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T-79.5103 Computational Complexity Theory (5 cr)
Second Midterm Exam, Mon 7 Apr 2014, 9–12 a.m.

Write down on each answer sheet:
- Your name, degree programme, and student number
- The text: "T-79.5103 Computational Complexity Theory 7.4.2014"
- The total number of answer sheets you are submitting for grading

Note: You can write down your answers in either Finnish, Swedish, or English.

1. Order the complexity classes **RP**, **EXP**, Σ_1^p, **L**, **NP**, **ZPP**, **NPSPACE**, **P**, and **PSPACE** by set inclusion (that is, write enough set inclusion statements of the form

$$X \subseteq Y$$

where X and Y are complexity classes given above such that all known set inclusions follow from the statements).

2. (a) Define the complexity classes **DP** and Δ_2^p, and show that $\mathbf{DP} \subseteq \Delta_2^p$.
   (b) Give an example of some "nontrivial" language L in the class **DP** (i.e. such that L is not obviously in either **NP** or **coNP**), and explain why it belongs to **DP**.

3. (a) Define the complexity classes Σ_i^p ($i \geq 0$) and **PH**.
   (b) Prove that $\mathbf{PH} \subseteq \mathbf{PSPACE}$. (*Hint:* Induction.)

4. Show that the following problem is **NP**-complete:

   DOMINATING SET
   INSTANCE: An undirected graph $G = (V, E)$ and an integer K.
   QUESTION: Does G contain a set of at most K vertices $U \subseteq V$, such that any vertex not in U is connected to a vertex in U by an edge (i.e. for any $v \in V \setminus U$ there is some $u \in U$ such that $\{u, v\} \in E$)?

   (*Hint:* Think of the vertices in $V \setminus U$ as being "covered" by vertices in U.)

Grading: Each problem 6p, total 24p.