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Department of Computer Science
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T-79.5103 Computational Complexity Theory (5 cr)
Second Midterm Exam, Tue 7 Apr 2015, 1–4 p.m.

Write down on each answer sheet:
- Your name, degree programme, and student number
- The text: "T-79.5103 Computational Complexity Theory 7.4.2015"
- The total number of answer sheets you are submitting for grading

Note: You can write down your answers in either Finnish, Swedish, or English.

1. Order the complexity classes **AP**, **EXP**, Π_2^p, **NC**, **NP**, **NL**, **RP**, **NPSPACE**, and **P**, by set inclusion (that is, write enough set inclusion statements of the form

   $$X \subseteq Y$$

   where X and Y are complexity classes given above such that all known set inclusions follow from the statements).

2. Prove that the following REACHABILITY problem is in complexity class **NL**: Given a graph $G = (V,E)$ and vertices $s,t \in V$, is there a path from s to t in G?

3. (a) Define the concepts of an oracle Turing machine and the relativised complexity classes $\mathbf{P}^A$ and $\mathbf{NP}^A$ for an "oracle language" A.

   (b) Prove that there exists an oracle language A for which $\mathbf{P}^A = \mathbf{NP}^A$.

4. Show that the following problem is NP-complete:

   DIRECTED CYCLE COVER
   INSTANCE: A directed graph $G = (V,E)$ and an integer K.
   QUESTION: Is there a subset of at most K arcs (directed edges) $F \subseteq E$, such that any directed cycle in the graph G contains at least one arc from the set F?[1]

   (*Hint:* Reduction from VERTEX COVER; represent vertices by arcs.)

Grading: Each problem 6p, total 24p.

[1] In other words, given a directed graph G, what is the smallest number of arcs that need to be removed to make it acyclic?