ELEC-C3210 Materiaalien ominaisuudet **4.harjoitus, malliratkaisut**

1. Isobaarinen prosessi ab: $W_{ab} = p_a(V_b - V_a)$ = 90 kJ. Ideaalikaasuyhtälöstä $T = pV/(nR)$ ja yksiatomisen kaasun $C_p = \frac{5}{2}R$, joten $Q_{ab} = nC_p(T_b - T_a) = nC_p\frac{p}{nR}(V_b - V_a) = \frac{5}{2}W_{ab}$ = 225 kJ. $\Delta U_{ab} = Q_{ab} - W_{ab}$ = 135 kJ. Isokoorinen prosessi bc: $dV = 0 \Rightarrow W_{bc} = 0 \Rightarrow$ $\Delta U_{bc} = Q_{bc} = nC_V(T_c - T_b) = nC_V\frac{V}{nR}(p_c - p_b) = \frac{3}{2}V(p_c - p_b)$ = –240 kJ. Koko kiertoprosessi: $W = (p_a - p_c)(V_c - V_a)/2$ = 30 kJ ja koska $\Delta U = 0$, niin Q = 30 kJ. Prosessi ca: $W_{ca} = (p_c + p_a)(V_a - V_c)/2$ = –60 kJ. $\Delta U_{ca} = -\Delta U_{ab} - \Delta U_{bc}$ = 105 kJ. $Q_{ca} = \Delta U_{ca} + W_{ca}$ = 45 kJ.

2. Merkitään: systeemin alkutila a, isobaarinen prosessi ab, adiabaattinen bc ja isokoorinen ca.
a) Isobaarisessa prosessissa p on vakio, joten ideaalikaasuyhtälöstä myös $p = nRT/V$ on vakio $\Rightarrow$ T/V on vakio. Kun $V_b = V_a/2$, niin $T_b = T_a/2$. Adiabaattisessa prosessissa $TV^{\gamma-1}$ on vakio $\Rightarrow$ $T_bV_b^{\gamma-1} = T_cV_c^{\gamma-1}$ $\Rightarrow$ $T_c = T_b(V_b/V_a)^{\gamma-1} = T_a/2\cdot(1/2)^{\gamma-1} = T_a(1/2)^{\gamma} = 114$ K ($\gamma = 1{,}40$). b) Tarkastellaan isokoorista prosessia ca $\Rightarrow$ V on vakio $\Rightarrow$ $V = nRT/p$ on vakio $\Rightarrow$ T/p on vakio $\Rightarrow$ $p_c = p_a T_c/T_a = 0{,}758\cdot 10^5$ Pa.

3. a) Carnot-sykli: isotermiset osat $W_{ab} = \int_{V_1}^{V_2}\frac{kNT}{V}dV = kNT_1\ln(V_2/V_1)$ ja $W_{cd} = kNT_2\ln(V_4/V_3)$
Adiabaattiset: $dQ = 0 \Rightarrow W_{bc} = -\Delta U_{bc} = \frac{3}{2}kN(T_1 - T_2)$ ja $W_{da} = \frac{3}{2}kN(T_2 - T_1)$.
b) $W = kNT_1\ln(V_2/V_1) + kNT_2\ln(V_4/V_3)$, missä $p_1V_1 = p_2V_2$, $p_3V_3 = p_4V_4$, $p_2V_2^{\gamma} = p_3V_3^{\gamma}$ ja $p_4V_4^{\gamma} = p_1V_1^{\gamma}$. $\Rightarrow$ $\frac{p_2}{p_1}\left(\frac{V_2}{V_1}\right)^{\gamma} = \frac{p_3}{p_4}\left(\frac{V_3}{V_4}\right)^{\gamma}$ $\Rightarrow$ $\frac{V_1}{V_2}\left(\frac{V_2}{V_1}\right)^{\gamma} = \frac{V_4}{V_3}\left(\frac{V_3}{V_4}\right)^{\gamma}$ $\Rightarrow$ $\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$ $\Rightarrow$ $\frac{V_2}{V_1} = \frac{V_3}{V_4}$ $\Rightarrow$ $W = kN[T_1 - T_2]\ln\left(\frac{V_2}{V_1}\right)$. c)
$$e = \frac{W}{Q_{ab}} = \frac{W}{W_{ab}} = \frac{kN(T_1 - T_2)\ln(V_2/V_1)}{kNT_1\ln(V_2/V_1)} = \frac{T_1 - T_2}{T_1}.$$

4. a) Kappaleiden entropian muutokset $\Delta S_1 = \int_1^{1'}\frac{dQ}{T} = m_1c_1\int_1^{1'}\frac{dT}{T} = m_1c_1\ln\frac{T}{T_1}$, ja

$\Delta S_2 = \int_2^{2'} \frac{dQ}{T} = m_2 c_2 \ln \frac{T'}{T_2}$. Eristetty systeemi $\Rightarrow \quad Q_1 = -Q_2 \quad \Rightarrow$

$m_1 c_1 (T - T_1) = -m_2 c_2 (T' - T_2) \quad \Rightarrow \quad T' = T_2 - \frac{m_1 c_1}{m_2 c_2}(T - T_1) \quad \Rightarrow$

$\Delta S_{tot} = m_1 c_1 \ln \frac{T}{T_1} + m_2 c_2 \ln \frac{T'}{T_2} \; = \; m_1 c_1 \ln \frac{T}{T_1} + m_2 c_2 \ln \left[1 - \frac{m_1 c_1}{m_2 c_2} \frac{T - T_1}{T_2} \right].$

b) Lausekkeen $\Delta S_{tot}(T, T')$ derivaatan 0-kohdasta:

$\frac{d(\Delta S_{tot})}{dT} = m_1 c_1 \cdot \frac{T_1}{T} \cdot \frac{1}{T_1} + m_2 c_2 \cdot \frac{T_2}{T'} \cdot \frac{1}{T_2} \cdot \frac{dT'}{dT} = 0, \quad \frac{dT'}{dT} = -\frac{m_1 c_1}{m_2 c_2} \quad \Rightarrow$

$T = T' = \frac{m_1 c_1 T_1 + m_2 c_2 T_2}{m_2 c_2 + m_1 c_1}$.