Aalto University
Department of Computer Science
Pekka Orponen

T-79.5103 Computational Complexity Theory (5 cr)
First Midterm Exam, Mon 2 Feb 2015, 10–12 a.m.

Write down on each answer sheet:
- Your name, degree programme, and student number
- The text: "T-79.5103 Computational Complexity Theory 2.2.2015"
- The total number of answer sheets you are submitting for grading

Note: You can write down your answers in either Finnish, Swedish, or English.

1. (a) Design (i.e. give the transition diagram for) a Turing machine M that computes the following function $f : \{0,1\}^* \longrightarrow \{0,1\}^*$:

$$f(x) = \begin{cases} 0 & \text{if the number of 1's in } x \text{ is even (incl. zero),} \\ 1 & \text{if the number of 1's in } x \text{ is odd.} \end{cases}$$

   Thus, for instance, $f(10100) = 0$, $f(01011) = 1$, and $f(\varepsilon) = 0$.

   (b) Give the computation sequences of your machine, i.e. the lists of configurations the machine passes through until it halts, on inputs 101, 0, and ε.

2. Which of the following claims are true and which are false? (No proofs are needed, just indicate your choice by the letter T or F.)

   (a) All languages accepted by deterministic Turing machines are recursive.

   (b) All languages accepted by nondeterministic Turing machines are recursively enumerable.

   (c) The complement of any recursive language is recursively enumerable.

   (d) The intersection of any two recursively enumerable languages is recursive.

   (e) The Turing machine Halting Problem belongs to the complexity class NP.

   (f) A problem A can be shown to be undecidable by devising a reduction mapping t from A to the Halting Problem.

   (g) The problem of determining if a Turing machine accepts all inputs x of length $|x| \leq 100$ is undecidable.

   (h) The problem of determining if a Turing machine runs for at least 100 steps on the input ε (empty string) is undecidable.

3. (a) Define the formal language L_{all} representing the decision problem:

   Given a Turing machine M; does M accept all possible input strings?

   (b) Prove, without appealing to Rice's theorem, that the language L_{all} is not recursive.

Grading: Each problem 4p, total 12p.