Economics of Strategy for Online and Digital Markets

Topics in Economic Theory and Policy, 31C01000

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Lecture 4: Auction theory

Outline

- Why auctions (price discovery, competition)
- Auction methods
- Private vs. common value auctions

Auctions as a tool for selling stuff

- We consider a situation where a seller has a single item for sale and there a number of potential buyers.
- What is the right price to ask?
  - It will depend on how much buyers are willing to pay.
  - Buyers are not going to tell your their true value.
  - Auction is a mechanism for price discovery.
- Auctions also create a competition between buyers.

Ascending auction

- Seller asks for bids from potential buyers.
- Several rounds of bidding are possible.
- Price starts low. Price is increased until only one bidder is left.
- Variation: Price starts high, is lowered until someone buys.
- Remaining bidder pays their bid.
- Common applications
  - Traditional auction houses selling all sorts of fancy stuff.
  - eBay, huuto.net (Finnish eBay) and the like.

Simple model

- Seller looking to sell one item.
- There are n buyers
  - Buyers have their own valuations for the item $v_1, v_2, \ldots, v_n$.
  - These valuations are private information.
- We assume that the valuations are drawn from an uniform distribution $[0, 100]$.
- Seller sets the rules for the auction.

Ascending auction

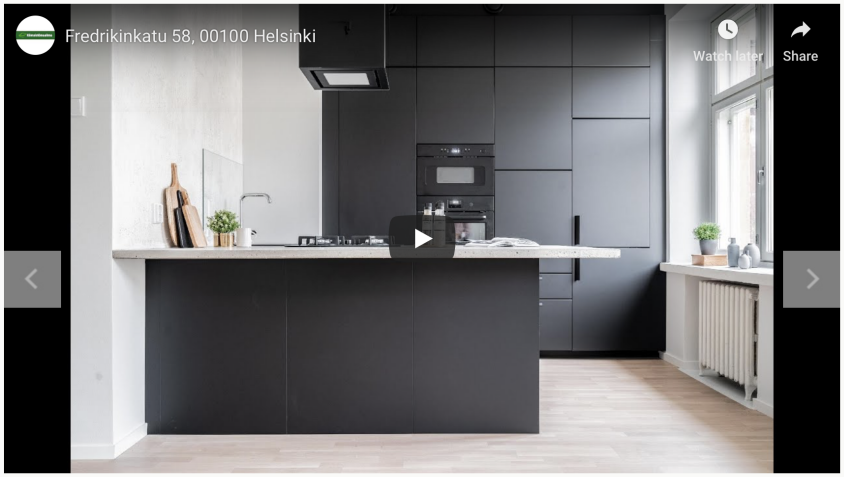
- Prices start at zero, and rises slowly.
- Buyers can bid at the current price or exit.
- Auction ends when just one bidder remains.
- Final bidder wins, and pays the price at which the second remaining bidder dropped out.

How should you bid?

Online auction for an apartment

Fredrikinkatu 58, Etu-Töölö, Helsinki

40 m² | KT, 1h, avok, kph, vh



Annettu	Asiakasnumero	Tyyppi	Summa
01.01.2019	**1493587**	**Kirjallinen tarjous**	**337 000,00 €**
30.12.2018	1500570	Kirjallinen tarjous	336 000,00 €
29.12.2018	1493587	Kirjallinen tarjous	335 000,00 €
27.12.2018	1500570	Kirjallinen tarjous	332 000,00 €
25.12.2018	1493587	Kirjallinen tarjous	331 000,00 €
25.12.2018	1500570	Kirjallinen tarjous	329 000,00 €
23.12.2018	1493587	Kirjallinen tarjous	328 000,00 €
22.12.2018	1500570	Kirjallinen tarjous	326 000,00 €
21.12.2018	1493587	Kirjallinen tarjous	324 000,00 €
20.12.2018	1499837	Kirjallinen tarjous	319 000,00 €
20.12.2018	1493587	Kirjallinen tarjous	318 000,00 €
19.12.2018	1500570	Kirjallinen tarjous	315 000,00 €
19.12.2018	1500082	Kirjallinen tarjous	307 000,00 €
19.12.2018	1493587	Kirjallinen tarjous	306 000,00 €

Source: Kiinteistömaailma.

Ascending auction

- In this auction it is optimal for you to bid until the price is higher than your private value:
  - If you continue bidding above your value and win, you need to pay a price that is higher than your private value.
  - If you stop bidding below your value, you lose the potential gain from buying the item below your value.
- Bidder with the highest valuation will win and pay the second highest value.
- Example with three bidders
  - Assume that the valuations are 25, 33, 75.
  - First exit at 25, second at 33 and the auction ends.

Expected revenue from the ascending auction

- Assume two bidders with random values.
- Expected revenue from the auction is equal to the expected value of the second highest bid.
- Expected values for the bids
  - Highest value is 66 ⅔.
  - Second highest value is 33 ⅓.
- So the expected revenue for the seller is 33 ⅓.

Role of competition

More competition increases the bids and the expected value for the seller:

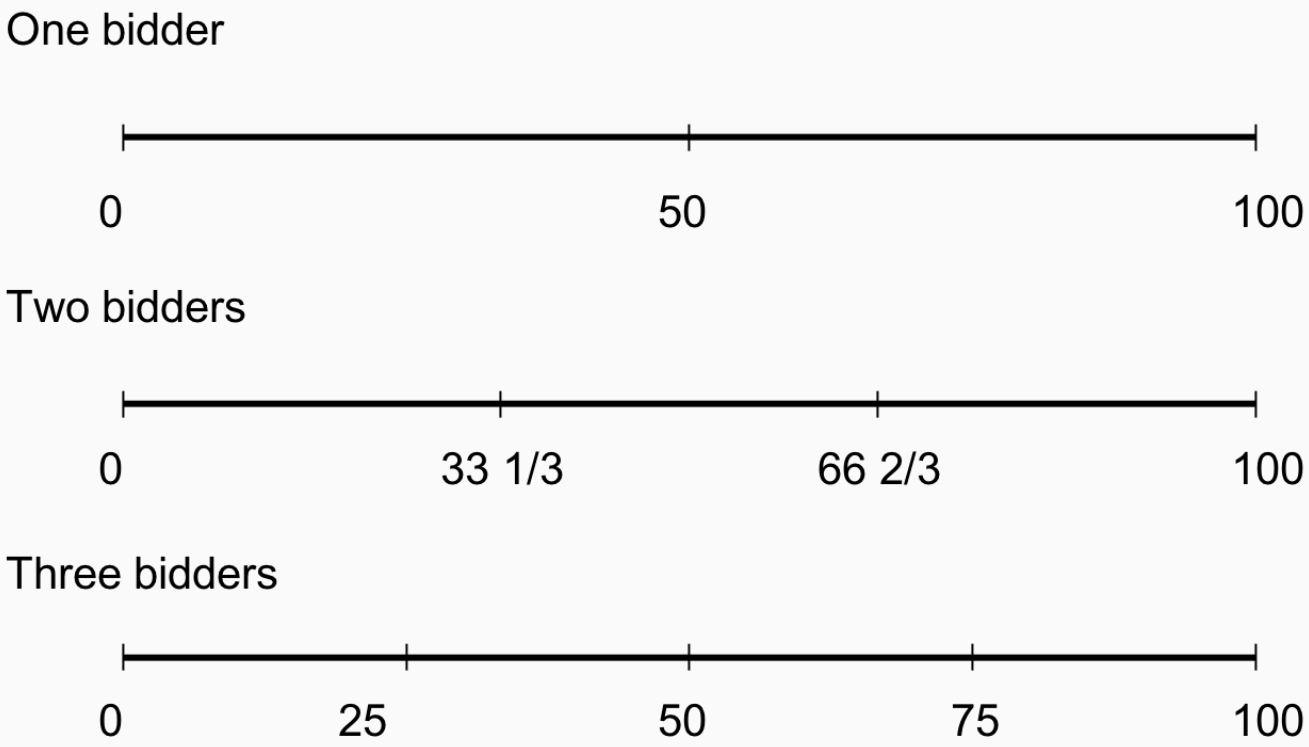


Figure. Expected bid values drawn from uniform $[0, 100]$ distribution as the number of bidders increases.

Why auctions have been popular online

- Arranging an auction is a trade-off between the benefit of price discovery vs. the cost of arranging an auction.
  - For example, think of a traditional auction selling arts.
- Online both of the advantage and disadvantage change:
  - Many more participants possible online vs. physically.
  - Lower costs of arranging.
  - Emergence of popular sites has made it easier for people to know where to find the auction action.

Online example

Watching DealDash website.

Sealed bidding

- Seller asks for bids from potential buyers.
- There is only one round of bidding.
- Buyers deliver their bids so that the other buyers do not observe their bids.
- Highest bidder wins and pays their bid.
- Or a variation: Winning bidder pays the second highest bid.
- Common applications
  - Procurement contracts for commodities and services.
  - Private M&A ownership transactions between firms.

Sealed bidding – Second price auction

- Buyers submit their bids.
- Seller opens the bids.
- Bidder who submitted the highest bid wins.
- Winner pays the second highest bid.

How should you bid?

Second price auction – Optimal bidding

- Again, it is optimal for you to bid your private value.
  - Proof is left to the exercises.
- In equilibrium, everyone bids their value.
- Bidder with the highest valuation will win and pay the second highest value, exactly as in the ascending auction.
- Example with three bidders
  - Assume that the valuations are 25, 33, 75.
  - Winner has value 75, pays 33, as in ascending auction.

Sealed bidding – First price auction

- Buyers submit their bids.
- Seller opens the bids.
- Bidder who submitted the highest bid wins.
- Winner pays her/his bid.

How does the optimal bidding change?

In-class exercise: Auction

First stage:

- Guess how long the line on the blackboard is in centimeters.
- Write down your guess on the piece of paper in front of you.

In-class exercise: Auction

Second stage:

- Now you will bid for a money prize that is worth 1 € times the true answer in centimeters.
  - E.g. if the line is 90 cm, the prize is worth 90 €.
- The highest bidder will win the prize.
- The winner will be be paid as follows:

$$\text{payment} = (1\,€ \times \text{the true answer}) - \text{winning bid}$$

- Write down your bid on the piece of paper in front of you.

In-class exercise:

Return the papers. Discussion on the results.

First price auction – Optimal bidding

- Optimal bid is less than your true value.
- A higher bid (but still below your value)
  - Increases your chances to win.
  - Decreases your profit if you do win.
- Optimal bid depends on what you think the others will bid.
- We need to consider an equilibrium analysis.

First price auction – Equilibrium

- Recall the concept of Nash equilibrium: the strategic interaction now takes place through bidding strategies.
- A set of bidding strategies is a Nash equilibrium if each bidder's strategy maximizes his expected payoff given the strategies of the others.
- Bidders do not know their opponent's values: this is called incomplete information.
- In equilibrium, all bidding strategies maximize the expected payoff of the bidder taking in to account the uncertainty about opponent values.

First price auction – Equilibrium

- In the above first price sealed bid auction example, there is an equilibrium in which both bidders bid half their value.
- Bidder with the highest value wins.
- In expectation, highest value is 66 ⅔, so the expected revenue is 33 ⅓. Which is the same as in the second price auction.
- Turns out this is a quite general result.

Auction efficiency

- Auction outcome is efficient if the high value bidder wins.
- The Nash equilibrium outcome is the same in several auction mechanisms:
  - The high value bidder wins.
  - The expected auction price equals the expected value of the second highest bidder.
- This is the essence of the revenue equivalence theorem:
  - Every auction that allocates the goods efficiently has the same expected profits for every bidder valuation and the same expected revenue for the seller.

How does the theory work in practice?

- Outcomes of the auction will depend on the implementation.
- Sellers can modify the basic auction forms and try to increase their revenue.
  - Attracting bidders to participate in the auction.
  - Setting minimum bid prices, i.e. reserve prices.
  - Subsidizing bidders to create competition.
- Sometimes the assumptions, rational behavior and rational expectations, may be too restrictive.
- In many settings, such as procurement, there is also the challenge of making sure bidders compete, i.e. avoid collusion.

Common values

- Previously we have assumed that the buyers have some *private* valuation for the item.
- Reverse is also possible, it may be that the value of the item once acquired is the same for all buyers, but the value during the auction is uncertain.
- These are named *common* value auctions.

Winner's curse

- In a common value auction the bidder with the highest valuation on the value of the item, i.e. the most optimistic bidder, wins.
- A bidder who fails to take this into account pays, on average, more than the item is worth.

Other Auction methods

- Second price auctions can be generalized to auctions when many identical items are sold simultaneously, so-called Vickrey auctions.
- All-pay auctions where bidders submit bids, highest bidder wins, and everyone has to pay their own bid.
  - Winner pays less in expectation, because everyone pays.
  - Not often seen in monetary auctions, but can be used to model e.g. R&D competition or lobbying

Learnings today

- Auctions are a method for price discovery and to induce competition, works well online.
- Several auction methods, but in theory often same expected revenues when an efficient method is employed.
- Details of the auction design matter in practice.

Readings for this lecture

- Milgrom, P. (1989) "Auctions and Bidding: A Primer", Journal of Economic Perspectives.
  - The proofs of the theorems are not required reading.
  - Also section "Correlated Bidder Information" and beyond may get too involved and is not essential for this course.
- Reiley, D. (2000) "Auctions on the Internet: What's Being Auctioned, and How?", Journal of Industrial Economics.
  - A lot of the detail is outdated and not essential. Focus on how the economic theory has been applied in real life cases.
  - Feel free to skip sections III, IV, VIII and IX.
- Ockenfels, A., Reiley, D. and Sadrieh, A. (2006) "Online auctions", NBER Working Paper 12785.
  - Another survey with some outdated links.
  - Sections 1, 2.1, 2.2 and 8 are the key parts for this course.

Exercises for Lecture 4

1. Find an example of an online auction (other than the examples in the lecture slides: Kiinteistömaailma, eBay, huuto.net, DealDash). Describe the auction mechanism that they are using.
2. Prove that in the second price sealed bid auction for a single indivisible good it is optimal for you to bid your private value v.

Next time

Guest lecture

- Economics of games
- Janne Peltola from Supercell