Aalto University
Department of Computer Science
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CS-E4530 Computational Complexity Theory (5 cr)
Second Midterm Exam, Tue 4 Apr 2017, 1–4 p.m.

Write down on each answer sheet:
- Your name, degree programme, and student number
- The text: "CS-E4530 Computational Complexity Theory 4.4.2017"
- The total number of answer sheets you are submitting for grading

Note: You can write down your answers in either Finnish, Swedish, or English.

1. Order the complexity classes **L**, **NP**, **NL**, **PSPACE**, **PH**, **P**, **RP**, Σ_2^p, and **NC**, by set inclusion (that is, write enough set inclusion statements of the form

   $$X \subseteq Y$$

   where X and Y are complexity classes given above such that all known set inclusions follow from the statements).

2. (a) Define the complexity classes Σ_i^p ($i \geq 0$) and **PH**.

   (b) Give an example of some "nontrivial" language L in the class Σ_2^p (i.e. one which is not obviously in Σ_1^p or Π_1^p). Show that L is in Σ_2^p, and explain why it does not seem to be in $\Sigma_1^p \cup \Pi_1^p$.

3. (a) For a complexity class $\mathcal{C}$ give the definition of a $\mathcal{C}$-complete language.

   (b) Define what it means for $\mathcal{C}$ to be closed under reductions.

   (c) Let L be an **NP**-complete language. Show that $L \in \mathbf{P}$ if and only if $\mathbf{P} = \mathbf{NP}$.

4. Show that the following problem is **NP**-complete:

   LONGEST PATH
   INSTANCE: An undirected graph $G = (V, E)$ and an integer K.
   QUESTION: Does G contain a simple path (that is, a path encountering no vertex more than once) with K or more edges?
   You may use the **NP**-complete problems given in the lectures.

Grading: Each problem 6p, total 24p.