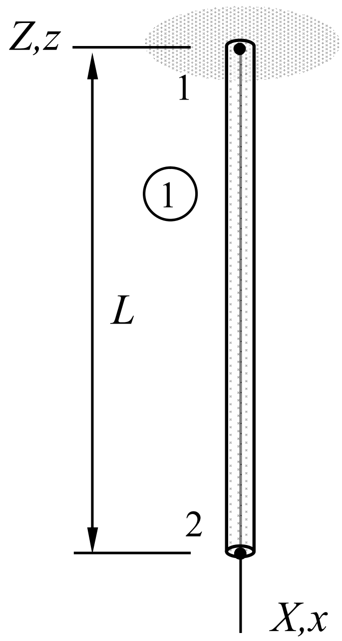
LECTURE ASSIGNMENT 1. In stationary thermo-elasticity without external forces, the virtual work density of the bar model is given by

$$\delta w_{\Omega} = -\frac{d\delta u}{dx} EA \frac{du}{dx} + \frac{d\delta u}{dx} EA\alpha\Delta\vartheta .$$

Assuming that E, A and α are constants, determine the displacement of node 2 at the constant temperature $2\vartheta°$. Use linear approximation. At the initial temperature $\vartheta°$, length of the bar is L and stress in the bar vanishes.



Name________________________ Student number_________________

- Approximation to $u(x)$ and temperature increase $\Delta\vartheta$ are

$$u=\frac{x}{L}u_{X2}, \qquad \delta u=\frac{x}{L}\delta u_{X2}, \qquad \frac{du}{dx}=\frac{u_{X2}}{L}, \qquad \frac{d\delta u}{dx}=\frac{\delta u_{X2}}{L}.$$

$$\Delta\vartheta=2\vartheta^{\circ}-\vartheta^{\circ}=\vartheta^{\circ}$$

- When the approximation to $u(x)$ and the temperature change are substituted there, virtual work density simplifies to

$$\delta w_{\Omega}=-\delta u_{X2}\frac{1}{L}EA\frac{1}{L}u_{X2}+\delta u_{X2}\frac{1}{L}EA\alpha\vartheta^{\circ}.$$

- Integration over the element gives

$$\delta W=-\delta u_{X2}(\frac{EA}{L}u_{X2}-EA\alpha\vartheta^{\circ}).$$

- Principle of virtual work $\delta W=0 \quad \forall \delta \mathrm{a}$ and the fundamental lemma of variation calculus imply the nodal displacement

$$u_{X2}=L\alpha\vartheta^{\circ}.$$ ←