Computational assignment 1

In the lectures you have derived the stochastic differential equation

$$d\boldsymbol{v} = \boldsymbol{\mu} dt + \boldsymbol{\sigma} \cdot d\boldsymbol{W} \tag{1}$$

for a particle in a Maxwellian background plasma based on the Coulomb collision operator. In this assignment your task is to write an algorithm for solving the velocity space evolution of a test particle and use it to explore the properties of the collision operator.

Start by explicitly deriving the expressions for evaluating the three-dimensional friction and diffusion coefficients $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ in a uniform Maxwellian plasma at a temperature T. You should get something like

$$\boldsymbol{\mu} = -\nu \boldsymbol{v} \tag{2}$$

$$\boldsymbol{\sigma} = \sqrt{2D_{\parallel}} \frac{\boldsymbol{v}\boldsymbol{v}}{v^2} + \sqrt{2D_{\perp}} \left(\boldsymbol{I} - \frac{\boldsymbol{v}\boldsymbol{v}}{v^2} \right) \tag{3}$$

where the friction and diffusion coefficients ν, $D_{\parallel}$ and $D_{\perp}$ are defined for example in [1].

For solving the stochastic equation you can use the Euler-Maruyama method

$$\boldsymbol{v}_{n+1} = \boldsymbol{v}_n + \boldsymbol{\mu} \Delta t + \boldsymbol{\sigma} \cdot \Delta \boldsymbol{W} \tag{4}$$

where the random variables $\boldsymbol{W}$ are normally distributed with expectation value of 0 and a variance of Δt.

Test your collision operator by simulating a single test particle starting at various energies and with different time steps. Then perform a simulation for a larger sample of particles and observe the evolution of their distribution.

In your report you should describe your implementation (include the source code) and discuss the behaviour of the collision operator and the convergence of your algorithm. You should also discuss the computation time and efficiency of the method.

[1] E. Hirvijoki 2014 https://aaltodoc.aalto.fi/handle/123456789/12614