

Aalto University
Department of Information and Computer Science
Christopher Purcell

CS-E4530 Computational Complexity Theory (5 cr)
First Midterm Exam, Wed 1 Feb 2017, 10.00–12.00

Write down on each answer sheet:

- Your name, degree programme, student number, and the date
- The course title: “CS-E4530 Computational Complexity Theory”

Note: You can write down your answers in Finnish, Swedish or English.

1. (a) Design (i.e. give the transition diagram for) a Turing machine M that removes trailing 0's from a binary input string, i.e. computes the following function $f : \{0, 1\}^* \rightarrow \{0, 1\}^*$:

$$f(x) = \begin{cases} y1 & \text{if } x = y10^k \text{ for some } y \in \{0, 1\}^*, k \geq 0, \\ \varepsilon & \text{if } x = 0^k \text{ for some } k \geq 0. \end{cases}$$

where ε denotes the empty string. (For instance, $f(10100) = 101$ and $f(000) = \varepsilon$.)

- (b) Give the computation sequences of your machine, i.e. the lists of configurations the machine passes through until it halts, on inputs 010, 00, and ε .
2. Which of the following claims are true and which are false? (No proofs are needed, just indicate your choice by the letter T or F.)
- (a) The computation of a deterministic Turing machine halts on every input.
 - (b) All languages accepted by deterministic Turing machines are recursive.
 - (c) Nondeterministic Turing machines can accept also nonrecursive languages.
 - (d) The complement of any language decided by a Turing machine is recursively enumerable.
 - (e) The intersection of any two recursively enumerable languages is recursive.
 - (f) The problem of determining if a Turing machine has at least 7 states is undecidable.
 - (g) The problem of determining if a Turing machine accepts at least 7 strings is undecidable.
 - (h) A problem A can be shown to be undecidable by devising a reduction mapping t from A to the Halting Problem.
3. (a) Define the formal language L_7 representing the decision problem:
- Given a Turing machine M ; does M accept all strings x of length $|x| \leq 7$,
and only those?
- (b) Prove, without appealing to Rice's theorem, that the language L_7 is not recursive.

Grading: Each problem 4p, total 12p.