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Origami and beyond

Crystal Flowers 14.2. 2019

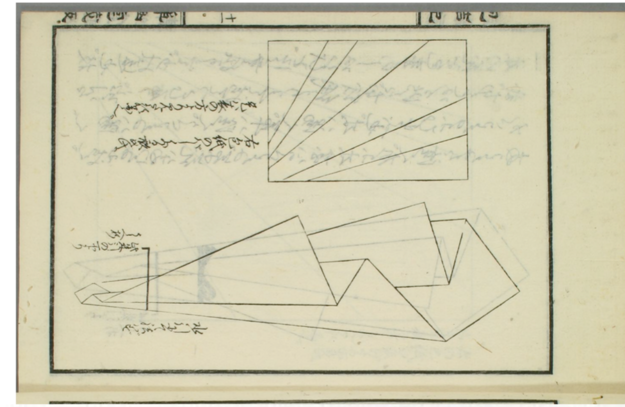
Kirsi Peltonen

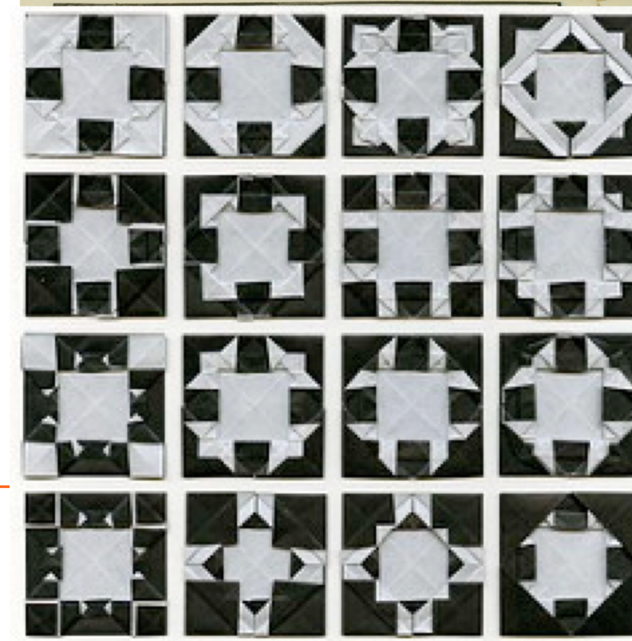
Folds by Megan McGlynn (Shapes in Action 2018)

Where did origami come from ?

‘*oru*’ = to fold , ‘*kami*’ =paper vs.

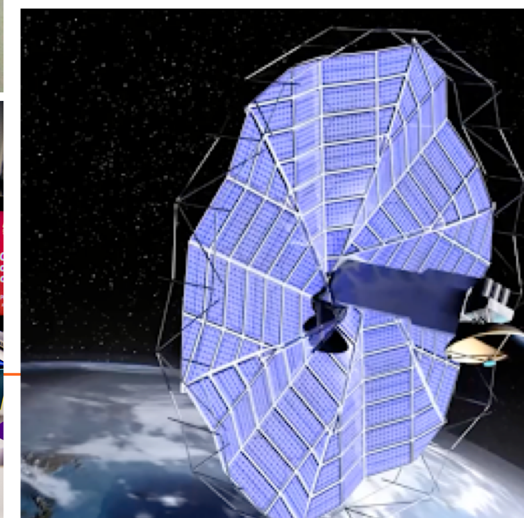
‘*origami tsuki*’ = certificate



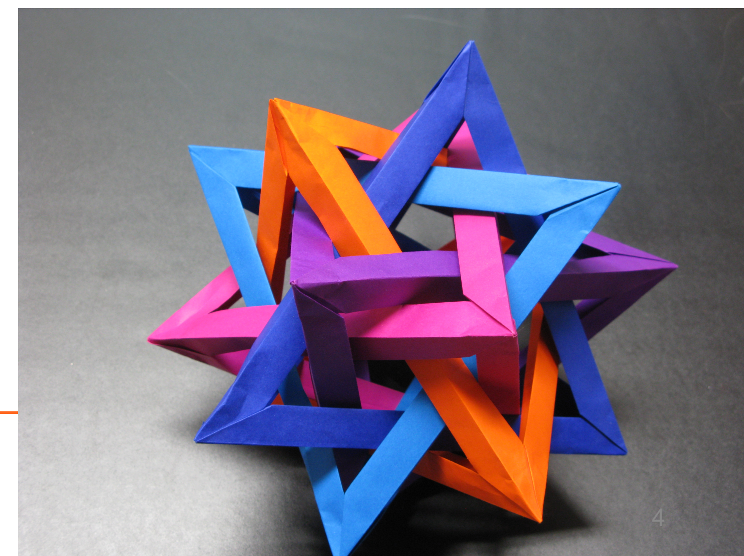


Why should we care about origami/folding ?

- Art/crafts form
- Architecture
- Educational tool
- Subfield of mathematics
- Engineering applications
- Material science
- Medical implants, ….

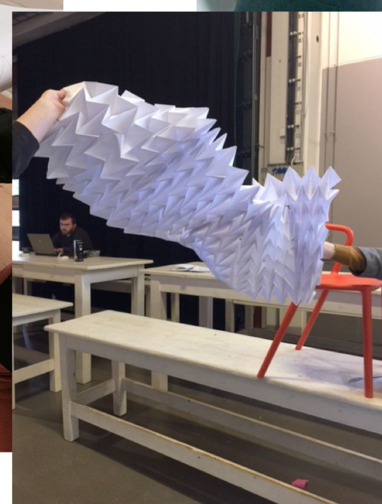


Origami and Crystal Flowers 2015, 2017



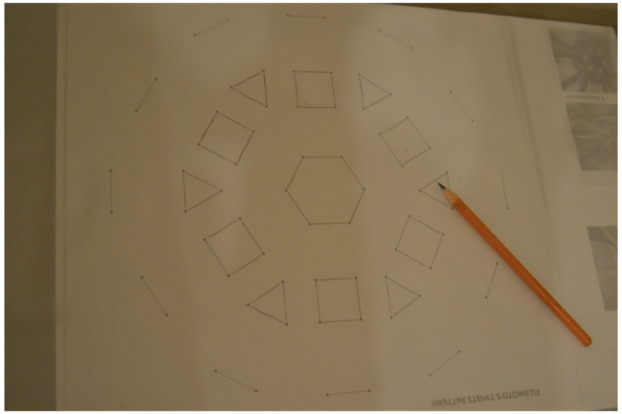
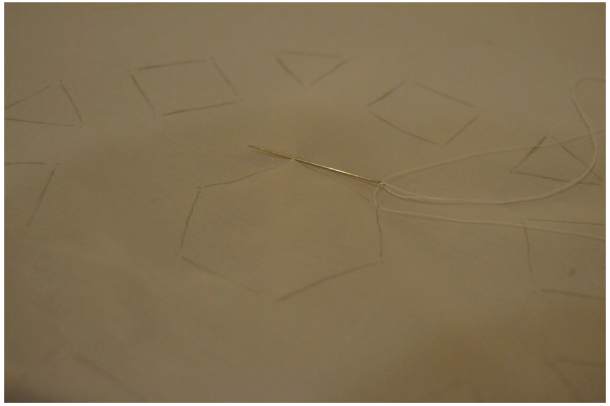
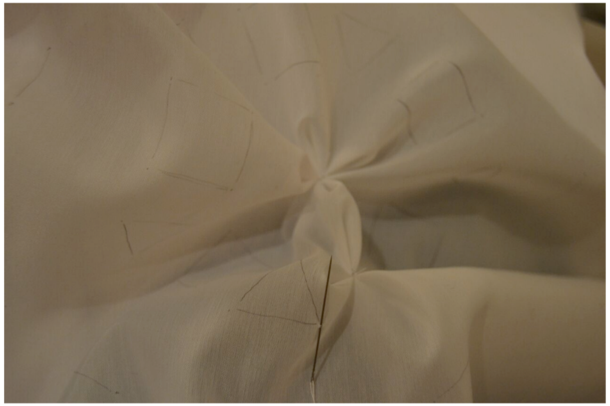
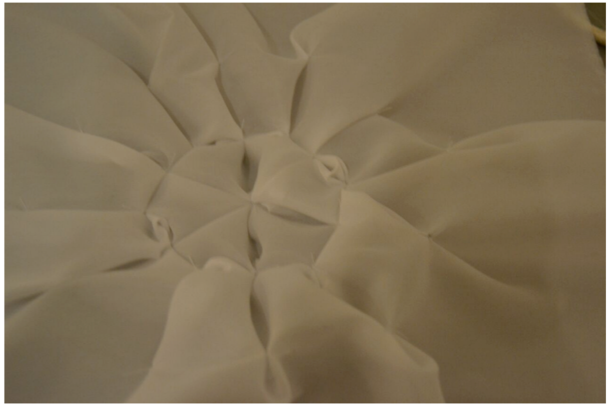
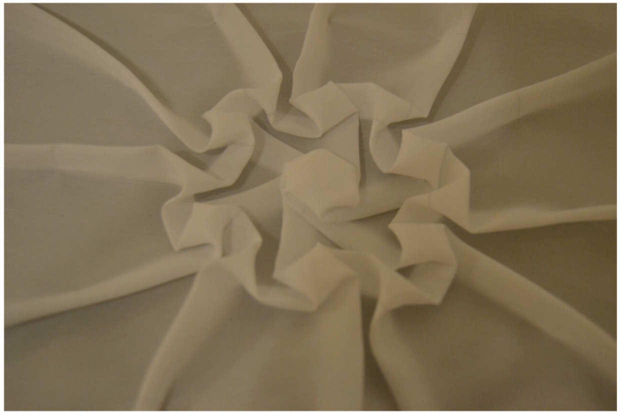
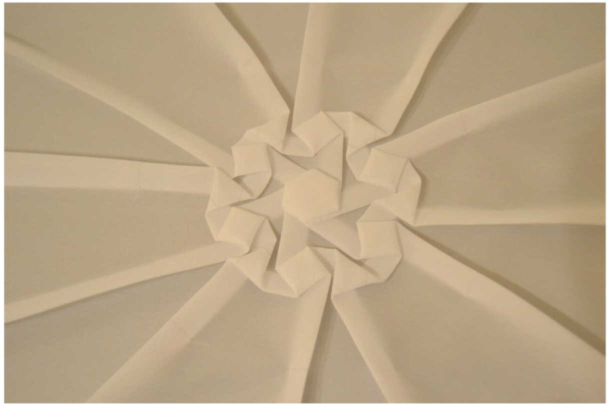
Origami/Folding from different perspectives

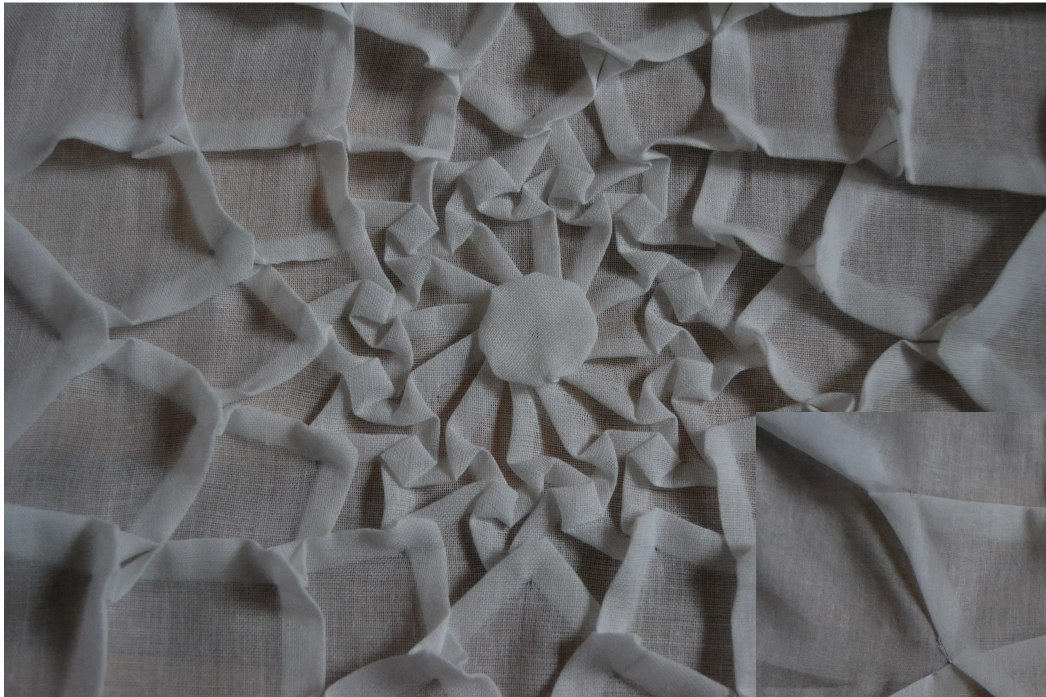
Paper tesselation by C. Bennes (ARTS), T. Hyppölä (CHEM), L. Laatu (ELEC), L. Lazarov (SCI), M. Taponen (ARTS), J. Rinta-Mänty (ARTS)



Crystal Flowers 2015

Fujimoto twist by Liisi Huotari (ARTS)



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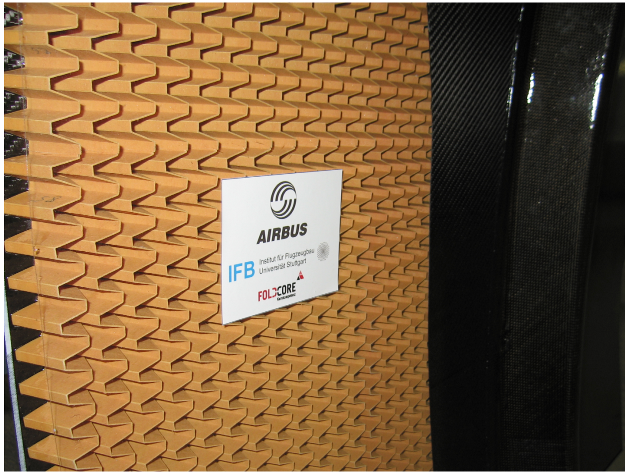
Meri Tuomela (ARTS)

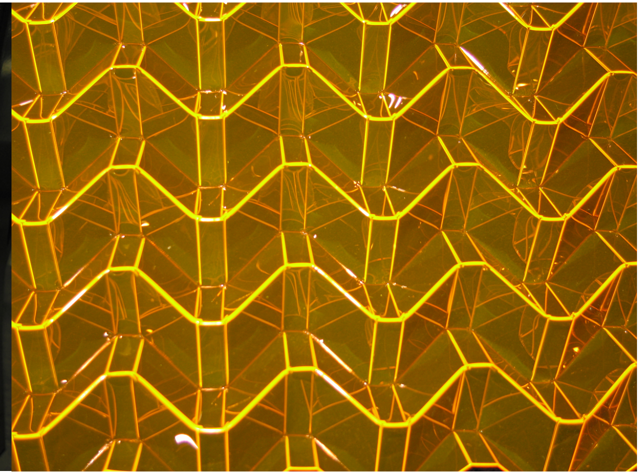
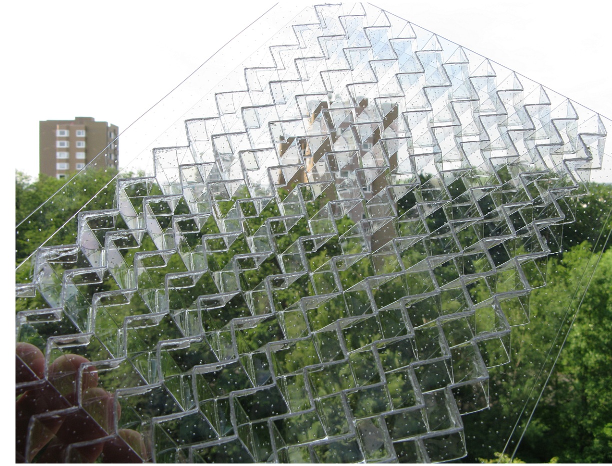
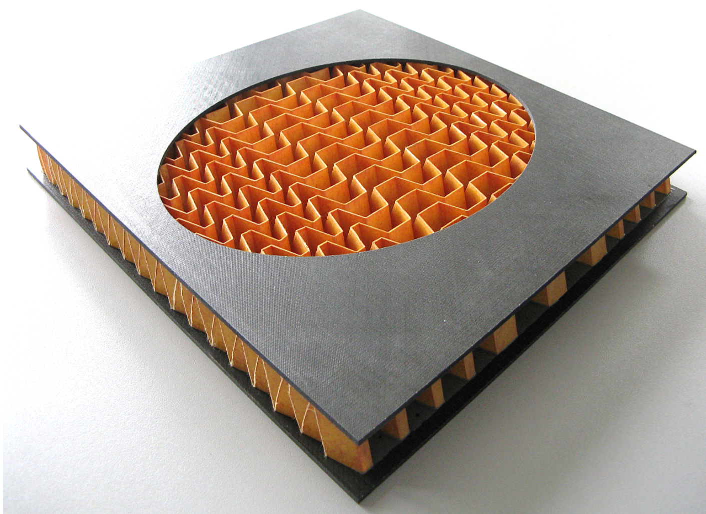
‘Fold, tye and dye’
Workshop by Laura Isoniemi

Riikka Schroderus (HU, Math)

Yves Klett : engineering perspective

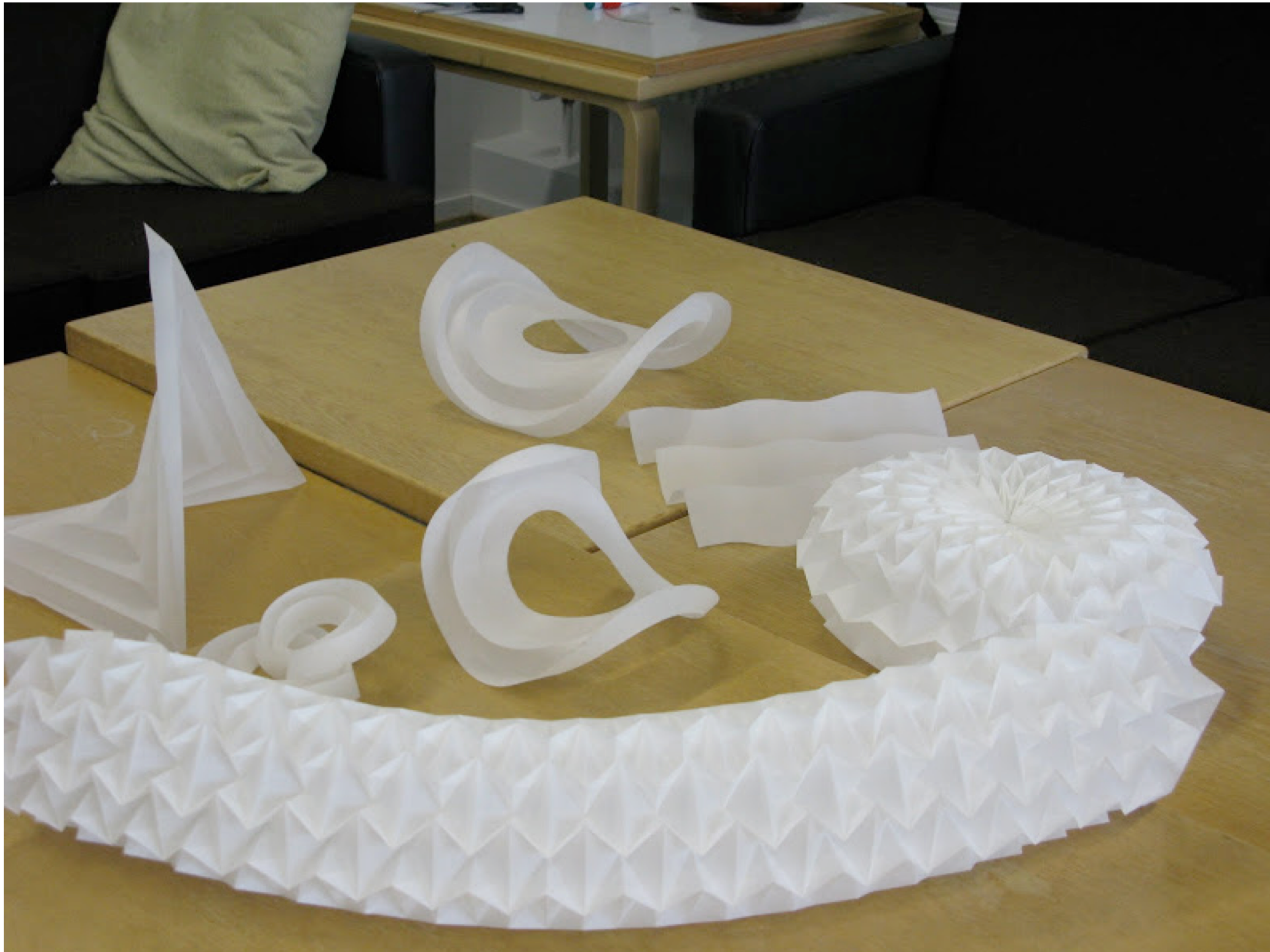
https://www.youtube.com/watch?v=xh6UNYjjjUA

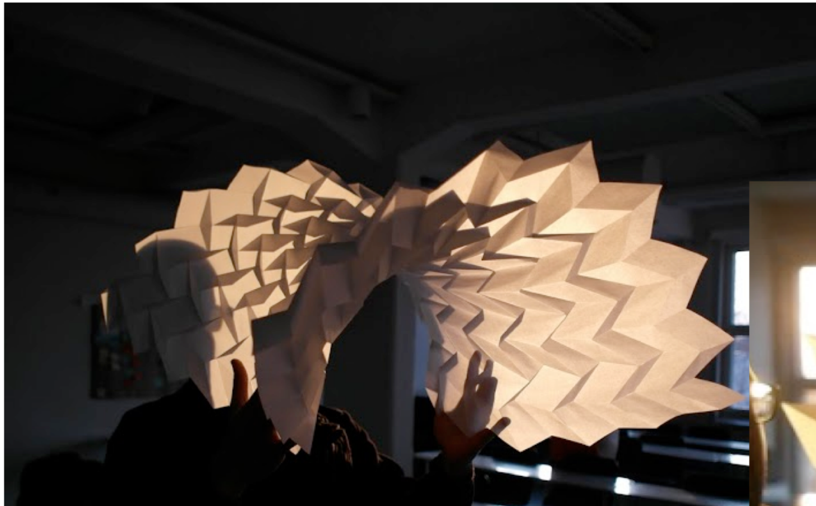


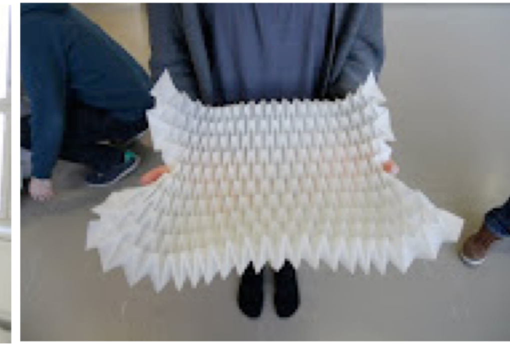
- Origami / Folding *is* important
- But it is also a lot of fun!
- There are many useful applications
- Most of which have not yet even been discovered!

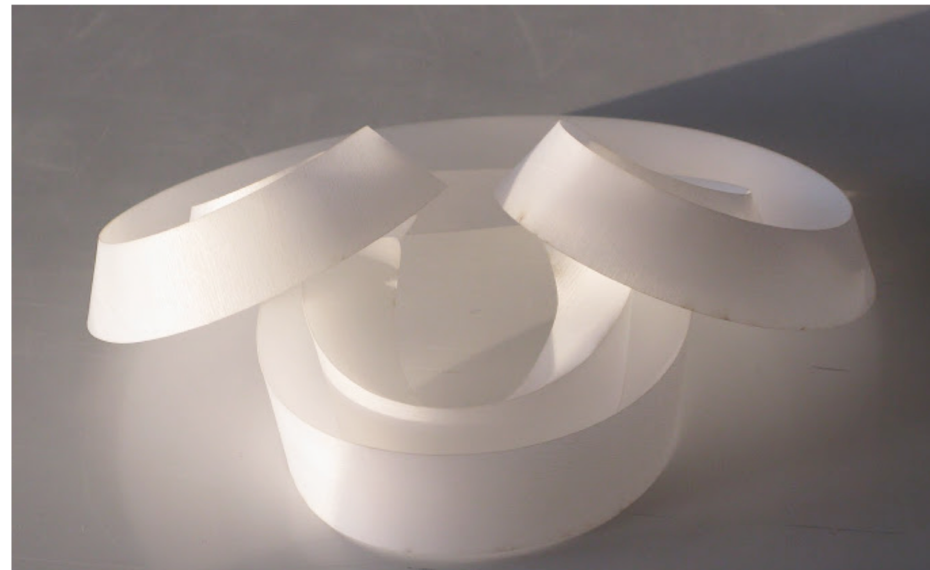
Laser cutting and folding in Aalto Arabia











Miura cube from cardboard

Tristan Hamel (Aalto ARTS)

Lauri Tervonen (Aalto SCI)

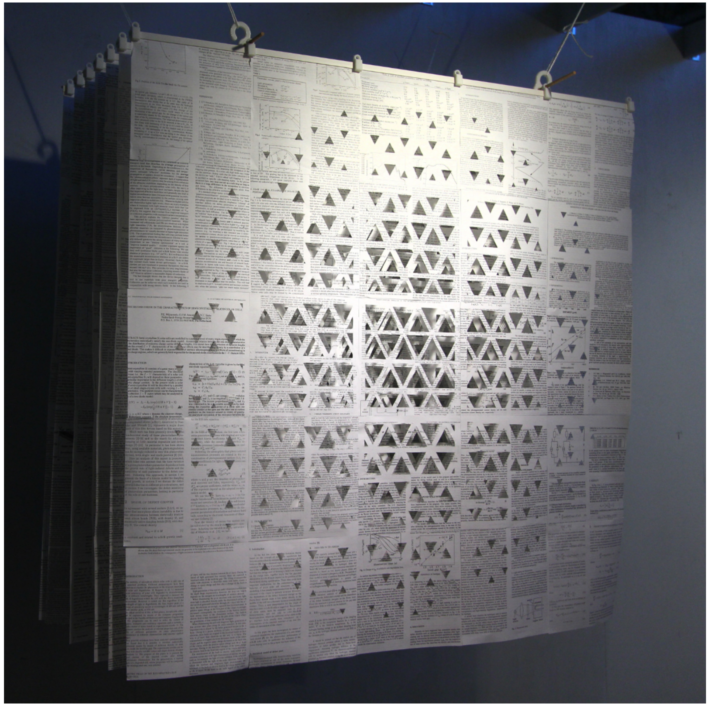
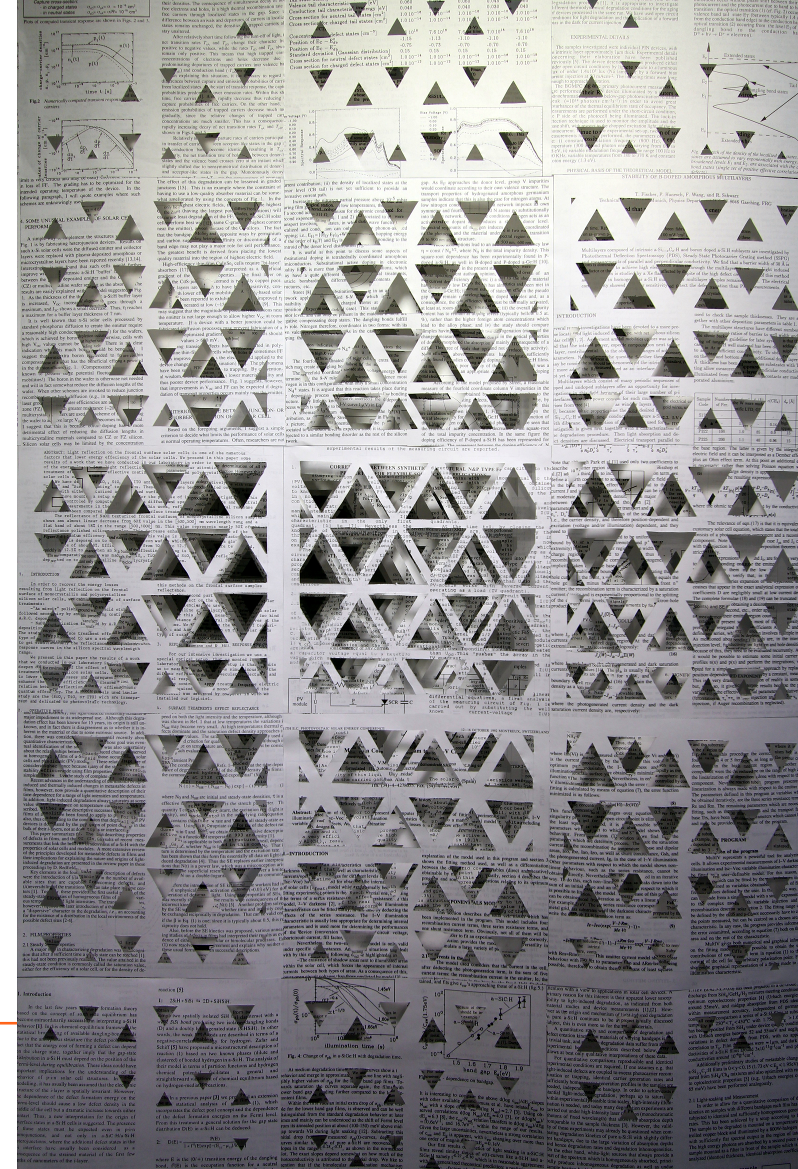


Photo: Lauri Tervonen, Habitare

Aaltoliike project (conceptual design teaching) Laura Isoniemi



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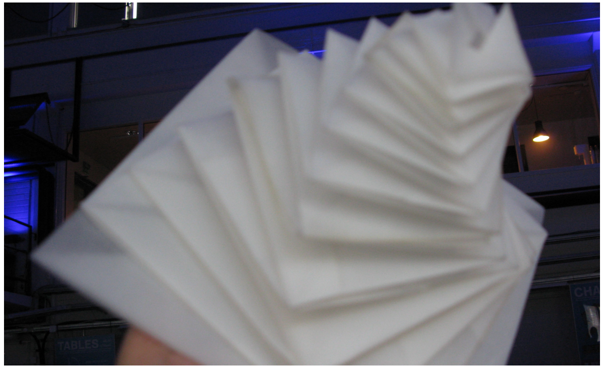
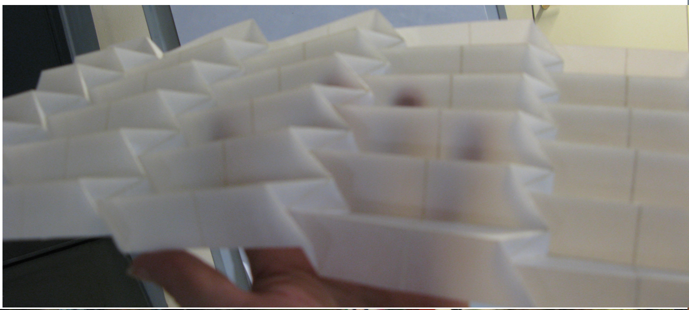
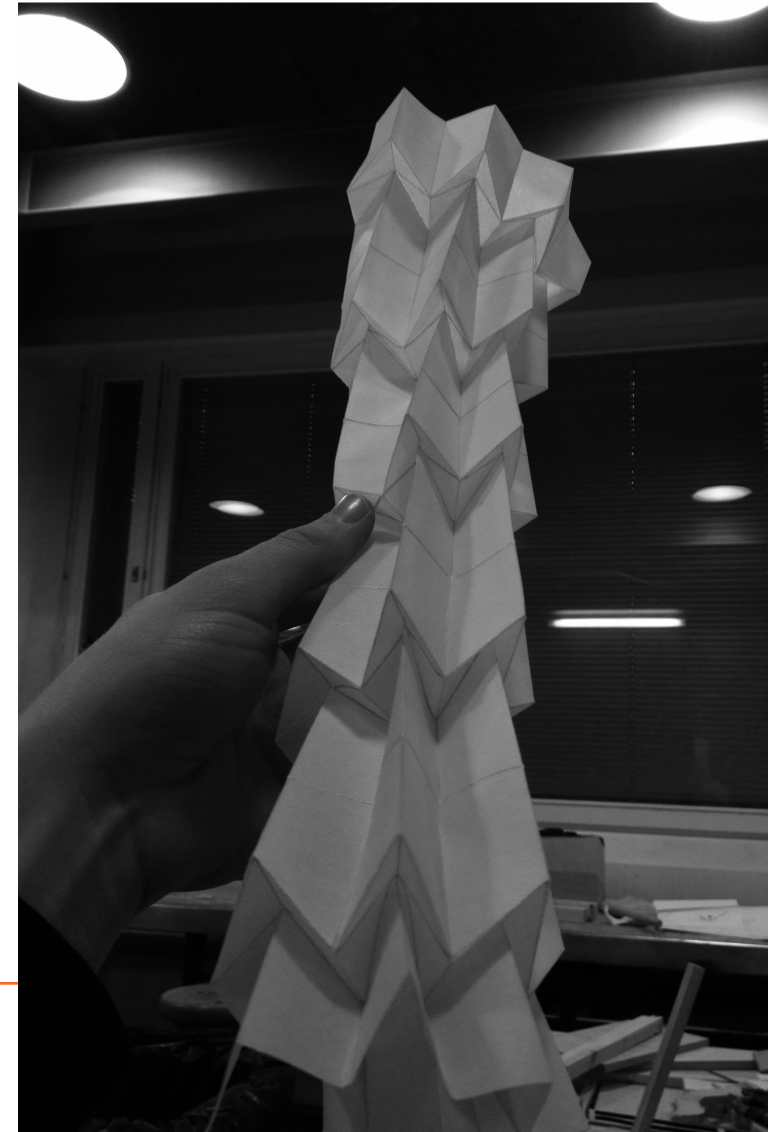
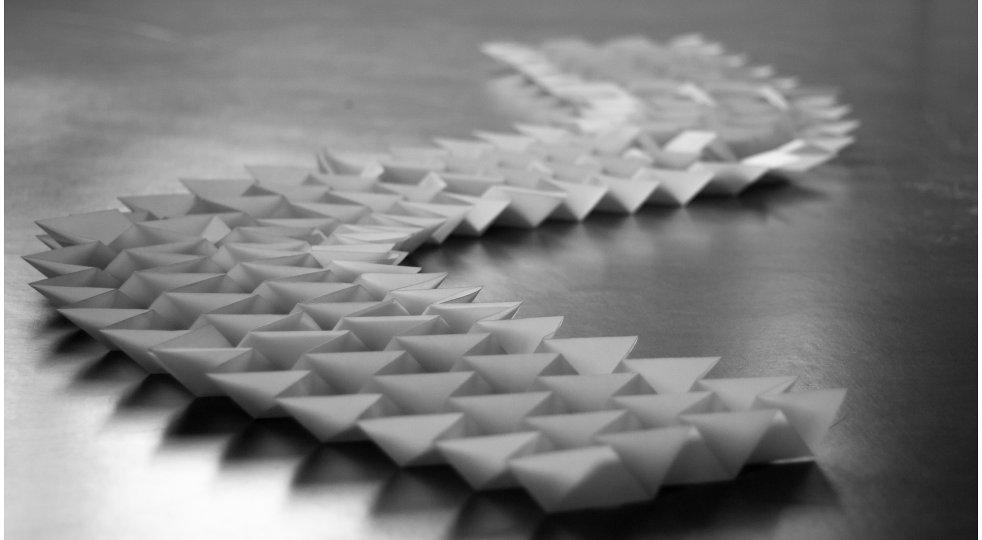
Origami in Crystal Flowers 2017







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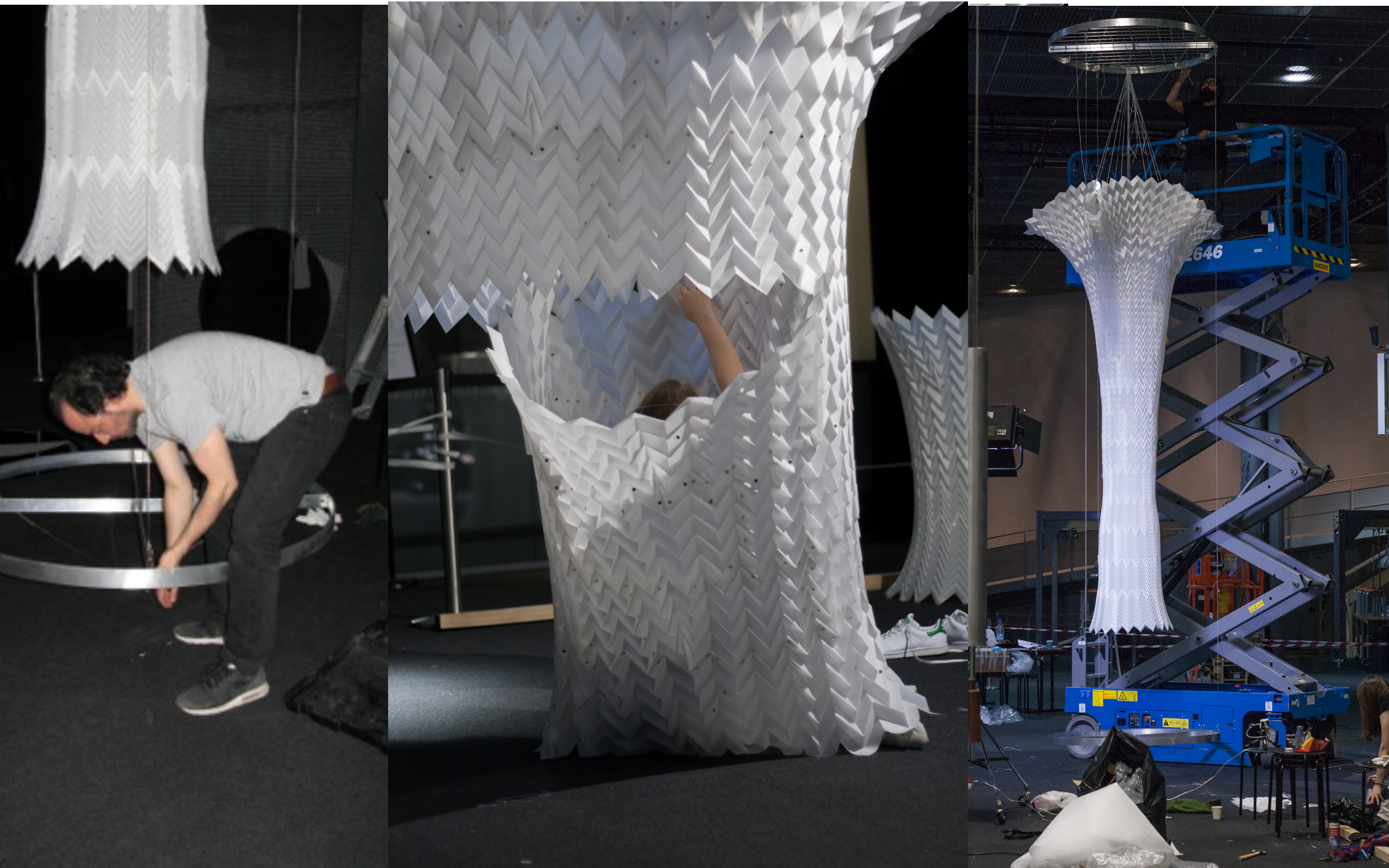
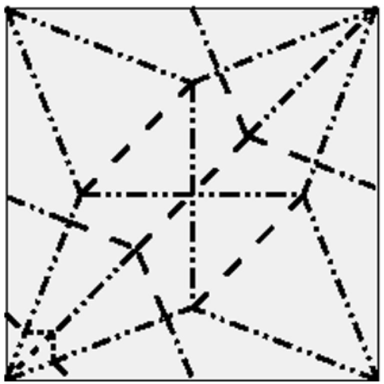
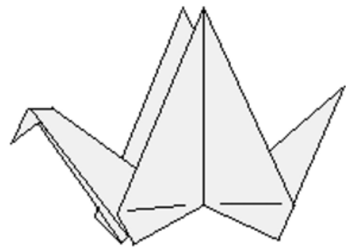
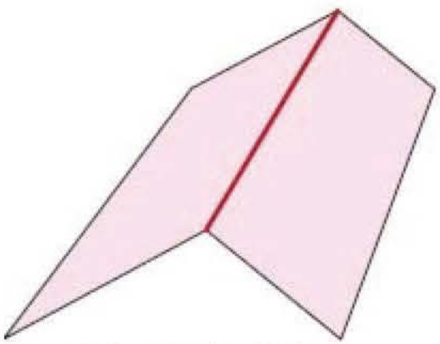


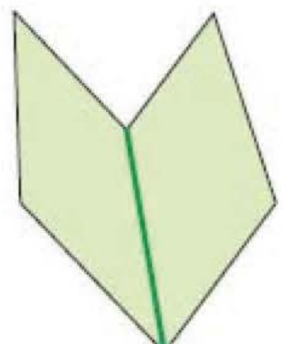
On flat foldable origami



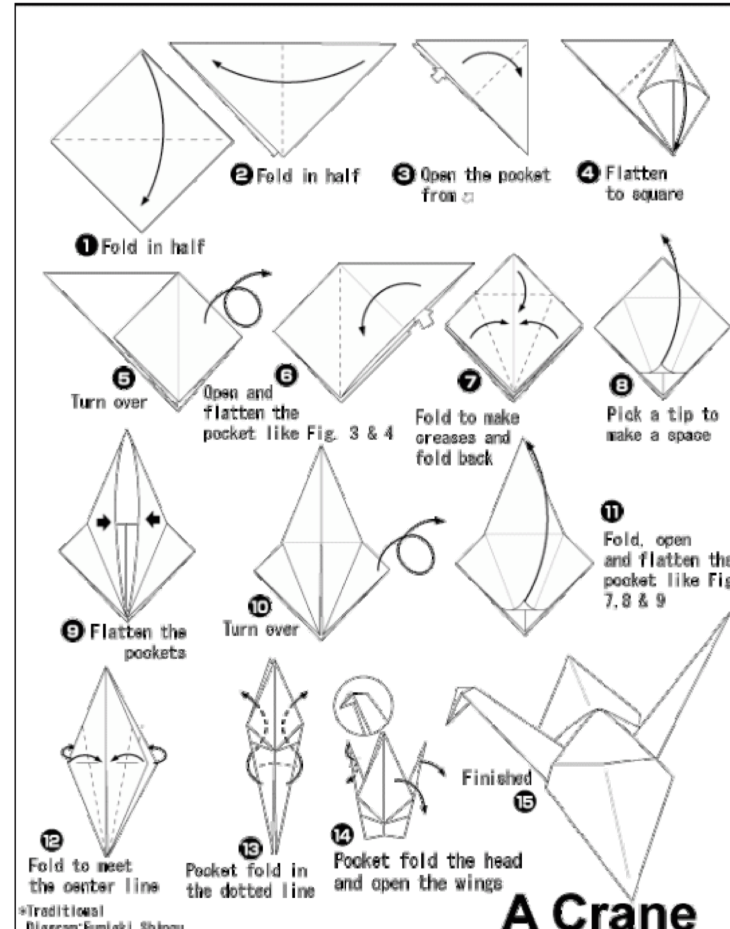




Mountain fold



Valley fold



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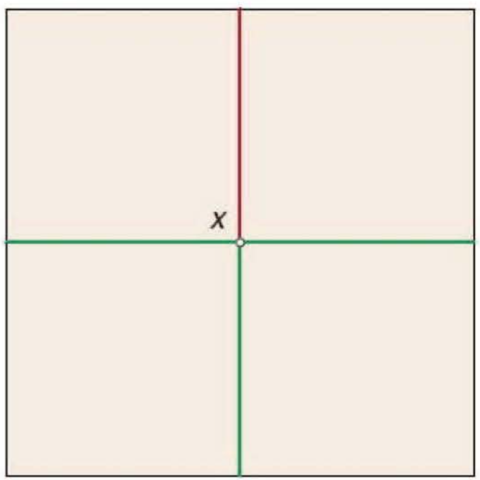
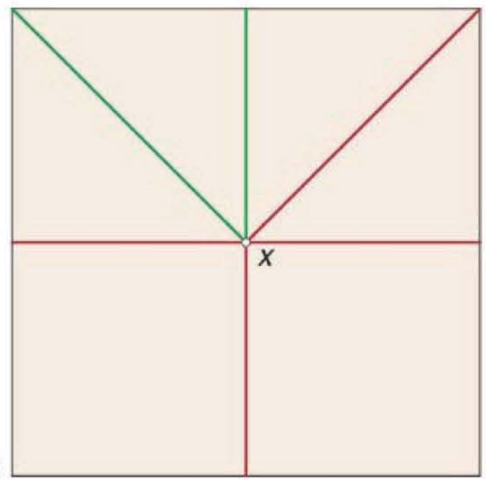
Folding maps vs. algorithms

A Vertex in a crease pattern

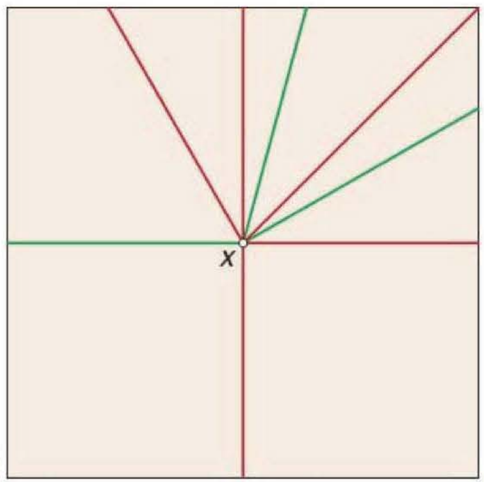
= any point not on the boundary of the paper at which two or more creases meet



Degree 4 vertex

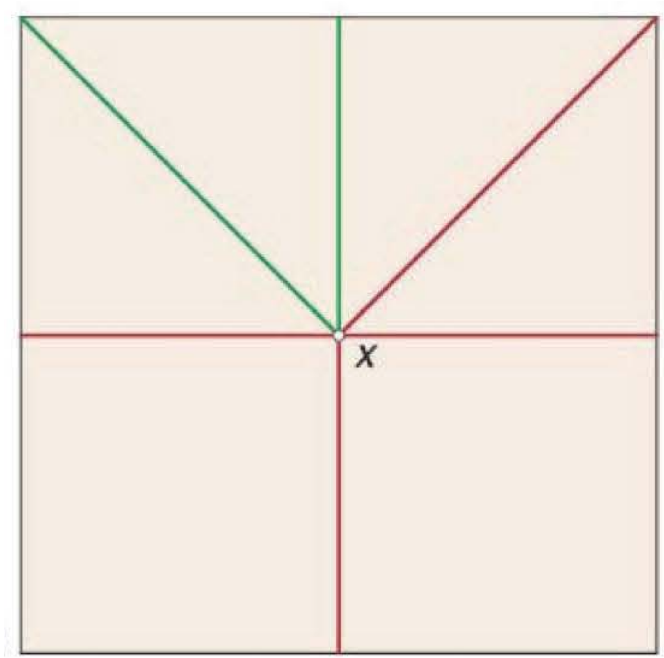


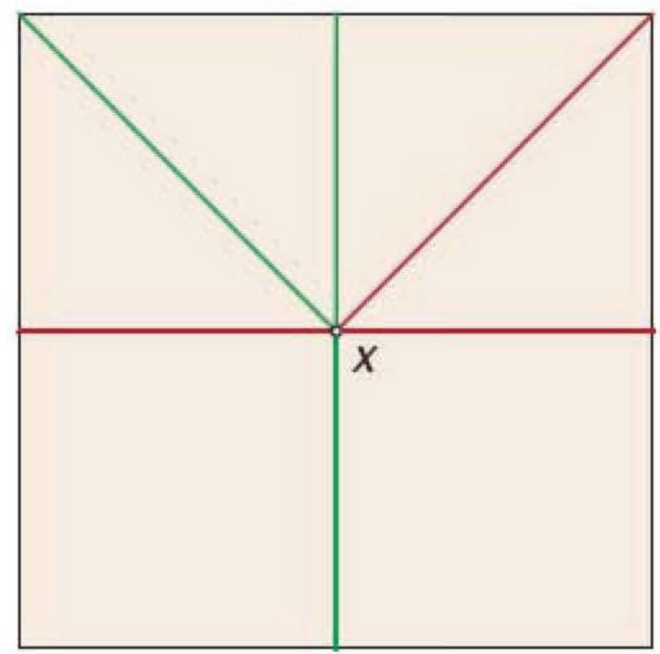
Degree 6 vertex



Degree 8 vertex

Which one is flat foldable pattern ?





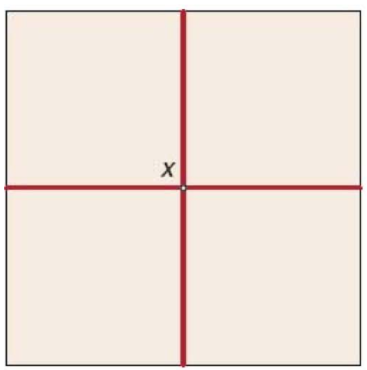
Is there a general pattern/rule for flat foldable vertices ?

Some *necessary* conditions

Even degree: *A vertex in a flat foldable pattern has even degree*

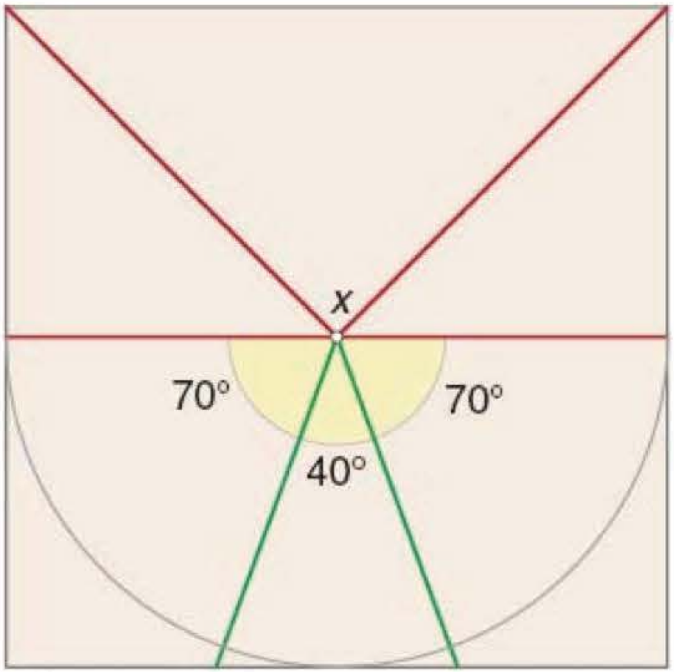
Follows from:

Maekawa-Justin (80's): *If **M** mountain creases and **V** valley creases meet at a vertex of a flat folding, then then **M=V+2** or **M=V-2**.*



Add folds to make vertex pattern flat

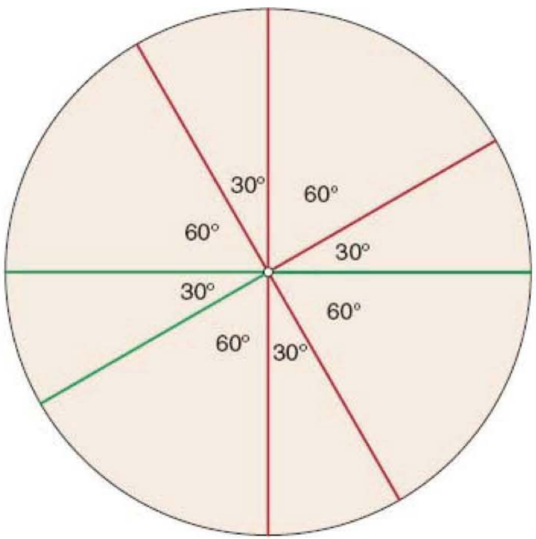
Maekawa-Justin not sufficient



Why does this not fold flat ?

Local minimum theorem

In any flat folding, any wedge whose angle is a local minimum (large-small-large wedge angles) **must be delimited by one mountain and one valley fold.**

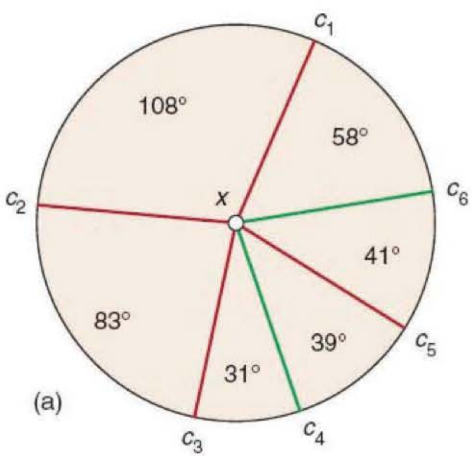


Is this flat foldable ?

Kawasaki-Justin characterization

A set of even number of creases meeting at a vertex folds flat if, and only if, the alternating sum of the determined wedge angles is zero: $\theta_1 - \theta_2 + \theta_3 - \theta_4 + \ldots + \theta_{n-1} - \theta_n = 0 \Leftrightarrow$

$\theta_1 + \theta_3 + \ldots + \theta_{n-1} = 180°$ (since $\theta_1 + \theta_2 + \theta_3 + \ldots + \theta_n = 360°$)



Note: ignores mountain-valley pattern !
=> There must *exist* a way to choose flat foldable mv pattern

This pattern: 39-41+58-108+83-31=0
Sufficiency ? This is the hard part !

Find other flat foldable mv configurations

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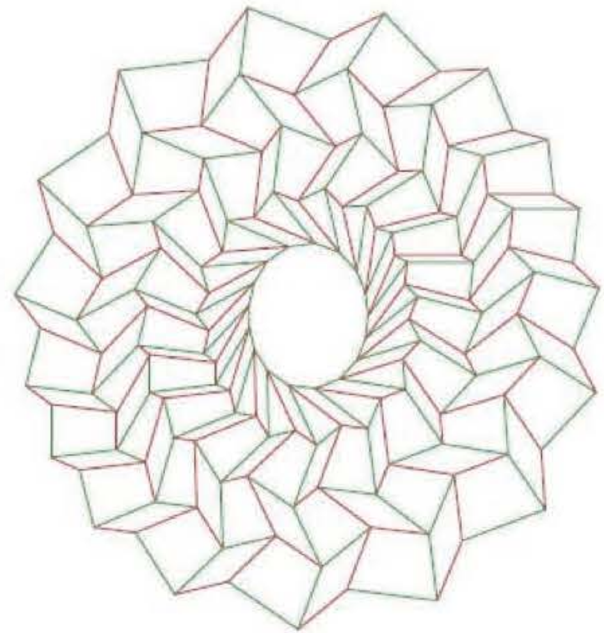
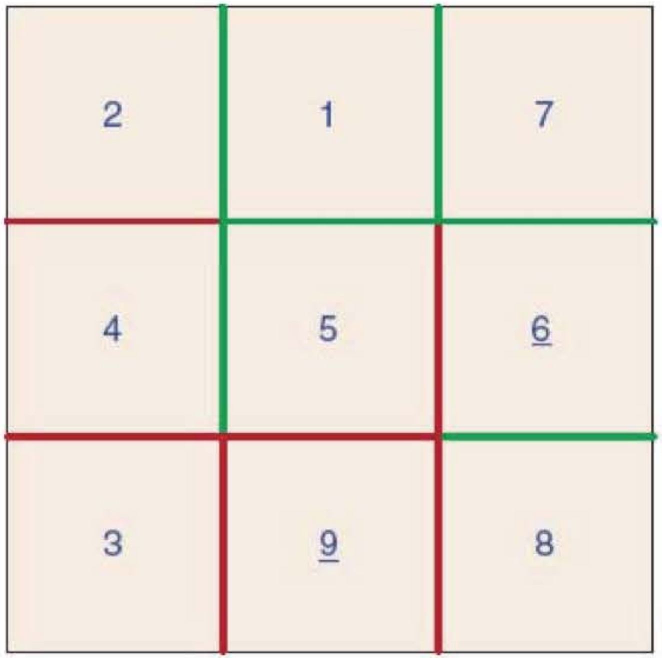
Stewart A. Robertson: Isometric folding of Riemannian manifolds, 1977 (!!!)

Global flat foldability



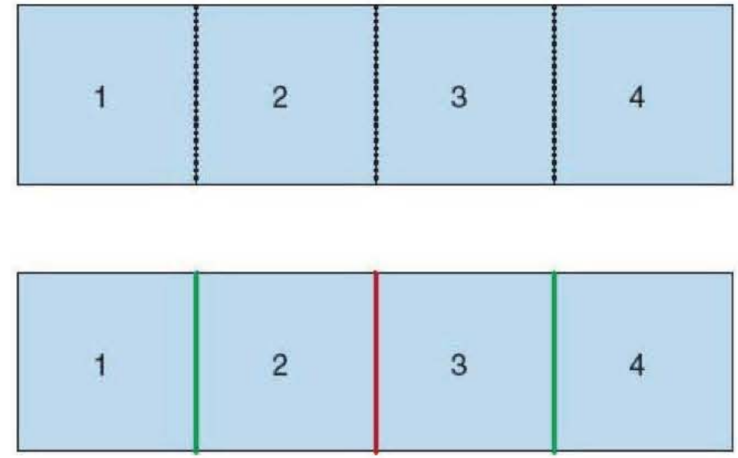
Robert Lang: Oval tessellation

An open problem: Map folding



Is there an efficient method (algorithm) for deciding weather or not a given rectangular map can be folded flat, with each grid crease segment pre-marked as either a mountain or valley fold ?

Stamp folding



How many different permutations can you fold ?

24=4! Permutations of 1234
Orientation by facing 1 upwards
(4321 in the figure)

References

M. Friedman: A History of Folding in Mathematics, 2018

R. Lang: Twists, Tilings and Tessellations: Mathematical Methods for Geometric Origami, 2018

E.D. Demaine, J. O'Rourke: Geometric Folding Algorithms: Linkages, Origami, Polyhedra, 2007

J. O'Rourke: How to Fold It: The Mathematics of Linkages, Origami, and Polyhedra, 2011