Diagnostic Tests

Some practical ways to check the residual assumptions of the simple Classical Linear Regression Model (CLRM)

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How to check graphically The Assumptions of the Simple Classical Linear Regression Model (CLRM)

- Consider the simple classical linear regression model $y_t = \alpha + \beta x_t + u_t$

estimated from a sample by $y_t = \hat{\alpha} + \hat{\beta} x_t + \hat{u}_t$, t=1, 2, …, T

- Let $\hat{y}_t = \hat{\alpha} + \hat{\beta} x_t$ be the predicted value for observation t

Assumption	Graphical method
1. $E(u_t) = 0$	Plot $\hat{u}_t$ against t (always true if estimated using OLS)
2. $Var(u_t) = \sigma^2$	Homoscedasticity: plot $(t, \hat{u}_t)$ and $(x_t, \hat{u}_t)$; look at the variation
3. $Cov(u_i, u_j) = 0$	Plot $(\hat{u}_t, \hat{u}_{t+1})$ to look for possible autocorrelation of first order May also plot $(\hat{u}_t, \hat{u}_{t+k})$ for higher order autocorrelation k>1
4. $Cov(u_t, x_t) = 0$	Plot $(x_t, \hat{u}_t)$; look for possible dependence
5. $u_t \sim N(0, \sigma^2)$	Histogram compared to normal curve or QQ-plot (straight line if normal)

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- Let $\hat{y}_t = \hat{\alpha} + \hat{\beta} x_t$ be the predicted value for observation t
- <u>Assumption</u> <u>Tests</u>

Assumption	Tests
1. $E(u_t) = 0$	Always true if estimated using OLS; good to check that $\Sigma \hat{u}_t = 0$
2. $Var(u_t) = \sigma^2$	White's test for heteroscedasticity:
	Estimate quadratic regression $\hat{u}_t^2 = \hat{\alpha}_0 + \widehat{\beta_0} x_t + \widehat{\beta}_1 x_t^2 + \hat{\varepsilon}_t$
	(should be insignificant, otherwise $Var(u_t)$ depends on x_t)
3. $Cov(u_i, u_j) = 0$	Durbin-Watson test for possible autocorrelation of first order
	Breusch-Godfrey for higher order autocorrelation
4. $Cov(u_t, x_t) = 0$	Calculate correlation (should be insignificant) or
	Estimate linear regression $\hat{u}_t = \hat{\alpha}_1 + \widehat{\beta}_1 x_t + \hat{\varepsilon}_t$ (should be insignificant)
5. $u_t \sim N(0, \sigma^2)$	Jarque-Bera-test or some other normality test (e.g. Shapiro-Wilk test)