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CS-E4530 Computational Complexity Theory (5 cr)
Second Midterm Exam, Tue 9 Apr 2019, 1–4 p.m.

Write down on each answer sheet:
- Your name, degree programme, and student number
- The text: "CS-E4530 Computational Complexity Theory 9.4.2019"
- The total number of answer sheets you are submitting for grading

Note: You can write down your answers in either Finnish, Swedish, or English.

1. Order the complexity classes P, $\mathsf{NPSPACE}$, NP, EXP, RP, PSPACE, Σ_1^p, NL and ZPP by set inclusion (that is, write enough set inclusion statements of the form

   $$X \subseteq Y$$

   where X and Y are complexity classes given above such that all known set inclusions follow from the statements). *6p.*

2. Show that if $\mathsf{P} = \mathsf{NP}$, then $\mathsf{EXP} = \mathsf{NEXP}$. *8p.*

3. (a) Define the complexity classes Σ_k^p ($k \geq 0$) and PH.

   (b) Prove that $\mathsf{PH} \subseteq \mathsf{PSPACE}$. (*Hint:* Induction.) *8p.*

4. Show that the following problem is NP-complete:

   At-Least-2-SAT
   Instance: A Boolean formula in conjunctive normal form (general CNF, i.e. not restricted to k-CNF for any fixed k) such that in each clause all the literals are distinct. (Thus, a clause such as e.g. $(x \vee \neg y \vee \neg y \vee z)$ is not allowed.)
   Question: Is there a truth assignment τ to the variables in the formula such that for each clause *at least two* literals in it are satisfied by τ? *8p.*

Total 30p.