1. *We consider a GaAs-pin-photodiode, with an intrinsic layer thickness of 1.0 μm and a surface of 10^{-4} cm^2. The detector is illuminated at a wavelength of 775 nm with a light intensity of 0.1 W cm^{-2}. The absorption coefficient of the active region is 10^4 cm^{-1} at the wavelength of interest. Calculate the induced current under the assumption that photons are absorbed only in the active region.*

Generation of charge carriers in active region is $G(x) = \alpha J_{ph}(x)$, where $J_{ph}(x)$ is photon density. Photon density flux will decrease as a function of depth x $J_{ph}(x) = J_{ph,0}e^{-\alpha x}$, where the photon flux in the surface (one assumes that the absorption on top of the active region is small) is $J_{ph,0} = \frac{I_0}{\omega} = \frac{I_0}{hc}\lambda$. Induced current is $I_L = Ae\int_0^W G(x)\,dx = Ae\int_0^W \alpha J_{ph}(x)\,dx =$

$$= \frac{Ae\alpha\lambda I_0}{hc}\int_0^W e^{-\alpha x}\,dx = \frac{Ae\lambda I_0}{hc}\left[1-e^{-\alpha W}\right] = 3.95\ \mu\text{A}.$$

2. *Estimate the bandwidth of a GaAs pin-photodiode, when the detector surface is 1 mm^2 and the thickness of the active region is 1.0 μm. The dielectric constant of GaAs is 12.3, the saturation velocity for holes is 3×10^6 cm/s and the detector load is 50 Ω.*

Detector frequency response may be limited either by electrical RC-circuit or drift time of charges. Frequency response's maximum angular frequency ω_m, ${}_m\ {}_d$ 1. Drifting time from the center of the active region towards edges is $\tau_d = \frac{d/2}{v_s} = \frac{0.5\cdot10^{-6}\,\text{m}}{3\cdot10^4\ \text{m s}^{-1}} \approx 17\ \text{ns}$, and then the maximum frequency is $f_m = \frac{1}{2\pi\tau_d} \approx 9\ \text{MHz}$. Diode junction capacitance is $C_j = \frac{A\varepsilon_r\varepsilon_0}{d} = 1.1\cdot10^{-10}\ \text{F}$ and in this case $\omega_m = \frac{1}{RC_j} \Rightarrow f_m = \frac{1}{2\pi RC_j} = 29\ \text{MHz}$.

3. *The active region of a p-i-n-fotodiode (silicon) is circular in shape with a diameter of 0.4 mm. At the wavelength of 700 nm and intensity 0.1 mW/cm² light induces a 56.6 nA current. Calculate the responsivity of the photodiode and the quantum efficiency at this wavelength.*

Incident light power

$$P_{inc} = I_{inc} \cdot A = I_{inc} \cdot \pi r^2 = 0.1 \, mW/cm^2 \cdot \pi \cdot (0.02cm)^2 \approx 126nW.$$

Responsivity

$$R = \frac{I_{ph}}{P_{inc}} = \frac{56.6nA}{126nW} \approx 0.45 \frac{A}{W}$$

Quantum efficiency

$$\eta = R \cdot \frac{h \cdot c}{q \cdot \lambda} = 0.45 \frac{A}{W} \cdot \frac{6.626 \cdot 10^{-34} J \cdot s \cdot 3 \cdot 10^{8} m \cdot s^{-1}}{1.6 \cdot 10^{-19} A \cdot s \cdot 700 \cdot 10^{-9} m} \left(\approx R \cdot \frac{1.24}{\lambda(\mu m)} \right) \approx 0.80$$

4. *A pin-photodiode has a responsivity of 0.6 A/W at a wavelength of 0.8 μm. The dark current value is 1 nA, the bandwidth is 10 MHz and the load resistance is 100 Ω. a) Calculate the signal-to-noise ratio when the input optical signal has a power of 1.0 μW. b) What is the NEP of the detector?*

a) Responsivity $R = I_{ph}/P(\omega)$, where $P(\omega) = P_{inc} e^{j\omega t}$ is light power. Hence $i_{ph,rms} = R \frac{P_{inc}}{\sqrt{2}}$.

Signal-to-noise- ratio

$$\left(\frac{S}{N}\right)_{power} = \frac{i_{ph}^2 R_{eq}}{\left[\overline{i_S}^2 + \overline{i_J}^2\right] R_{eq}} = \frac{\frac{1}{2}(RP_{inc})^2}{\left[2q\left(I_{ph} + I_B + I_D\right)B + \frac{4k_B TB}{R_{eq}}\right]} = 109 \approx 40.7 \text{ db} \quad (I_B = 0).$$

b) $NEP = (P_{inc})_{\min}$, when $\frac{S}{N} = 1$ and $B = 1$ Hz.

$$\Rightarrow \quad \left(\frac{S}{N}\right)_{power} = 1 = \frac{\frac{1}{2}(RP_{inc})^2}{\left[2q\left(I_{ph} + I_B + I_D\right)B + \frac{4k_B TB}{R_{eq}}\right]}$$

$$NEP = P_{inc}\left(S/N = 1, B = 1 \text{ Hz}\right) \approx \frac{\sqrt{2}}{R}\left[2qI_D B + \frac{4k_B TB}{R_{eq}}\right]^{1/2} = 3.0 \times 10^{-11} \text{ W}.$$

5. *The quantum efficiency of an InGaAsP/InP avalanche detector is 0.8 at a wavelength of 1.3 μm. When the detector is illuminated with a power if 1.0 μW, a current of 20 μA is measured from the detector. The thickness of the multiplication region is 1.5 μm. a) What is the multiplication coefficient of the photodiode? b) How large is the ionisation coefficient if we suppose that only electrons multiply themselves?*

a) The current I_{ph} generated by the light:

$$\eta = \frac{I_{ph}}{q} \cdot \frac{h\nu}{P_{inc}} \quad \Rightarrow \quad I_{ph} = \eta \frac{q\lambda}{hc} P_{inc} = 0.839\ \mu\mathrm{A}.$$

Avalanche amplification is $M_e = \frac{J_{eo}(W)}{J_{ei}} = \frac{I_o}{I_{ph}} = 23.8$, where $I_0 = 20$ μA.

b) Current density increases within distance Δx by $\Delta J_e = J_e \alpha_e \Delta x$

From here one gets a differential equation $\frac{dJ_e}{dx} = J_e \alpha_e$, which can be integrated

$$\int_{J_{ei}}^{J_{eo}(W)} \frac{dJ_e}{J_e} = \int_0^W \alpha_e dx \quad \Rightarrow \quad \ln \frac{J_{eo}(W)}{J_{ei}} = \ln M = \alpha_e W.$$

Impact ionization coefficient $\alpha_e = \frac{\ln M}{W} = 2.1 \cdot 10^4 \frac{1}{\mathrm{cm}}$.