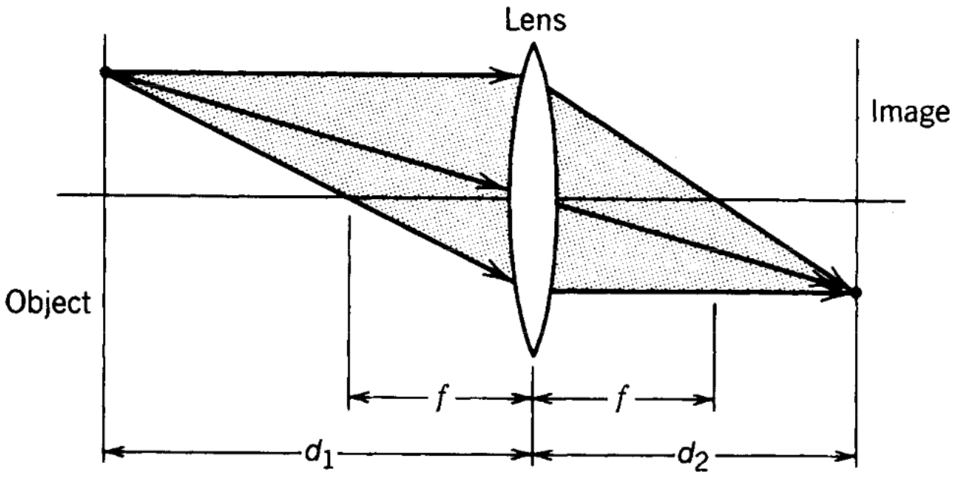
Chapter 4

FOURIER OPTICS II

Image formation

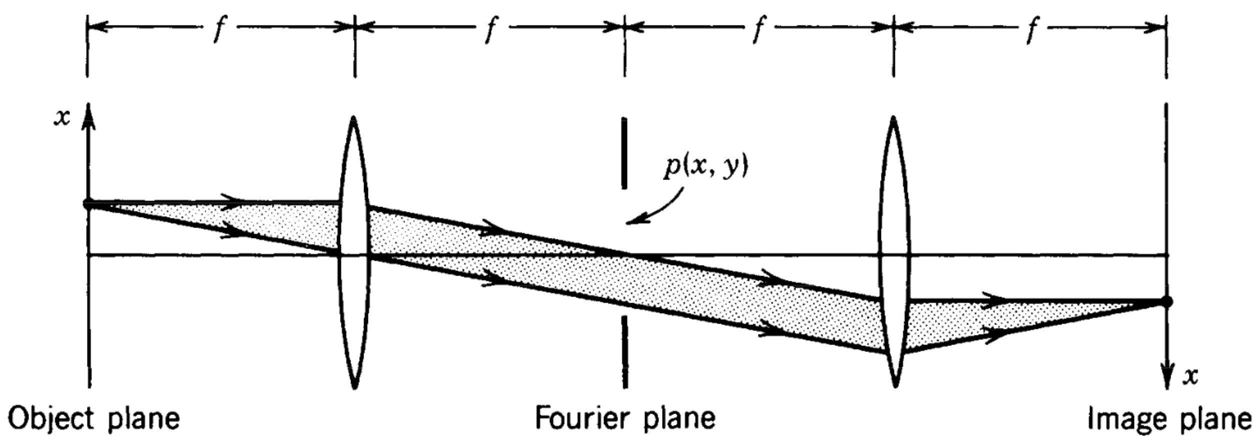


Perfect non-diffracted images are calculated using the ray-optics lens equations:

$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f},$$

$$M = -\frac{d_2}{d_1}.$$

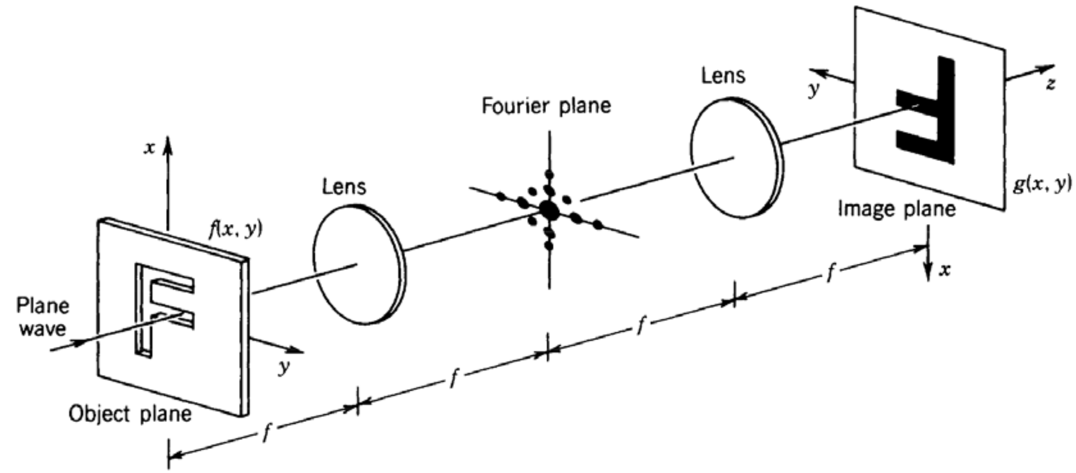
For a **4-f imaging system**, $M = -1$. In the central, Fourier plane, however, the spatial spectrum can be filtered, e.g., with an aperture of

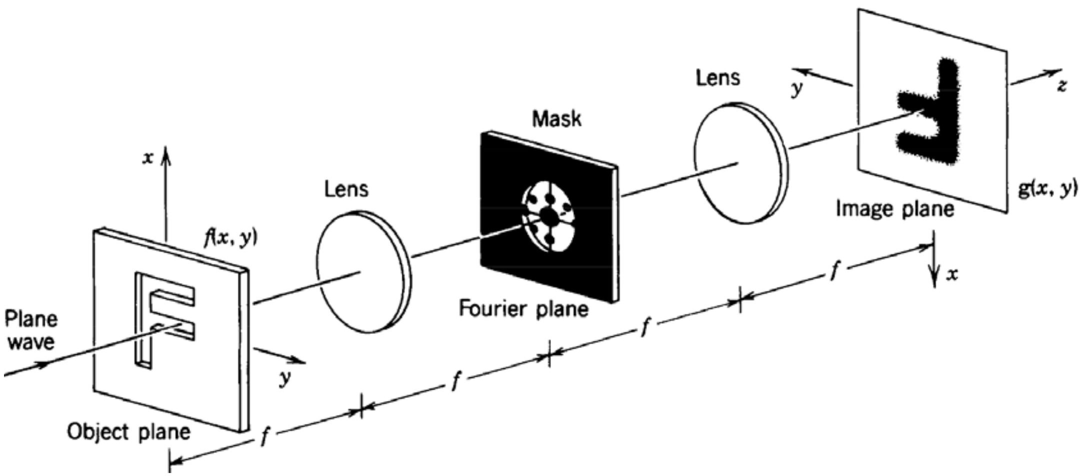


$$p(x,y) = \mathrm{circ}\left(\frac{x}{a}, \frac{y}{a}\right)$$

$$= \begin{cases} 1, x^2 + y^2 \le a^2 \\ 0, \quad \text{otherwise} \end{cases}$$

Spatial filtering





In the Fourier plane,

$$x = \frac{\lambda}{\Lambda_x} f = \lambda f \nu_x,$$

$$y = \frac{\lambda}{\Lambda_y} f = \lambda f \nu_y.$$

With the aperture, the transfer function becomes

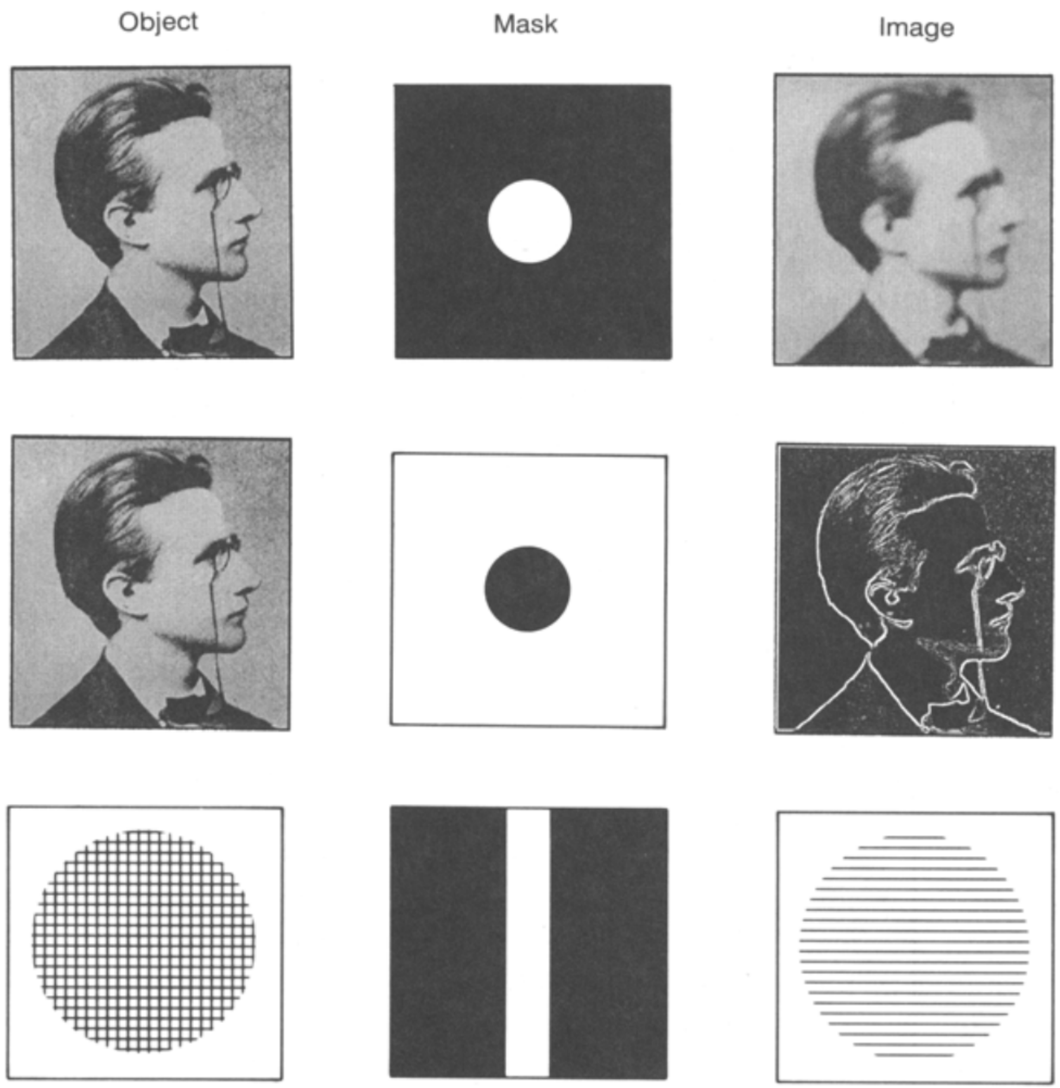
$$H(\nu_x, \nu_y) = p(\lambda f \nu_x, \lambda f \nu_y)$$

Its inverse Fourier transform gives the impulse-response function

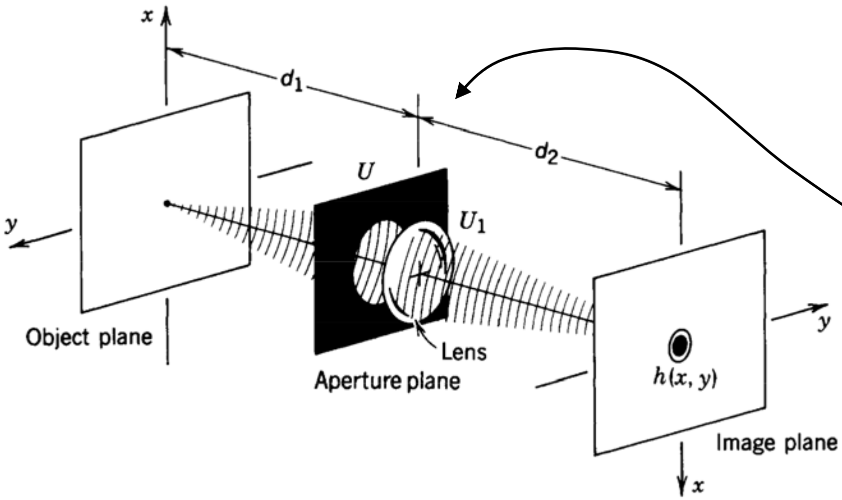
$$h(x, y) = \frac{1}{(\lambda f)^2} P\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right)$$

For a circular aperture, $p(x, y) = \mathrm{circ}\left(\frac{x}{a}, \frac{y}{a}\right) \Rightarrow P(\rho) = a^2 \frac{J_1(2\pi a \rho)}{a\rho}$, where $\rho = \frac{\sqrt{x^2+y^2}}{\lambda f}$

Low-pass, high-pass, and vertical-pass spatial filters



Wave optics of a single-lens imaging system



The impulse-response function $h(x, y)$ is the image of a point source in the object plane:

$$U(x, y) \approx h_1 \exp\left[-jk\frac{x^2+y^2}{2d_1}\right],$$

$$U_1(x, y) = U(x, y) \exp\left(jk\frac{x^2+y^2}{2f}\right) p(x, y)$$

$$h(x, y) = h_2 \iint_{-\infty}^{\infty} U_1(x', y') \exp\left[-j\pi\frac{(x-x')^2+(y-y')^2}{\lambda d_2}\right] dx'\,dy', \text{ where } h_2 = (j/\lambda d_2)\exp(-jkd_2).$$

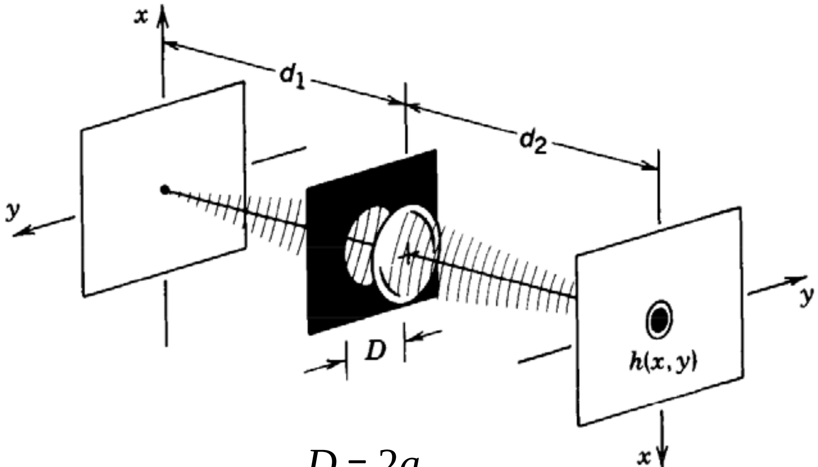
$$\Rightarrow h(x, y) = h_1 h_2 \exp\left(-j\pi\frac{x^2+y^2}{\lambda d_2}\right) P_1\left(\frac{x}{\lambda d_2}, \frac{y}{\lambda d_2}\right),$$

where P_1 is the FT of the *generalized pupil function* $p_1(x, y) = p(x, y) \exp\left(-j\pi\epsilon\frac{x^2+y^2}{\lambda}\right).$

Since in practice, $h(x,y)$ is very localized, we have $h(x, y) \approx \frac{1}{\lambda d_1}\frac{1}{\lambda d_2} P\left(\frac{x}{\lambda d_2}, \frac{y}{\lambda d_2}\right).$

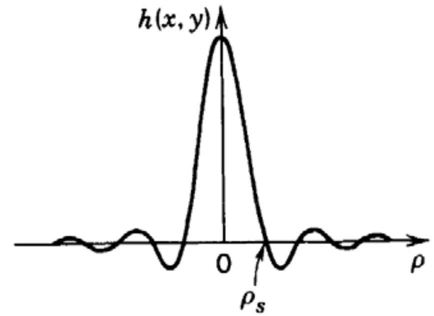
The *image field* is the convolution of the *magnified object field* (ray-optics image) with $h(x, y)$.

A single-lens imaging system with a circular aperture



$D = 2a$

$$h(x,y) \approx \frac{1}{\lambda d_1}\frac{1}{\lambda d_2} P\left(\frac{x}{\lambda d_2}, \frac{y}{\lambda d_2}\right)$$

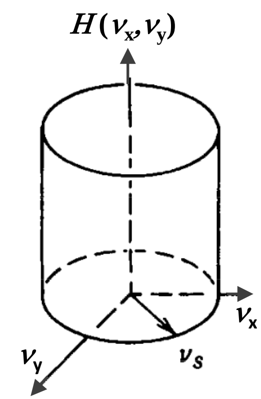


$$\rho_s = 1.22\frac{\lambda d_2}{D}$$

For a circular aperture, $p(x,y) = \mathrm{circ}\left(\frac{x}{a}, \frac{y}{a}\right) \Rightarrow P(\rho) = a^2 \frac{J_1(2\pi a\rho)}{a\rho}$.

With $\rho = \frac{r}{\lambda d_2}$, we have

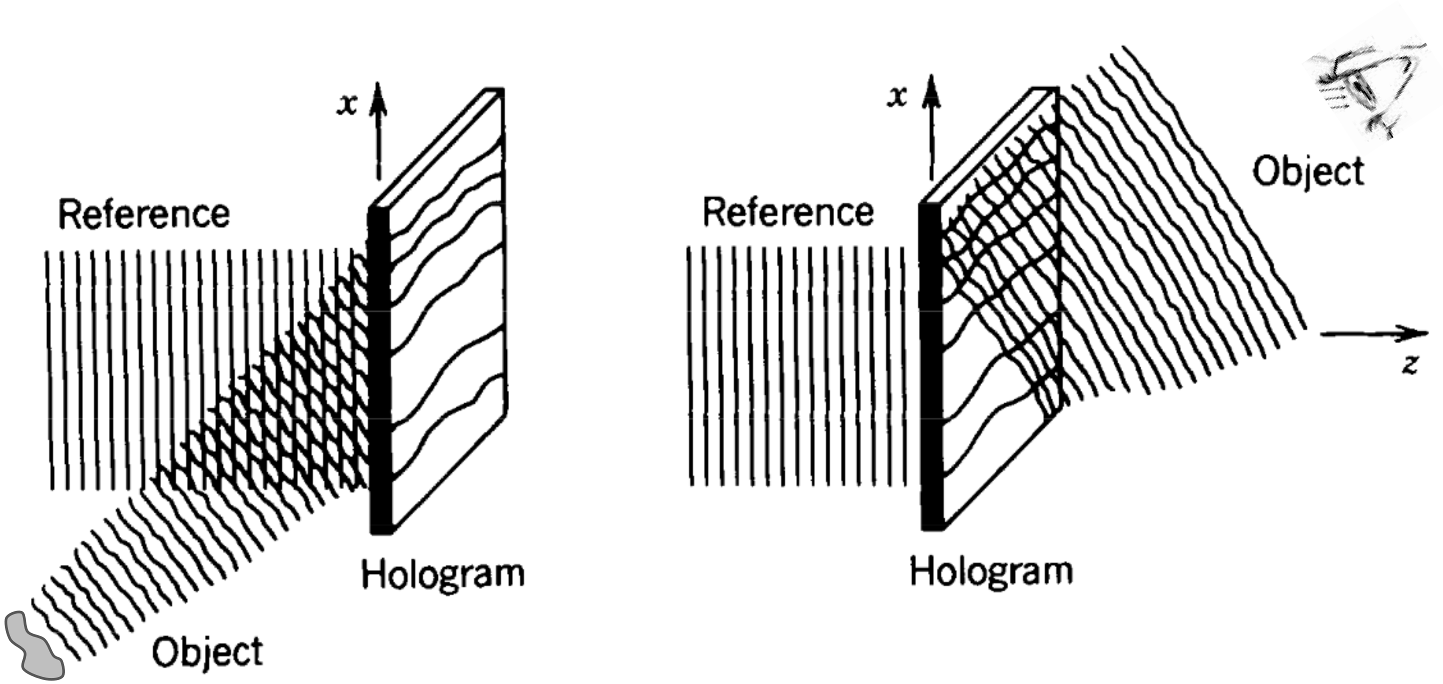
$$h(r) \approx \frac{1}{\lambda d_1}\frac{1}{\lambda d_2} a^2 \frac{J_1\left(\frac{2\pi a r}{\lambda d_2}\right)}{\frac{ar}{\lambda d_2}} = \frac{2\pi a^2}{\lambda^2 d_1 d_2}\frac{J_1\left(\frac{2\pi a r}{\lambda d_2}\right)}{\frac{2\pi a r}{\lambda d_2}} \Rightarrow r_s = 0.61\frac{\lambda d_2}{a}.$$



The ***transfer function*** is **FT**$\{h(r)\} \Rightarrow H(\nu_x, \nu_y) \approx p(\lambda d_2 \nu_x, \lambda d_2 \nu_y)$.

$$\nu_s = \frac{a}{\lambda d_2}.$$

Holography



Recording:

$$t \propto |U_o + U_r|^2 = |U_r|^2 + |U_o|^2 + U_r^* U_o + U_r U_o^*$$

$$= I_r + I_o + U_r^* U_o + U_r U_o^*$$

Reconstruction:

$$U = t U_r \propto U_r I_r + U_r I_o + I_r U_o + U_r^2 U_o^*$$

Divide by $\sqrt{I_r}$:

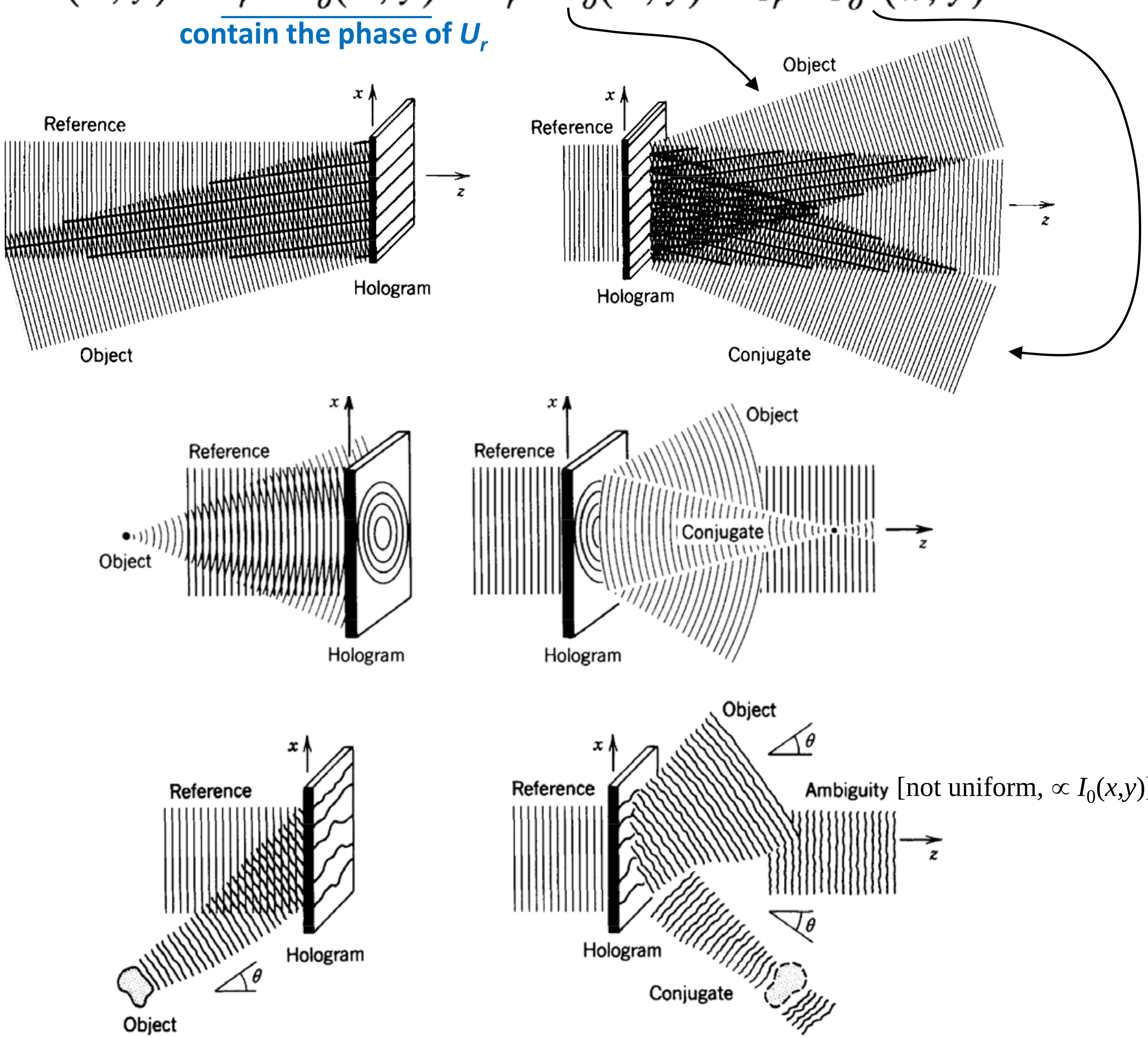
$$U(x, y) \propto I_r + I_o(x, y) + \underline{I_r^{1/2} U_o(x, y)} + I_r^{1/2} U_o^*(x, y)$$

also the phase

The reconstructed field components

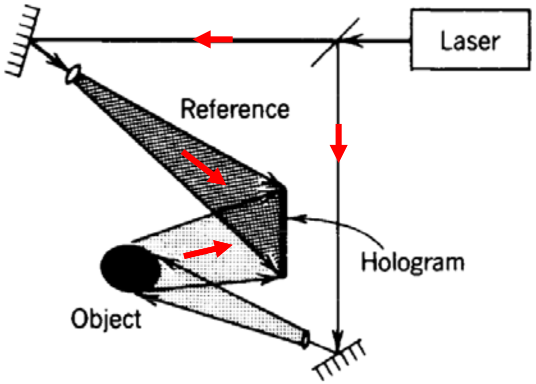
$$U(x,y) \propto I_r + I_o(x,y) + I_r^{1/2}U_o(x,y) + I_r^{1/2}U_o^*(x,y)$$

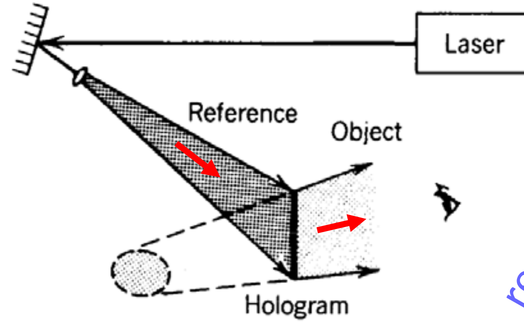
contain the phase of U_r



The holographic apparatus

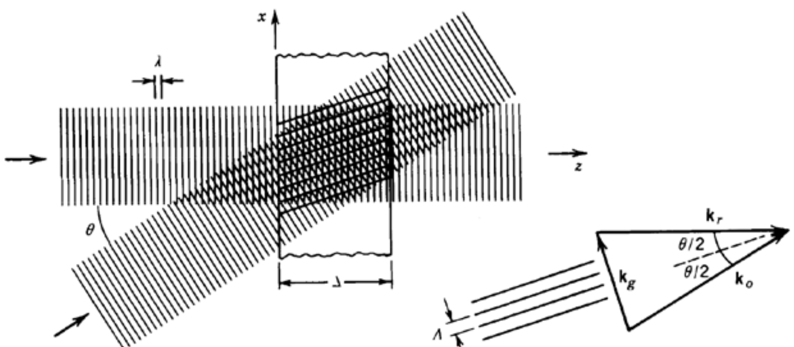
recording

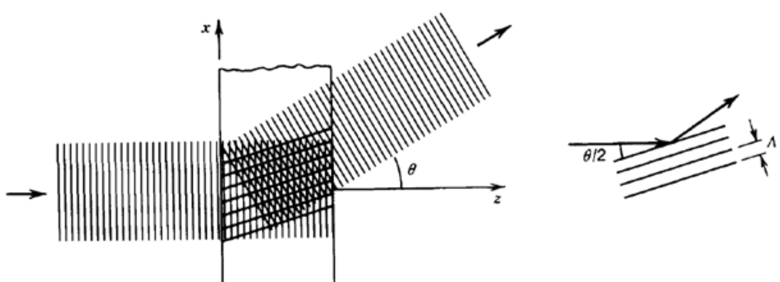




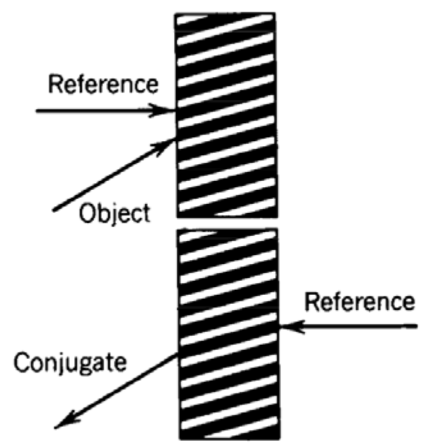
reconstruction

Volume holography (*also white-light reconstruction*)

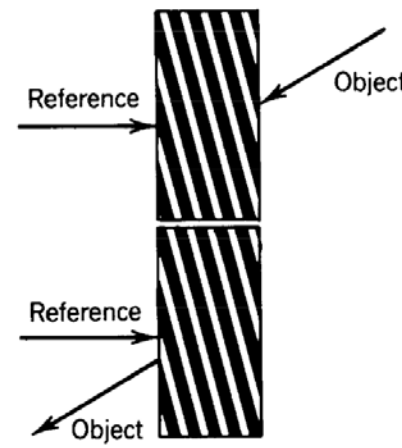




Other options:



Transmission hologram



Reflection hologram