual work density is constant so that it is enough to multiply by the area. Virtual work expressions of element 1 and 2 (point force) become

$$\delta W^1 = \delta w_\Omega \frac{L^2}{2} = -\delta u_{X1}\left(\frac{1}{2}\frac{Et}{1-\nu^2}u_{X1} + \frac{1}{2}\frac{Et}{1-\nu}L\alpha\vartheta^\circ\right),$$

$$\delta W^2 = \delta u_{X1} F.$$

Virtual work expression of the structure $\delta W = \delta W^1 + \delta W^2$, principle of virtual work, and the fundamental lemma of variation calculus imply the equilibrium equation

$$\frac{1}{2}\frac{Et}{1-\nu^2}u_{X1} + \frac{1}{2}\frac{Et}{1-\nu}L\alpha\vartheta^\circ - F = 0 \quad \Leftrightarrow \quad u_{X1} = \frac{1-\nu^2}{Et}\left(2F - \frac{Et}{1-\nu}L\alpha\vartheta^\circ\right).$$

Displacement vanishes with the force (this is also the horizontal constraint force when the node is fixed)

$$F = \frac{1}{2}\frac{Et}{1-\nu}L\alpha\vartheta^\circ.$$