All the procedures assume triaxial tests

Schemes for calculation of the elastic stress (using incremental procedure)

a) drained triaxial test

Drained triaxial test is stress driven. That means we know the stress increment (we can assume $\Delta p'$, and $\Delta q = 3\Delta p'$), we know elastic parameters, and we calculate strain increments, using the equation below.

$$\begin{Bmatrix} \Delta p' \\ \Delta q \end{Bmatrix} = \begin{bmatrix} K' & 0 \\ 0 & 3G' \end{bmatrix} \begin{Bmatrix} \Delta \varepsilon_v^e \\ \Delta \varepsilon_q^e \end{Bmatrix} \qquad (1)$$

In each increment

1. Assume $\Delta p'$

2. Compute $\Delta q = 3\Delta p'$

3. With equation 1, compute strain increments

4. Check the value of the yield function. If the value is negative, continue with the increments, if the value switches to positive, it means in the given strain / stress increment we switch to elasto-plasticity.

b) undrained triaxial test

Undrained triaxial test is strain driven. We know that the total volumetric strain change is zero $\Delta\varepsilon_v = 0$, and we know that $\Delta\varepsilon_v = \Delta\varepsilon_1 + 2\Delta\varepsilon_3$. We also know that there is no plastic strain.

In each increment

1. Assume $\Delta\varepsilon_1$

2. Compute $\Delta\varepsilon_3$

3. Compute $\Delta\varepsilon_q^e = 2/3\left(\Delta\varepsilon_{11} - \Delta\varepsilon_{33}\right)$

4. With equation 1, compute the stress increments.

5. Check the value of the yield function. If the value is negative, continue with the increments, if the value switches to positive, it means in the given strain / stress increment we switch to elasto-plasticity.

Schemes for calculation of the elasto-plastic increment (using incremental procedure)

a) drained triaxial test

Mohr-Coulomb model is an elastic - perfectly plastic model. As such, when we enter plasticity, and we have drained test, the strain increments are perfectly plastic, i.e. we do not expect any stress change. However, we will test that assumption.

The plastic multiplier, for the case of non-associated flow rule, the equations are:

$$F\left(p',q\right)=q-\eta p'-c^{*}=0$$

$$\frac{\partial F}{\partial p'}dp'+\frac{\partial F}{\partial q}dq=0 \quad : \quad consistency\ condition$$

$$dp'=K\left(d\varepsilon_{v}-d\varepsilon_{v}^{p}\right) \quad : \quad Hooks'\ law$$

$$dq=3G\left(d\varepsilon_{q}-d\varepsilon_{q}^{p}\right)$$

$$d\varepsilon_{v}^{p}=d\lambda\frac{\partial Q}{\partial p'} \quad : \quad flow\ rule$$

$$d\varepsilon_{q}^{p}=d\lambda\frac{\partial Q}{\partial q} \qquad (2)$$

$$\lambda=\frac{\frac{\partial F}{\partial p'}Kd\varepsilon_{v}+\frac{\partial F}{\partial q}3Gd\varepsilon_{q}}{\left(\frac{\partial F}{\partial p'}K\frac{\partial Q}{\partial p'}+\frac{\partial F}{\partial q}3G\frac{\partial Q}{\partial q'}\right)}=\frac{-\eta Kd\varepsilon_{v}+3Gd\varepsilon_{q}}{\left(\eta K\xi+3G\right)} \qquad (3)$$

and the set of equation for the elasto-plastic matrix, which replaces the elastic matrix is

$$\frac{\partial F}{\partial p'}=-\eta,\quad \frac{\partial F}{\partial q}=1,\frac{\partial Q}{\partial p'}=-\zeta,\frac{\partial Q}{\partial q}=1 \qquad (4)$$

$$D^{ep}=\frac{D^{e}-D^{e}\frac{\partial F}{\partial\sigma}\left(\frac{\partial Q}{\partial\sigma}\right)^{T}D^{eT}}{\left(\frac{\partial F}{\partial\sigma}\right)^{T}D^{e}\frac{\partial Q}{\partial\sigma}}$$

with

$\eta=\dfrac{6\sin\varphi'}{3-\sin\varphi'}$ and $\xi=\dfrac{6\sin\psi}{3-\sin\psi}$

The stress increments are given in equation (1), where we need to remember that the elastic strain increment is the difference between total strain increment and plastic strain increment.

If the test would be still stress driven, we could still assume $\Delta p'$, and $\Delta q=3\Delta p'$. That would allow us to compute the strains using the elasto-plastic matrix, as well as the plastic strains using equations (2). However, you may find that the calculation is not possible, and that the only possible increment in stress would follow the yield surface. If you find that, the only possible increment of $\Delta p'$ - and hence $\Delta q=3\Delta p'$ is zero, as it will stay on the yield surface. Knowing this, and assuming strain increment $\Delta\varepsilon_{1}$, we can compute the remaining strain $\Delta\varepsilon_{3}$. Note that as stress increments are zero, whole $\Delta\varepsilon_{1}$ is plastic, which allows us to compute the remaining strain component. You may need to compute plastic multiplier first, and use that plastic multiplier to compute the strain. You may need to use solve function in Excel (you may need the solver add-on enabled, see appendix).

Note that you may need the extra equation of the yield surface, as the stress state after each increment is on the yield surface.

b) undrained triaxial test

In the undrained triaxial test we have an extra unknown, which is pore pressure. Assume that the test is strain driven, so assume increment $\Delta\varepsilon_1$. As we know that the volume change is zero, we can compute corresponding strain $\Delta\varepsilon_3$. Those strains will have elastic and plastic components, which are unknown.

As the plastic multiplier can be computed with total strain, we can compute the plastic multiplier, as well as the plastic strains. Having plastic strains, we can compute elastic strains, leading to the stress increments (effective). Alternatively, one can use the elasto-plastic matrix to get those increments.

Having the stress increments, the extra equation is due to the fact that the total stress increment Δp and the deviatoric stress increment are related $\Delta q = 3\Delta p$. This allows us to compute pore pressure u.

Check whether the stress state after each strain increment is on the yield surface.

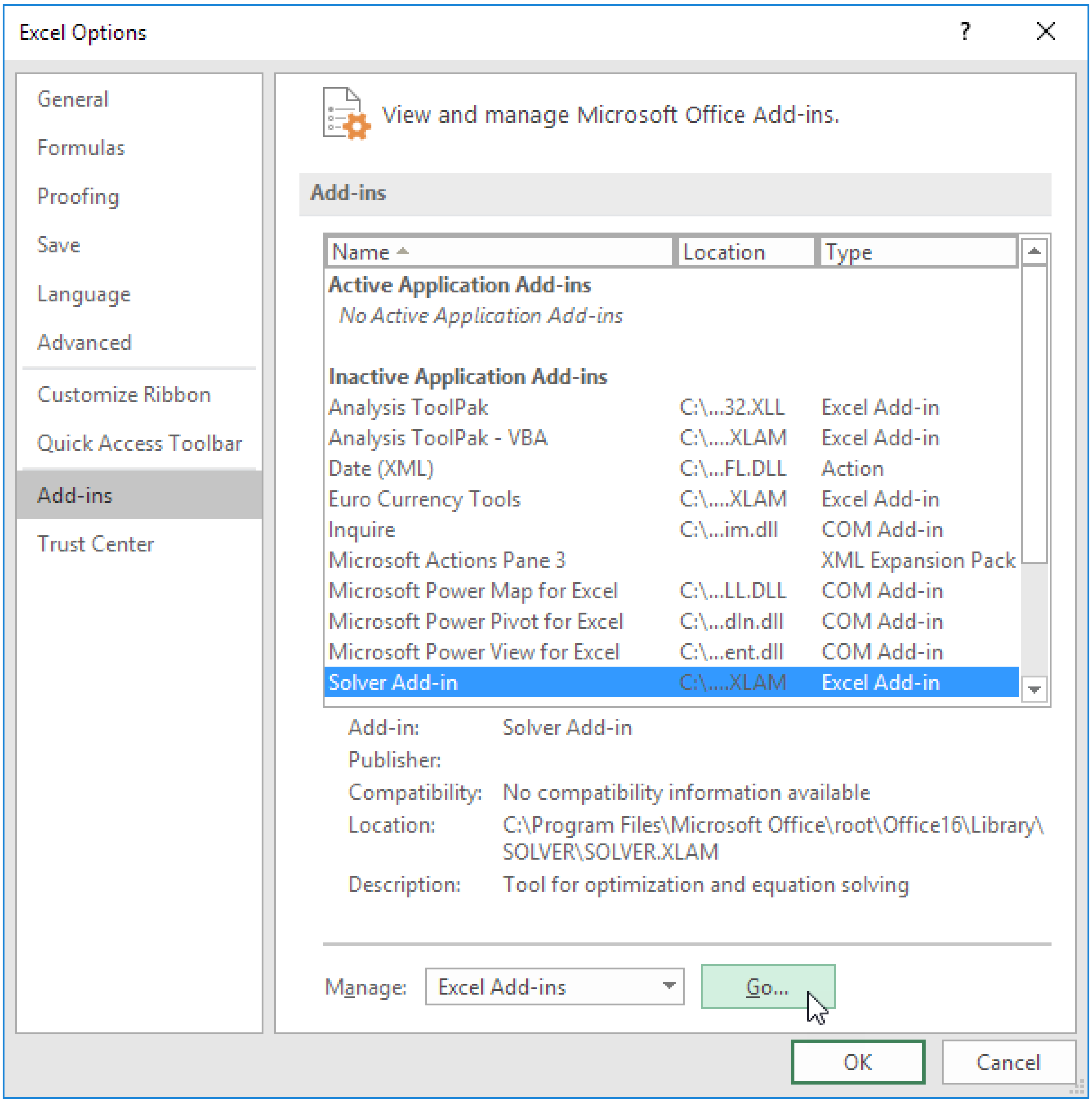
Check whether the effective stress and deviatoric stress are increasing for the dilation angle different to zero, leading to increase of strength of soil during undrained shearing.

Appendix: Load the Solver Add-in in Excel

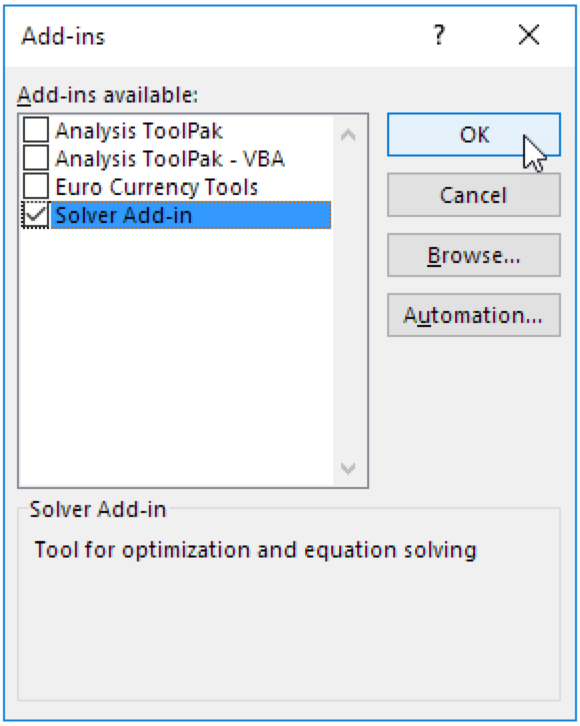
To load the solver add-in, execute the following steps.

1. On the File tab, click Options.

2. Under Add-ins, select Solver Add-in and click on the Go button.



3. Check Solver Add-in and click OK.



4. You can find the Solver on the Data tab, in the Analyze group.