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T-79.5103 Computational Complexity Theory (5 cr)
First Midterm Exam, Mon 1 Feb 2016, 10–12 a.m.

Write down on each answer sheet:
- Your name, degree programme, and student number
- The text: "T-79.5103 Computational Complexity Theory 1.2.2016"
- The total number of answer sheets you are submitting for grading

Note: You can write down your answers in either Finnish, Swedish, or English.

1. (a) Design (i.e. give the transition diagram for) a Turing machine M that computes the following function $f : \{1\}^* \longrightarrow \{1\}^*$:

$$f(x) = \begin{cases} 1^{n-1} & \text{if } x = 1^n \text{ and } n \text{ is odd} \\ \varepsilon & \text{otherwise} \end{cases}$$

   where ε denotes the empty string. Thus, for instance, $f(111) = 11$ and $f(11) = \varepsilon$.

   (b) Give the computation sequences of your machine, i.e. the lists of configurations the machine passes through until it halts, on inputs 111, 11, and ε.

2. Which of the following claims are true and which are false? (No proofs are needed, just indicate your choice by the letter T or F.)

   (a) The computation of a deterministic Turing machine halts on every input.

   (b) All languages accepted by nondeterministic Turing machines are recursively enumerable.

   (c) The Turing machine Halting Problem belongs to the complexity class NP.

   (d) The complement of any language accepted by a deterministic Turing machine is recursively enumerable.

   (e) A problem A can be shown to be undecidable by devising a reduction mapping t from A to the Halting Problem.

   (f) The problem of determining if a Turing machine accepts at least 7 strings is undecidable.

   (g) The problem of determining if a Turing machine has at least 7 states is undecidable.

   (h) The problem of determining if a Turing machine runs for at least 7 steps on all inputs of length $|x| \leq 7$ is undecidable.

3. (a) Define the formal language L_7 representing the decision problem:

   Given a Turing machine M; does M accept all strings x of length $|x| \leq 7$, and only those?

   (b) Prove, without appealing to Rice's theorem, that the language L_7 is not recursive.

Grading: Each problem 4p, total 12p.