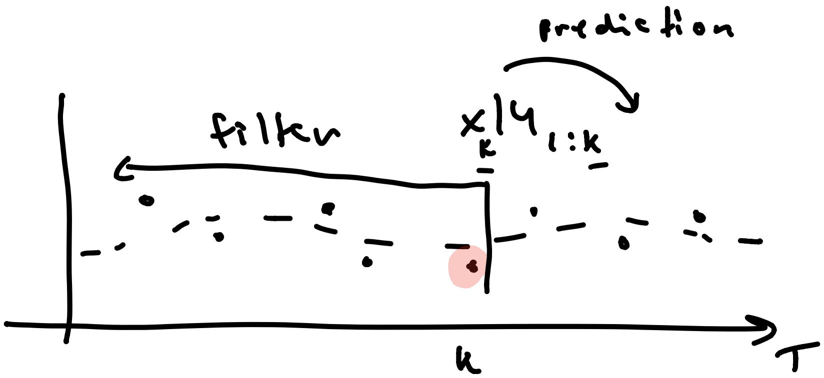

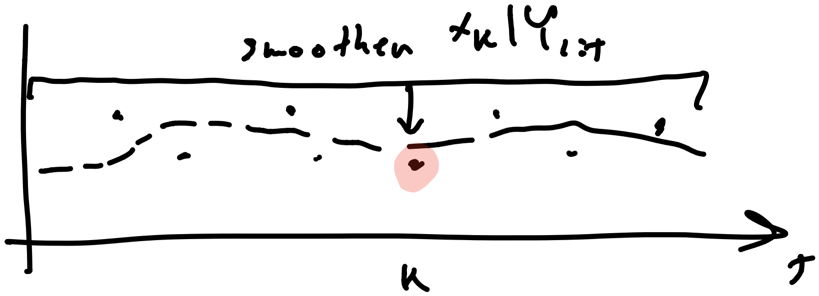

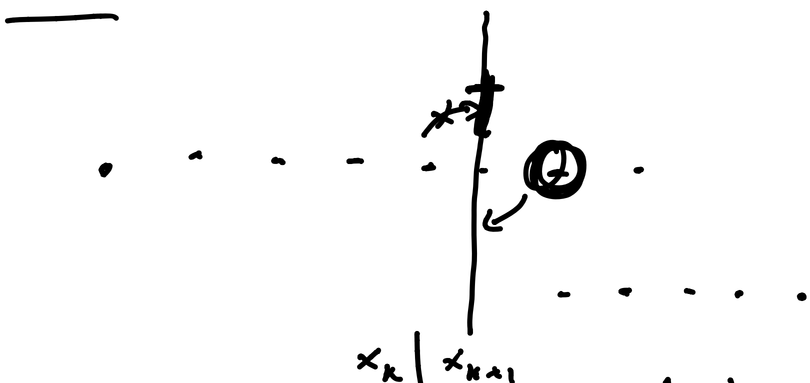

$$p(B|A) = \frac{p(B,A)}{p(A)} = \frac{p(A|B)\,p(B)}{p(A)}$$

$$
\begin{aligned}
& p(x_k \mid x_{k+1}, \underline{y_{1:T}}) \\
&= p(x_k \mid x_{k+1}, y_{1:k}) \\
&= \frac{p(x_k, x_{k+1} \mid y_{1:k})}{p(x_{k+1} \mid y_{1:k})} \\
&= \frac{p(x_{k+1} \mid x_k, \cancel{y_{1:k}})\, p(x_k \mid y_{1:k})}{p(x_{k+1} \mid y_{1:k})} \\
&= \frac{p(x_{k+1} \mid x_k)\, p(x_k \mid y_{1:k})}{p(x_{k+1} \mid y_{1:k})}
\end{aligned}
$$

assume that we know

$$p(x_{k+1} | y_{1:T})$$

we wish to find

$$p(x_k | y_{1:T})$$

$$p(x_k, x_{k+1} | y_{1:T})$$
$$= p(x_k | x_{k+1}, y_{1:T})\, p(x_{k+1} | y_{1:T})$$
$$= \frac{p(x_{k+1} | x_k)\, p(x_k | y_{1:k})}{p(x_{k+1} | y_{1:k})} \cdot p(x_{k+1} | y_{1:T})$$

$\int (\cdot)\, dx_{k+1}$ both sides:

$$p(x_k | y_{1:T}) = \int \frac{p(x_{k+1} | x_k)\, p(x_k | y_{1:k})}{p(x_{k+1} | y_{1:k})} \cdot p(x_{k+1} | y_{1:T})\, dx_{k+1}$$
$$= p(x_k | y_{1:k}) \cdot \int \frac{p(x_{k+1} | x_k)\, p(x_{k+1} | y_{1:T})}{p(x_{k+1} | y_{:k})}\, dx_{k+1}$$

RTS: to get $p(x_k | x_{k+1}, y_{1:k})$ we first form $p(x_k, x_{k+1} | y_{1:k})$ and condition on x_{k+1}

$$p(x_k, x_{k+1} | y_{1:k})$$
$$= p(x_{k+1} | x_k, \cancel{y_{1:k}})\, p(x_k | y_{1:k})$$
$$= N(x_{k+1} | A x_k, Q)\, N(x_k | m_k, P_k)$$

$$\underbrace{\qquad\qquad}_{\text{"}p(y\mid x)\text{"}} \qquad \underbrace{\qquad\qquad}_{\text{"}p(x)\text{"}}$$

$\text{"}x_{k+1}\text{"} \nearrow y \qquad \text{"}x_k\text{"} \nearrow x \qquad \text{"}x_k\text{"} \nearrow x$

$$p\left(\begin{pmatrix} x_k \\ x_{k+1} \end{pmatrix} \middle| y_{1:k}\right)$$

$$= N\left(\begin{pmatrix} x_k \\ x_{k+1} \end{pmatrix} \middle| \begin{pmatrix} m_k \\ A m_k \end{pmatrix}, \begin{pmatrix} P_k & P_k A^T \\ A P_k & A P_k A^T + Q \end{pmatrix}\right)$$

$$p(x_k \mid x_{k+1}, y_{1:k})$$

$$= N\big(x_k \mid m_k + P_k A^T (A P_k A^T + Q)^{-1}(x_{k+1} - A m_k),$$

$$P_k - P_k A^T (A P_k A^T + Q)^{-1} A P_k \big)$$

$$= p(x_k \mid x_{k+1}, y_{1:T}) \qquad (\ldots) = P_k - G_k (A P_k A^T + Q) \cdot G_k^T$$

$$p(x_{k+1}, x_k \mid y_{1:T})$$

$$= p(x_k \mid x_{k+1}, y_{1:T})\, p(x_{k+1} \mid y_{1:T})$$

$$= N\big(x_k \mid P_k A^T (A P_k A^T + Q)^{-1} x_{k+1} + \underbrace{m_k - P_k A^T (A P_k A^T + Q)^{-1} A m_k}, (\ldots)\big) \cdot \underbrace{N(x_{k+1} \mid m^s_{k+1}, P^s_{k+1})}_{\text{assumption}}$$

$G_k = P_k A^T (A P_k A^T + Q)^{-1}$

$U_k = m_k - G_k A m_k$

$$= \underbrace{N(x_k \mid G_k x_{k+1} + U_k, (\ldots))}_{\text{"}p(y \mid Hx + U)\text{"}} \; \underbrace{N(x_{k+1} \mid m^s_{k+1}, P^s_{k+1})}_{\text{"}p(x)\text{"}}$$

$$= N\left(\begin{pmatrix} x_{k+1} \\ x_k \end{pmatrix} \middle| \begin{pmatrix} m^s_{k+1} \\ G_k m^s_{k+1} + U_k \end{pmatrix}, \begin{pmatrix} P^s_{k+1} & P^s_{k+1} G_k^T \end{pmatrix}\right)$$

$$
\begin{pmatrix} G_k P^s_{k+1} & G_k P^s_{k+1} G_k^T + \\ & P_k - G_k (A P_k A^T + Q) G_k^T \end{pmatrix}
$$

$$
\begin{aligned}
&p(x_k \mid y_{1:T}) \\
&= \mathcal{N}\Big(x_k \;\Big|\; G_k m^s_{k+1} + \overbrace{m_k - G_k A m_k}^{v_k}, \\
&\qquad G_k P^s_{k+1} G_k^T + P_k - G_k (A P_k A^T + Q) G_k^T\Big) \\
&= \mathcal{N}\Big(x_k \;\Big|\; m_k + G_k (m^s_{k+1} - \underbrace{A m_k}_{m^-_k}), \\
&\qquad P_k + G_k \big(P^s_{k+1} - \underbrace{[A P_k A^T + Q]}_{P^-_k}\big) G_k^T\Big)
\end{aligned}
$$

$$
G_k = P_k A^T (A P_k A^T + Q)^{-1}
$$