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CS-E4530 Computational Complexity Theory (5 cr)
First Midterm Exam, Mon 18 Feb 2019, 9–12 a.m.

Write down on each answer sheet:
- Your name, degree programme, and student number
- The text: "CS-E4530 Computational Complexity Theory 18.2.2019"
- The total number of answer sheets you are submitting for grading

Note: You can write down your answers in either Finnish, Swedish, or English.

1. Which of the following claims are true and which are false? (No proofs are needed, just indicate your choice by the letter T or F.)
   (a) The computation of a deterministic Turing machine halts on every input.
   (b) The complement of any decidable language is semidecidable.
   (c) The intersection of any two semidecidable languages is decidable.
   (d) The problem of determining if a Turing machine accepts at least 7 strings is undecidable.
   (e) The problem of determining if a Turing machine has at least 7 states is undecidable.
   (f) The problem of determining if a Turing machine runs for at least 7 steps on all inputs of length $|x| \leq 7$ is undecidable.
   (g) The Turing machine Halting Problem belongs to the class NP.
   (h) All problems in the complexity class NP can be reduced to the Turing machine Halting Problem. *2p.*

2. Prove that the complexity class NP is closed under unions and intersections. *2p.*

3. Prove that the following decision problem **TMSAT** is NP-complete:
   - **Instance:** A tuple $(\alpha, x, 1^n, 1^t)$, where $\alpha, x \in \{0,1\}^*$
   - **Question:** Is there a string $u \in \{0,1\}^*$ with $|u| \leq n$ such that the Turing machine M_α outputs 1 on input (x,u) within t steps? $(*)$
   - $\mathsf{TMSAT} = \big\{(\alpha, x, 1^n, 1^t)\colon \text{Condition } (*) \text{ holds for } (\alpha, x, 1^n, 1^t)\big\}$ *3p.*

4. (a) Define the language L_{ne} representing the decision problem:

   Given a Turing machine M; does M accept *some* input string, i.e. is the language accepted by M nonempty?

   (b) Prove, by a reduction from the Halting Problem, that the language L_{ne} is not decidable. Is the language semidecidable? (Justify your answer.) *3p.*

Total 10p.