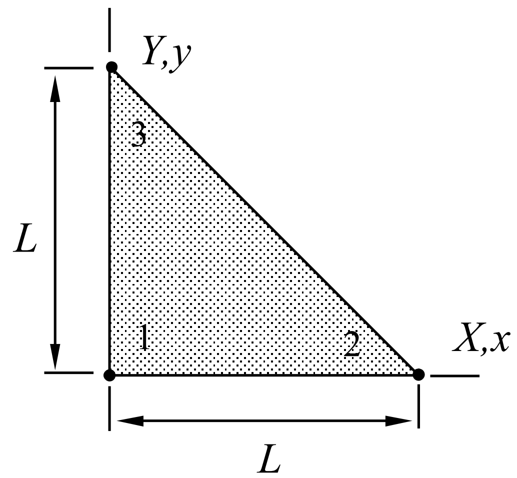
LECTURE ASSIGNMENT 2. In stationary thermo-elasticity, the variational densities of the thin slab mode of the plate model

$$\delta p_{\Omega}^{\text{int}} = -\begin{Bmatrix} \partial\delta\vartheta / \partial x \\ \partial\delta\vartheta / \partial y \end{Bmatrix}^{\mathrm{T}} kt \begin{Bmatrix} \partial\vartheta / \partial x \\ \partial\vartheta / \partial y \end{Bmatrix}, \quad \delta p_{\Omega}^{\text{ext}} = \delta\vartheta s$$

represent the energy balance. Assuming that the thermal conductivity k and thickness t of the element are constants, derive the element contribution δP^{int} for the linear triangle element shown. Temperature at nodes 1 and 3 is known to be $\vartheta°$ and the unknown nodal temperature is ϑ_2.



Name________________________ Student number_________________

- The linear shape functions can be deduced from the figure

$$N_2 = \frac{x}{L}, \qquad N_3 = \frac{y}{L}, \qquad N_1 = 1 - N_2 - N_3 = 1 - \frac{x}{L} - \frac{y}{L}$$

- Approximation to $\vartheta(x,y)$ and its variation $\delta\vartheta(x,y)$ (notice that the variation of a given quantity vanishes)

$$\vartheta = (1 - \frac{x}{L})\vartheta^\circ + \frac{x}{L}\vartheta_2, \qquad \frac{\partial\vartheta}{\partial x} = \frac{1}{L}(\vartheta_2 - \vartheta^\circ), \qquad \frac{\partial\vartheta}{\partial y} = 0$$

$$\delta\vartheta = \frac{x}{L}\delta\vartheta_2, \qquad \frac{\partial\delta\vartheta}{\partial x} = \frac{\delta\vartheta_2}{L}, \qquad \frac{\partial\delta\vartheta}{\partial y} = 0$$

- When the approximation is substituted there, the variational density simplifies to

$$\delta p_\Omega^{\text{int}} = -\frac{\delta\vartheta_2}{L} kt \frac{1}{L}(\vartheta_2 - \vartheta^\circ).$$

- Integration over the element gives

$$\delta P^{\text{int}} = \int_\Omega \delta p_\Omega^{\text{int}} dA = -\delta\vartheta_2 \frac{kt}{2}(\vartheta_2 - \vartheta^\circ). \quad \leftarrow$$