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T-79.5103 Computational Complexity Theory (5 cr)
Second Midterm Exam, Tue 5 Apr 2016, 1–4 p.m.

Write down on each answer sheet:
- Your name, degree programme, and student number
- The text: "T-79.5103 Computational Complexity Theory 5.4.2016"
- The total number of answer sheets you are submitting for grading

Note: You can write down your answers in either Finnish, Swedish, or English.

1. Order the complexity classes **PH**, $\mathbf{NC}_2$, **ZPP**, $\mathbf{P}^{\mathbf{PP}}$, **AP**, $\mathbf{NP} \cap \mathbf{coNP}$, **PSPACE**, **NL**, and **P**, by set inclusion (that is, write enough set inclusion statements of the form

$$X \subseteq Y$$

where X and Y are complexity classes given above such that all known set inclusions follow from the statements).

2. (a) Define the complexity classes Σ_i^p ($i \geq 0$) and **PH**.

   (b) Give an example of some "nontrivial" language L in the class Σ_2^p (i.e. one which is not obviously in Σ_1^p or Π_1^p). Show that L is in Σ_2^p, and explain why it does not seem to be in $\Sigma_1^p \cup \Pi_1^p$.

3. Let us define the following linear-exponential time complexity classes:

$$\mathbf{E} = \bigcup_{c>0} \mathbf{TIME}(2^{cn}), \qquad \mathbf{NE} = \bigcup_{c>0} \mathbf{NTIME}(2^{cn}).$$

Show that if $\mathbf{P} = \mathbf{NP}$, then also $\mathbf{E} = \mathbf{NE}$. You may assume, without loss of generality, that all the languages to be considered are over the binary alphabet. (*Hint:* For any language $L \in \mathbf{NE}$, consider the language $T_L = \{1^{n(1x)} \mid x \in L\}$, where $n(1x)$ denotes the numerical value of the binary string $1x$.)

4. Show that the following problem is NP-complete:

CLIQUE COVER
INSTANCE: An undirected graph $G = (V, E)$ and an integer K.
QUESTION: Can the vertices of G be partitioned into at most K sets $V_1, \ldots, V_K$, so that each V_i forms a clique in G? (*Hint:* Reduction from GRAPH COLOURING.)

Grading: Each problem 6p, total 24p.