Aalto University
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Geo-E2010
Advanced Soil Mechanics L

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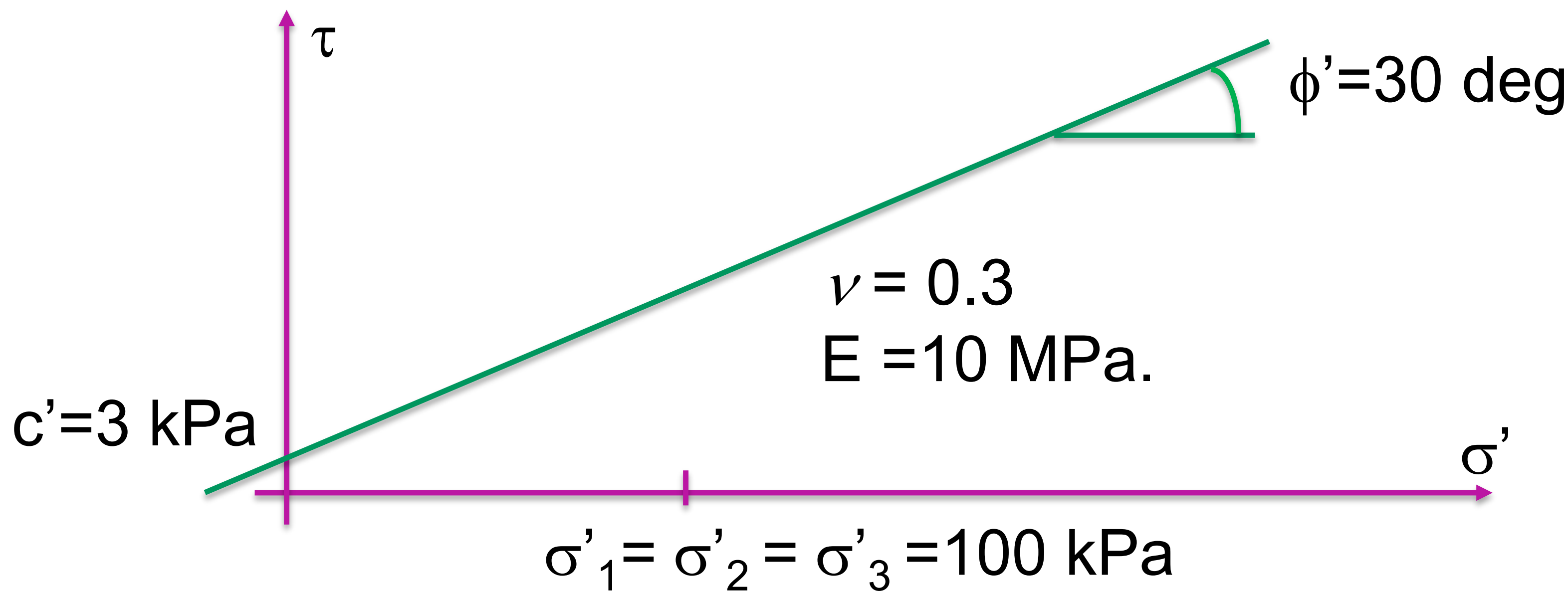
Tutorial 1
Mohr - Coulomb model

Tutorial 1

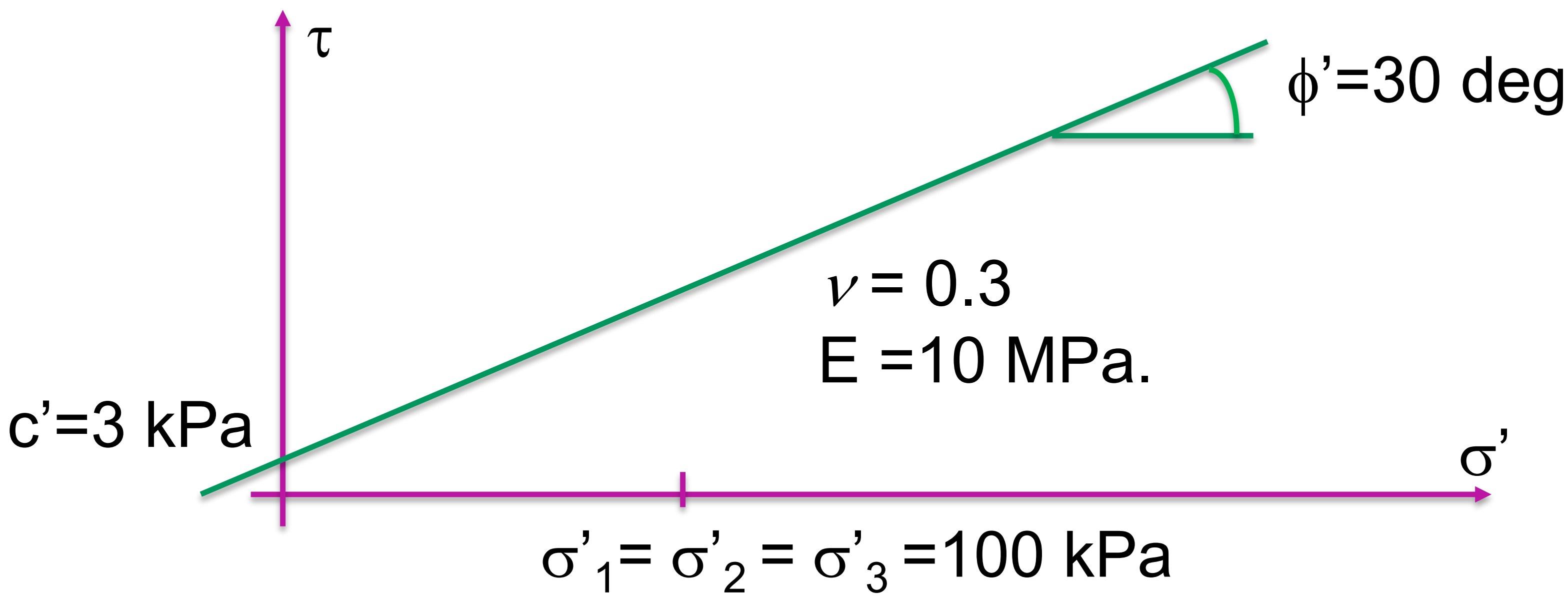
Simulate standard triaxial drained and undrained tests, assuming that soil is behaving as a Mohr-Coulomb material.

Assume initial condition p=100 kPa, q=0.
Assume Mohr-Coulomb soil parameters:

- friction angle 30 degrees
- dilation angle: 0 degrees, 3 degrees, 30 degrees
- cohesion c' equal to 3 kPa
- Poisson's ratio 0.3
- Young's modulus 10 MPa.

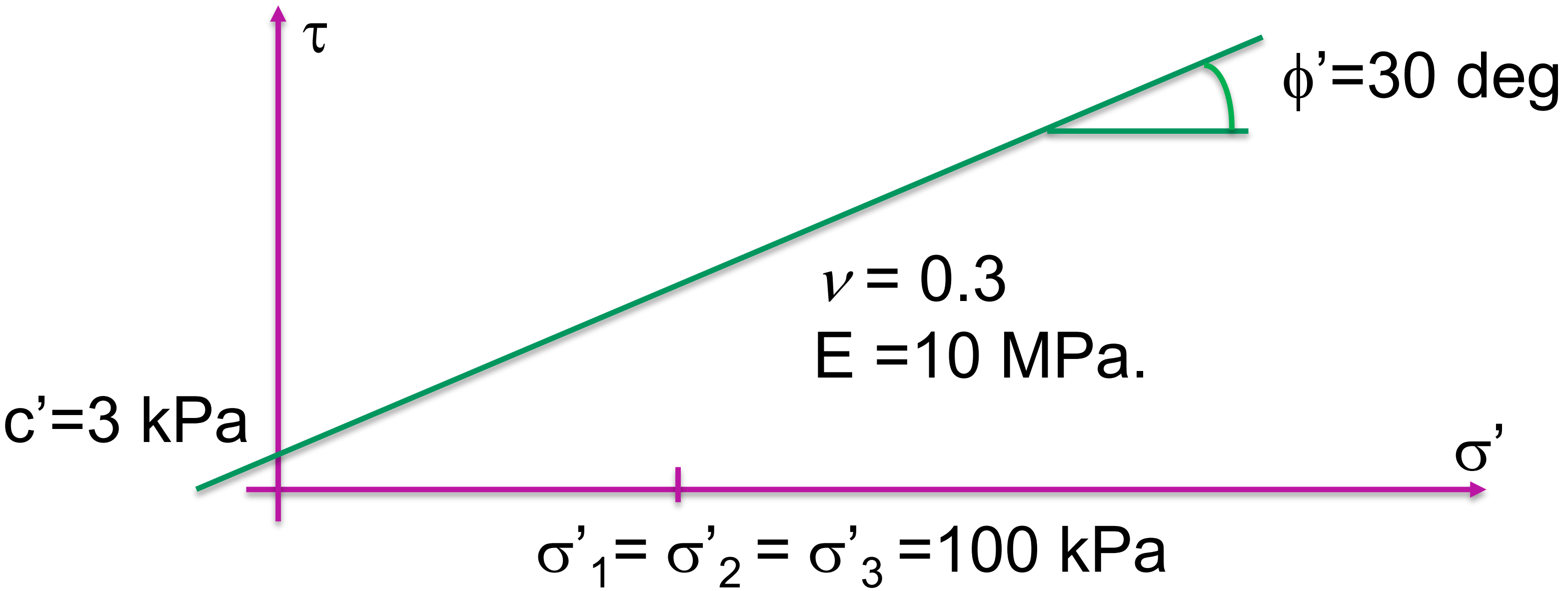

τ
φ'=30 deg
ν = 0.3
E =10 MPa.
c'=3 kPa
σ'
σ'1= σ'2 = σ'3 =100 kPa

Drained test



σ'_1 – increases, $\sigma'_2 = \sigma'_3$ = const.

Drained test



$$F = (\sigma_1' - \sigma_3') - (\sigma_1' + \sigma_3')\sin\phi' - 2c'\cos\phi' = 0$$

Drained...

p'=100 kPa and q=0 kPa. Hence $\sigma_1'=\sigma_3'=100$ kPa

In triaxial test cell pressure is constant, hence σ_3'=const.=100 kPa

How high σ_1' needs to be to reach yielding?

σ_1' - 100 – (σ_1' + 100)0.5 – 2*3*0.5 sqrt 3= 0

0.5 σ_1'=150+5,2=155.2 σ_1'=310.4 [kPa]

p'=(100+100+310.4)/3=170.1 kPa q=310.4-100=210.4 kPa

$$F = (\sigma_1' - \sigma_3') - (\sigma_1' + \sigma_3')\sin\phi' - 2c'\cos\phi' = 0$$

Drained...

Δp'=170,1-100=70,1 kPa and Δq=210,4 kPa.

Strains:

$$
\left\{
\underbrace{\begin{bmatrix} \Delta\varepsilon_{11} \\ \Delta\varepsilon_{22} \\ \Delta\varepsilon_{33} \\ \Delta\varepsilon_{12} \\ \Delta\varepsilon_{13} \\ \Delta\varepsilon_{23} \end{bmatrix}}_{\Delta\boldsymbol{\varepsilon}}
= \overbrace{\frac{1}{E}\begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ & & & 1+\nu & 0 & 0 \\ & & & & 1+\nu & 0 \\ . & & & & & 1+\nu \end{bmatrix}}^{\mathbf{D}^{el^{-1}}}
\underbrace{\begin{bmatrix} \Delta\sigma_{11} \\ \Delta\sigma_{22} \\ \Delta\sigma_{33} \\ \Delta\sigma_{12} \\ \Delta\sigma_{13} \\ \Delta\sigma_{23} \end{bmatrix}}_{\Delta\boldsymbol{\sigma}}
\right.
$$

$$F = (\sigma_1' - \sigma_3') - (\sigma_1' + \sigma_3')\sin\phi' - 2c'\cos\phi' = 0$$

Drained...

Δp'=170,1-100=70,1 kPa and Δq=210,4 kPa.

Strains:

$$\left\{ \overbrace{\begin{bmatrix} \Delta\varepsilon_{11} \\ \Delta\varepsilon_{22} \\ \Delta\varepsilon_{33} \\ \Delta\varepsilon_{12} \\ \Delta\varepsilon_{13} \\ \Delta\varepsilon_{23} \end{bmatrix}}^{\Delta\boldsymbol{\varepsilon}} = \overbrace{\frac{1}{10000}\begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ & & & 1+\nu & 0 & 0 \\ & & & & 1+\nu & 0 \\ . & & & & & 1+\nu \end{bmatrix}}^{\mathbf{D}^{el^{-1}}} \overbrace{\begin{bmatrix} 210.4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}^{\Delta\boldsymbol{\sigma}} \right.$$

Drained...

$\Delta p'=170,1-100=70,1$ kPa and $\Delta q=210,4$ kPa.

Strains:

$$\left\{\underbrace{\begin{bmatrix}\Delta\varepsilon_{11}\\ \Delta\varepsilon_{22}\\ \Delta\varepsilon_{33}\\ \Delta\varepsilon_{12}\\ \Delta\varepsilon_{13}\\ \Delta\varepsilon_{23}\end{bmatrix}}_{\Delta\boldsymbol{\varepsilon}} = \overbrace{\frac{1}{E}\begin{bmatrix}1 & -0.3 & -0.3 & 0 & 0 & 0\\ -0.3 & 1 & -0.3 & 0 & 0 & 0\\ -0.3 & -0.3 & 1 & 0 & 0 & 0\\ & & & 1+0.3 & 0 & 0\\ & & & & 1+0.3 & 0\\ . & & & & & 1+0.3\end{bmatrix}}^{\mathbf{D}^{el^{-1}}}\overbrace{\begin{bmatrix}210.4\\ 0\\ 0\\ 0\\ 0\\ 0\end{bmatrix}}^{\Delta\boldsymbol{\sigma}}\right.$$

$$F=(\sigma_1'-\sigma_3')-(\sigma_1'+\sigma_3')\sin\phi'-2c'\cos\phi'=0$$

Drained...

$$\left\{ \overbrace{\begin{bmatrix} \Delta\varepsilon_{11} \\ \Delta\varepsilon_{22} \\ \Delta\varepsilon_{33} \\ \Delta\varepsilon_{12} \\ \Delta\varepsilon_{13} \\ \Delta\varepsilon_{23} \end{bmatrix}}^{\Delta\boldsymbol{\varepsilon}} = \overbrace{\frac{1}{10000} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ & & & 1+\nu & 0 & 0 \\ & & & & 1+\nu & 0 \\ . & & & & & 1+\nu \end{bmatrix}}^{\mathbf{D}^{el^{-1}}} \overbrace{\begin{bmatrix} 210.4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}^{\Delta\boldsymbol{\sigma}} \right.$$

$$\Delta\varepsilon_{11} = 210.4/10000 = 0{,}02104 = 2.104\%$$

$$\Delta\varepsilon_{22} = -0.3 * 210.4/10000 = -0.006312 = -0.631\%$$

$$\Delta\varepsilon_{33} = -0.3 * 210.4/10000 = -0.006312 = -0.631\%$$

$$\Delta\varepsilon_{v} = \Delta\varepsilon_{11} + \Delta\varepsilon_{22} + \Delta\varepsilon_{33} = 0.842\%$$

$$\Delta\varepsilon_{q} = 2/3(\Delta\varepsilon_{11} - \Delta\varepsilon_{33}) = 1{,}823\%$$

Drained...

$$\left\{ \overbrace{\begin{bmatrix} p \\ q \end{bmatrix}}^{\Delta\boldsymbol{\sigma}} = \overbrace{\begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix}}^{\mathbf{D}^{el}} \overbrace{\begin{bmatrix} \Delta\varepsilon_v \\ \Delta\varepsilon_q \end{bmatrix}}^{\Delta\boldsymbol{\varepsilon}} \right.$$

$$K = \frac{E}{3(1-2\nu)} = \frac{10000}{3(1-0.6)} = 8333{,}33$$

$$G = \frac{E}{2(1+\nu)} = \frac{10000}{2(1+0.3)} = 3846{,}15$$

$$\Delta\varepsilon_v = \frac{1}{K}\Delta p = 70{,}1/8333{,}33 = 0{,}841\%$$

$$\Delta\varepsilon_q = \frac{1}{3G}\Delta q = 210.4/3846{,}15 = 1{,}82\%$$

Let’s check in Excel

Driver:

Strain increment $d\varepsilon_{11}$

$$\left\{ \overbrace{\begin{bmatrix} p \\ q \end{bmatrix}}^{\Delta\boldsymbol{\sigma}} = \overbrace{\begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix}}^{\mathbf{D}^{el}} \overbrace{\begin{bmatrix} \Delta\varepsilon_v \\ \Delta\varepsilon_q \end{bmatrix}}^{\Delta\boldsymbol{\varepsilon}} \right.$$

We know that:

p/q = 1/3 = K / 3G * $d\varepsilon_v$ / $d\varepsilon_q$ → G / K = $d\varepsilon_v$ / $d\varepsilon_q$

$$\Delta\varepsilon_v = \Delta\varepsilon_{11} + 2\Delta\varepsilon_{33}$$

$$\Delta\varepsilon_q = 2/3(\Delta\varepsilon_{11} - \Delta\varepsilon_{33})$$

-- we have 1 eq + 2 eq connecting strains and stress+ 2 eq defining strains

So – 5 unknowns – (p, q), $d\varepsilon_q$, $d\varepsilon_v$, $d\varepsilon_{33}$

Stop condition:

$$F = (\sigma_1' - \sigma_3') - (\sigma_1' + \sigma_3')\sin\phi' - 2c'\cos\phi' = 0$$

MC model p'- q- space

$$F = (\sigma_1' - \sigma_3') - (\sigma_1' + \sigma_3')\sin\phi' - 2c'\cos\phi' = 0$$

$$\sigma'_1 = 3p' - 2\sigma'_3 \qquad \sigma'_3 = \sigma'_1 - q$$

$$\sigma'_1 = 3p' - 2\sigma'_1 + 2q \quad = \quad \frac{3p' + 2q}{3}$$

$$\sigma'_3 = 3p' - 2\sigma'_3 - q \quad = \quad \frac{3p' - q}{3}$$

$$\sigma'_1 + \sigma'_3 = \frac{6p' + q}{3}$$

MC model p'- q- space

$$or \quad q = \sin\phi\left(\frac{6p' + q}{3}\right) + 2c\cos\phi$$

$$3q = 6p'\sin\phi + q\sin\phi + 6c\cos\phi$$

$$q = \frac{6\sin\phi'}{3 - \sin\phi'}p' + \frac{6c\cos\phi'}{3 - \sin\phi'}$$

$$q = \eta p' + c^*$$

$$where \quad \eta = \frac{6\sin\phi'}{3 - \sin\phi'}, c^* = \frac{6c\cos\phi'}{3 - \sin\phi'}$$

$$F = q - \eta p' - c^* = 0$$

MC - Model formulated in p' - q

MC model p'-q- space

$$\begin{Bmatrix} \Delta p' \\ \Delta q \end{Bmatrix} = \begin{bmatrix} K' & 0 \\ 0 & 3G' \end{bmatrix} \begin{Bmatrix} \Delta \varepsilon_v^e \\ \Delta \varepsilon_q^e \end{Bmatrix}$$

Assuming associated flow rule and ideal plasticity

$$F\left(p', q\right) = q - \eta p' - c^* = 0$$

$$\frac{\partial F}{\partial p'} dp' + \frac{\partial F}{\partial q} dq = 0 \quad :$$ **consistency condition**

Formulation of D^{ep} for MC

$$dp' = K\left(d\varepsilon_v - d\varepsilon_v^p\right)$$

$$dq = 3G\left(d\varepsilon_q - d\varepsilon_q^p\right)$$

$$d\varepsilon_v^p = d\lambda \frac{\partial F}{\partial p'}$$

$$d\varepsilon_q^p = d\lambda \frac{\partial F}{\partial q}$$

Substituting into consistency condition leads to:

$$\frac{\partial F}{\partial p'} K d\varepsilon_v - \frac{\partial F}{\partial p'} K d\lambda \frac{\partial F}{\partial p'} + \frac{\partial F}{\partial Q} 3G d\varepsilon_q - \frac{\partial F}{\partial q} 3G d\lambda \frac{\partial F}{\partial q} = 0$$

Formulation of D^{ep} for MC

$$d\lambda = \frac{\frac{\partial F}{\partial p'} K d\varepsilon_v + \frac{\partial F}{\partial q'} 3G d\varepsilon_q}{\frac{\partial F}{\partial p'} K \frac{\partial F}{\partial p'} + \frac{\partial F}{\partial q} 3G \frac{\partial F}{\partial q}} = \frac{\left[\frac{\partial F}{\partial p'}, \frac{\partial F}{\partial q}\right] \begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} \begin{Bmatrix} d\varepsilon_v \\ d\varepsilon_q \end{Bmatrix}}{\left[\frac{\partial F}{\partial p'}, \frac{\partial F}{\partial q}\right] \begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} \begin{Bmatrix} \frac{\partial F}{\partial p'} \\ \frac{\partial F}{\partial q} \end{Bmatrix}}$$

Formulation of D^{ep} for MC

$$\frac{\partial F}{\partial p'} = -\eta, \quad \frac{\partial F}{\partial q} = 1, \quad d\lambda = \frac{-\eta K d\varepsilon_v + 3G d\varepsilon_q}{\eta^2 K + 3G}$$

$$d\varepsilon_v^p = -d\lambda\eta, \quad d\varepsilon_q^p = d\lambda$$

$$F = q - \eta p' - c^* = 0$$

$$\begin{Bmatrix} dp' \\ dq \end{Bmatrix} = \begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} \begin{Bmatrix} d\varepsilon_v - d\varepsilon_v^p \\ d\varepsilon_q - d\varepsilon_q^p \end{Bmatrix}$$

$$= \begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} \begin{Bmatrix} d\varepsilon_v \\ d\varepsilon_q \end{Bmatrix} - \begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} d\lambda \begin{Bmatrix} -\eta \\ 1 \end{Bmatrix}$$

$$= \begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} \begin{Bmatrix} d\varepsilon_v \\ d\varepsilon_q \end{Bmatrix} - \begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} \begin{Bmatrix} -\eta \\ 1 \end{Bmatrix} \frac{\begin{bmatrix} -\eta K & 3G \end{bmatrix}}{\eta^2 K + 3G} \begin{Bmatrix} d\varepsilon_v \\ d\varepsilon_q \end{Bmatrix}$$

Formulation of D^{ep} for MC

$$= \left[\begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} - \frac{1}{\eta^2 K + 3G} \begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} \begin{bmatrix} \eta^2 K & -3G\eta \\ -\eta K & 3G \end{bmatrix} \right] \begin{Bmatrix} d\varepsilon_v \\ d\varepsilon_q \end{Bmatrix}$$

$$= \left[\begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} - \frac{1}{\eta^2 K + 3G} \begin{bmatrix} \eta^2 K^2 & -3GK\eta \\ -3GK\eta & 9G^2 \end{bmatrix} \right] \begin{Bmatrix} d\varepsilon_v \\ d\varepsilon_q \end{Bmatrix}$$

$$= \begin{bmatrix} K - \dfrac{\eta^2 K^2}{\eta^2 K + 3G} & \dfrac{3GK\eta}{\eta^2 K + 3G} \\ \dfrac{3GK\eta}{\eta^2 K + 3G} & 3G - \dfrac{9G^2}{\eta^2 K + 3G} \end{bmatrix} \begin{Bmatrix} d\varepsilon_v \\ d\varepsilon_q \end{Bmatrix}$$

Formulation of D^{ep} for MC

$$D^{ep} = \begin{bmatrix} K - \dfrac{\eta^2 K^2}{\eta^2 K + 3G} & \dfrac{3GK\eta}{\eta^2 K + 3G} \\ \dfrac{3GK\eta}{\eta^2 K + 3G} & 3G - \dfrac{9G^2}{\eta^2 K + 3G} \end{bmatrix}$$

$$\det\left|D^{ep}\right| = 0$$ → **perfect plasticity**

In general form :

for non associated flow:

$$\frac{\partial Q}{\partial \sigma}$$

$$D^{ep} = \frac{D^e - D^e \dfrac{\partial F}{\partial \underset{\sim}{\sigma}} \left(\dfrac{\partial F}{\partial \underset{\sim}{\sigma}}\right)^T D^{eT}}{\left(\dfrac{\partial F}{\partial \underset{\sim}{\sigma}}\right)^T D^e \dfrac{\partial F}{\partial \underset{\sim}{\sigma}}}$$

MC model p'-q- space

$$\begin{Bmatrix} \Delta p' \\ \Delta q \end{Bmatrix} = \begin{bmatrix} K' & 0 \\ 0 & 3G' \end{bmatrix} \begin{Bmatrix} \Delta \varepsilon_v^e \\ \Delta \varepsilon_q^e \end{Bmatrix}$$

Assuming associated flow rule and ideal plasticity

$$F\left(p', q\right) = q - \eta p' - c^* = 0$$

$$\frac{\partial F}{\partial p'} dp' + \frac{\partial F}{\partial q} dq = 0 \quad :$$ **consistency condition**

Formulation of D^{ep} for MC

$$dp' = K\left(d\varepsilon_v - d\varepsilon_v^p\right)$$

$$dq = 3G\left(d\varepsilon_q - d\varepsilon_q^p\right)$$

$$d\varepsilon_v^{pl} = \lambda \frac{\partial Q}{\partial p'}$$

$$d\varepsilon_q^{pl} = \lambda \frac{\partial Q}{\partial q'}$$

Substituting into consistency condition leads to:

$$dF = \frac{\partial F}{\partial p'} dp' + \frac{\partial F}{\partial q} dq = 0$$

Formulation of D^{ep} for MC

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$$dF = \frac{\partial F}{\partial p'} dp' + \frac{\partial F}{\partial q} dq = 0$$

$$dF = \frac{\partial F}{\partial p'} K\left(d\varepsilon_v - d\varepsilon_v^{pl}\right) + \frac{\partial F}{\partial q} 3G\left(d\varepsilon_q - d\varepsilon_q^{pl}\right) = 0$$

$$dF = \frac{\partial F}{\partial p'} K\left(d\varepsilon_v - \lambda \frac{\partial Q}{\partial p'}\right) + \frac{\partial F}{\partial q} 3G\left(d\varepsilon_q - \lambda \frac{\partial Q}{\partial q'}\right) = 0$$

$$\frac{\partial F}{\partial p'} K d\varepsilon_v - \frac{\partial F}{\partial p'} K\lambda \frac{\partial Q}{\partial p'} + \frac{\partial F}{\partial q} 3G d\varepsilon_q - \frac{\partial F}{\partial q} 3G\lambda \frac{\partial Q}{\partial q'} = 0$$

Formulation of D^{ep} for MC

$$dp' = K\left(d\varepsilon_v - d\varepsilon_v^p\right)$$

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$$\frac{\partial F}{\partial p'} K d\varepsilon_v - \frac{\partial F}{\partial p'} K\lambda \frac{\partial Q}{\partial p'} + \frac{\partial F}{\partial q} 3G d\varepsilon_q - \frac{\partial F}{\partial q} 3G\lambda \frac{\partial Q}{\partial q'} = 0$$

$$\frac{\partial F}{\partial p'} K d\varepsilon_v + \frac{\partial F}{\partial q} 3G d\varepsilon_q - \lambda\left(\frac{\partial F}{\partial p'} K \frac{\partial Q}{\partial p'} + \frac{\partial F}{\partial q} 3G \frac{\partial Q}{\partial q'}\right) = 0$$

$$\lambda = \frac{\dfrac{\partial F}{\partial p'} K d\varepsilon_v + \dfrac{\partial F}{\partial q} 3G d\varepsilon_q}{\left(\dfrac{\partial F}{\partial p'} K \dfrac{\partial Q}{\partial p'} + \dfrac{\partial F}{\partial q} 3G \dfrac{\partial Q}{\partial q'}\right)}$$

Formulation of D^ep for MC

$$F = q - \eta p' - c^*$$

$$\eta = \frac{6\sin\phi'}{3-\sin\phi'}$$

$$Q = q - \xi p'$$

$$\xi = \frac{6\sin\psi}{3-\sin\psi}$$

$$\lambda = \frac{\frac{\partial F}{\partial p'} K d\varepsilon_v + \frac{\partial F}{\partial q} 3G d\varepsilon_q}{\left(\frac{\partial F}{\partial p'} K \frac{\partial Q}{\partial p'} + \frac{\partial F}{\partial q} 3G \frac{\partial Q}{\partial q'}\right)}$$

$$\frac{\partial F}{\partial p'} = -\eta \qquad \frac{\partial F}{\partial q} = 1$$

$$\frac{\partial Q}{\partial p'} = -\xi \qquad \frac{\partial Q}{\partial q} = 1$$

Formulation of D^{ep} for MC

$$\frac{\partial F}{\partial p'} = -\eta \qquad \frac{\partial F}{\partial q} = 1$$

$$\frac{\partial Q}{\partial p'} = -\xi \qquad \frac{\partial Q}{\partial q} = 1$$

$$\lambda = \frac{\frac{\partial F}{\partial p'} K d\varepsilon_v + \frac{\partial F}{\partial q} 3G d\varepsilon_q}{\left(\frac{\partial F}{\partial p'} K \frac{\partial Q}{\partial p'} + \frac{\partial F}{\partial q} 3G \frac{\partial Q}{\partial q'} \right)}$$

$$\lambda = \frac{\frac{\partial F}{\partial p'} K d\varepsilon_v + \frac{\partial F}{\partial q} 3G d\varepsilon_q}{\left(\frac{\partial F}{\partial p'} K \frac{\partial Q}{\partial p'} + \frac{\partial F}{\partial q} 3G \frac{\partial Q}{\partial q'} \right)} = \frac{-\eta K d\varepsilon_v + 3G d\varepsilon_q}{\left(\eta K \xi + 3G \right)}$$

Formulation of D^{ep} for MC

$$\lambda = \frac{\frac{\partial F}{\partial p'} K d\varepsilon_v + \frac{\partial F}{\partial q} 3G d\varepsilon_q}{\left(\frac{\partial F}{\partial p'} K \frac{\partial Q}{\partial p'} + \frac{\partial F}{\partial q} 3G \frac{\partial Q}{\partial q'} \right)} = \frac{-\eta K d\varepsilon_v + 3G d\varepsilon_q}{(\eta K \xi + 3G)}$$

$$\eta = \frac{6 \sin\phi'}{3 - \sin\phi'}$$

$$\xi = \frac{6 \sin\psi}{3 - \sin\psi}$$

$$dp' = K\left(d\varepsilon_v - d\varepsilon_v^p \right)$$

$$dq = 3G\left(d\varepsilon_q - d\varepsilon_q^p \right)$$

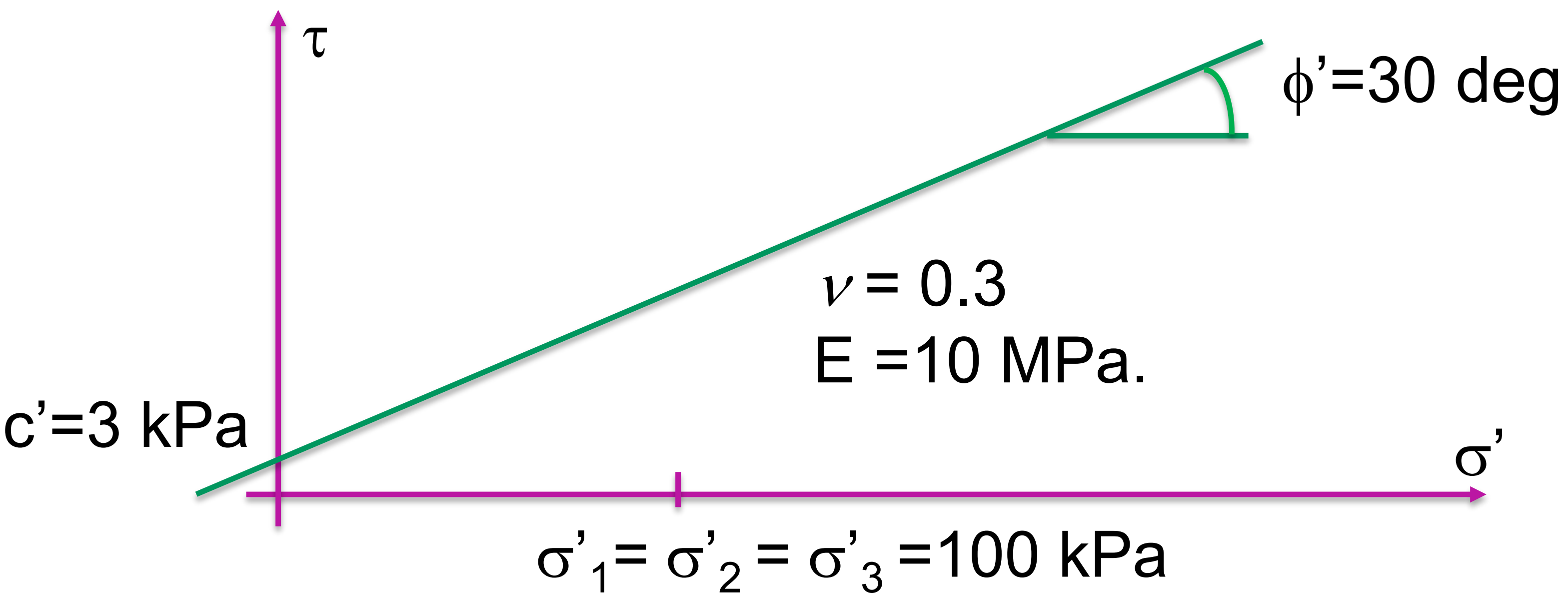
Task for the PhD students (MSc students: 5% extra at the Lecture Test 1 for each case)

Simulate standard triaxial drained and **undrained tests**, assuming that soil is behaving as a Mohr-Coulomb material.

Assume initial condition p=100 kPa, q=0.
Assume Mohr-Coulomb soil parameters:

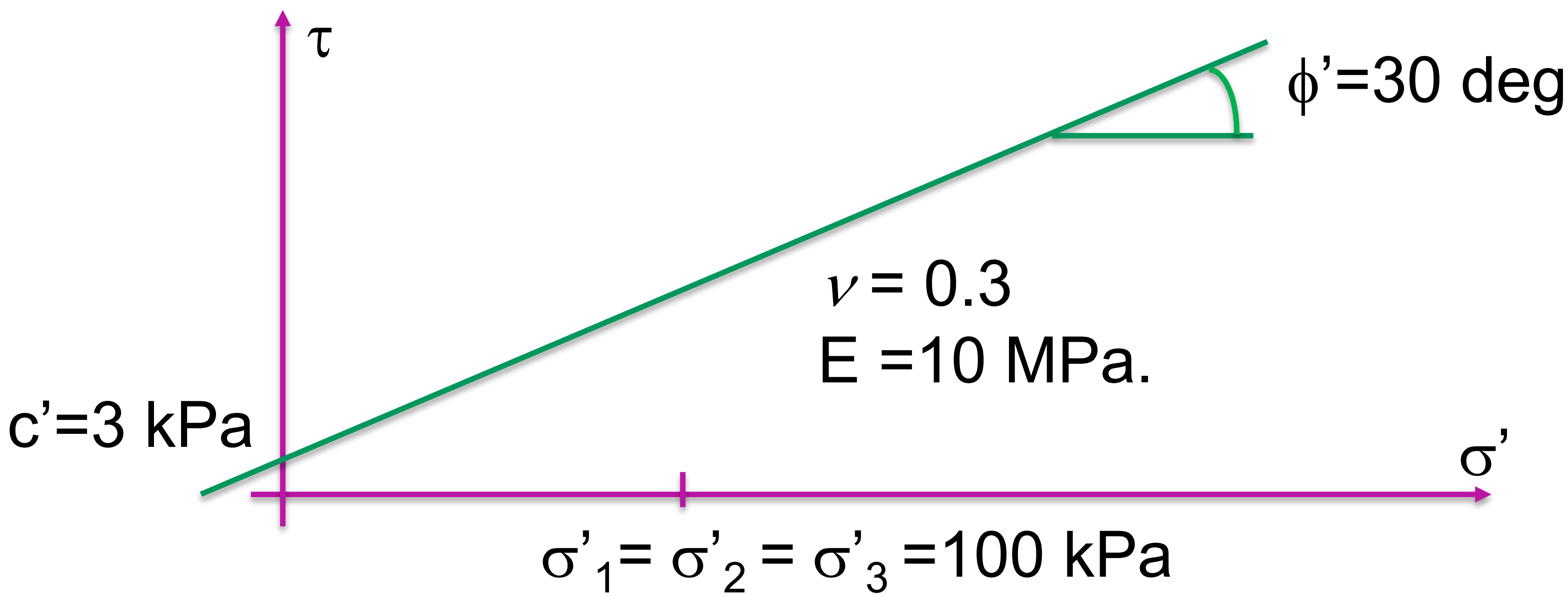
- friction angle 30 degrees
- dilation angle: 0 degrees, 3 degrees, 30 degrees
- cohesion c’ equal to 3 kPa
- Poisson’s ratio 0.3
- Young’s modulus 10 MPa.

Undrained Test



In undrained test, volume is constant

Undrained Test

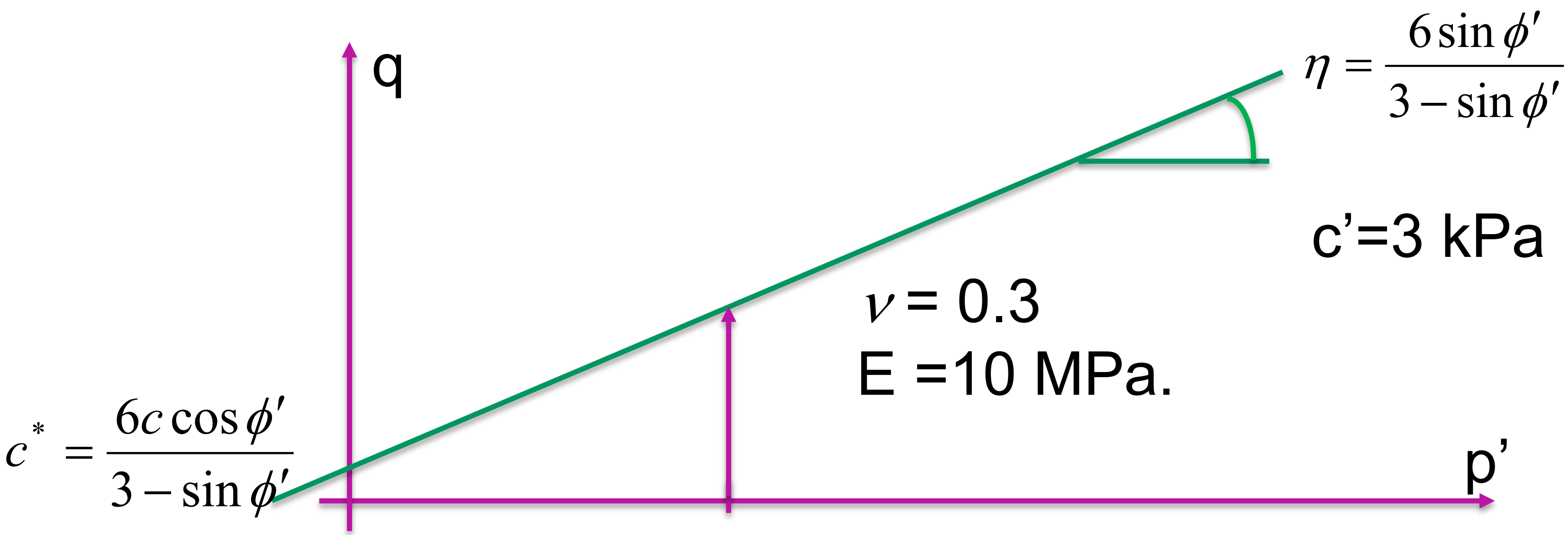


In elasticity no plastic strain, but

$$dp' = K\left(d\varepsilon_v - d\varepsilon_v^p\right)$$

$$dq = 3G\left(d\varepsilon_q - d\varepsilon_q^p\right)$$

Undrained Test



$$F = q - \eta p' - c^*$$

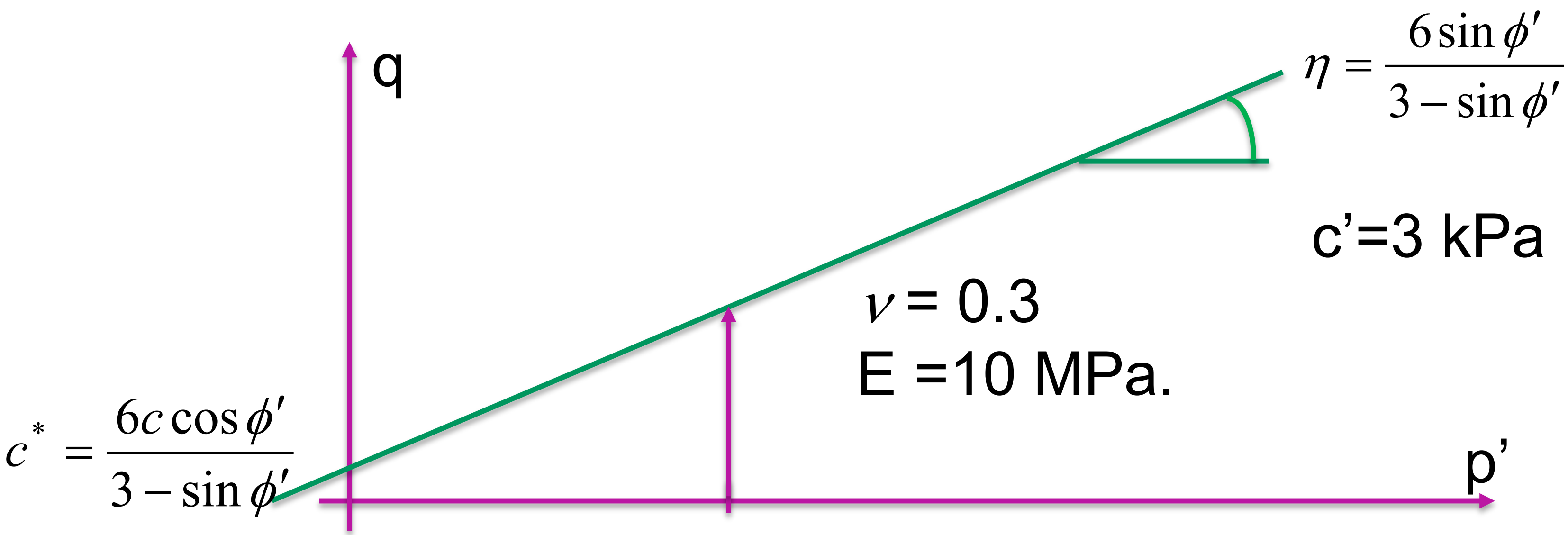
So, dp' must be zero...

All the change in p, must be turned into u

$$dp' = K\left(d\varepsilon_v - d\varepsilon_v^p\right)$$

$$dq = 3G\left(d\varepsilon_q - d\varepsilon_q^p\right)$$

Undrained Test



$$F = q - \eta p' - c^*$$

So, at yield, p’= 100 kPa, q=ηp’+c*

Undrained Test

After yield, same equations, but we demand that

$$d\varepsilon_v = 0 = d\varepsilon_v^{el} + d\varepsilon_v^{pl}$$

Instead of having q/p'=3

Undrained Test

After yield, same equations, but we demand that

$$d\varepsilon_v = 0 = d\varepsilon_v^{el} + d\varepsilon_v^{pl}$$

$$\eta = \frac{6\sin\phi'}{3-\sin\phi'}$$

$$\xi = \frac{6\sin\psi}{3-\sin\psi}$$

$$\lambda = \frac{-\eta K d\varepsilon_v + 3G d\varepsilon_q}{(\eta K \xi + 3G)}$$

$$dp' = K\left(d\varepsilon_v - d\varepsilon_v^p\right)$$

$$dq = 3G\left(d\varepsilon_q - d\varepsilon_q^p\right)$$

we also know that u=p-p' and q/p=3

Thank you