# Traditional vehicle localization with Extended Kalman filter
# Case: Tractor-trailer

Arto Visala 1.3.2017
From dissertation of Juha Backman 2014

# Content

- **Introduction to Extended Kalman Filter**
  - Markov process
  - State estimation

- **Applications**
  - Tractor-trailer state estimate
  - Mass flow estimation

# Introduction to Extended Kalman Filter



Juha Backman
1.3.2017

# Markov process

- **Assuming a state vector** *x* **that completely describes the system**

$$x_k$$

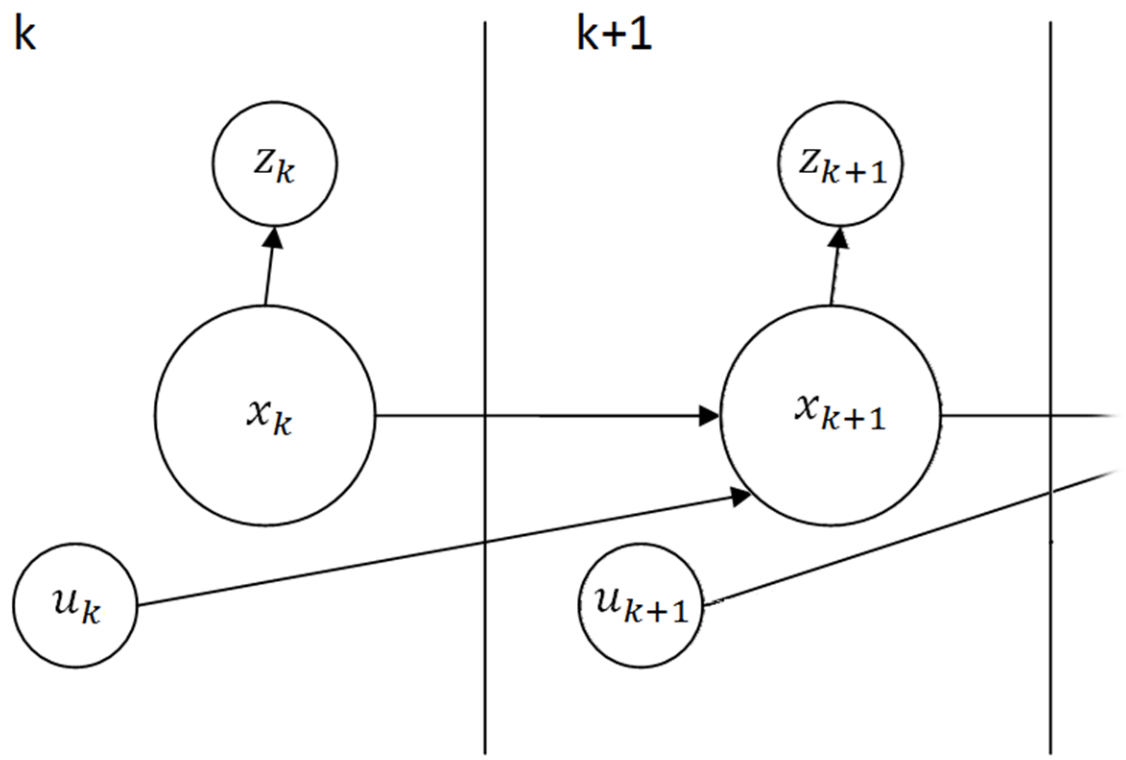
- **The state of the system depends only on the state at the previous instant of time, not on the entire past.**

$$p(x_{k+1}|x_k, x_{k-1}, x_{k-2}, \dots) = p(x_{k+1}|x_k)$$

- **More general presentation is when input** *u* **is affecting the next state and measurement** *z* **provides information on the present state.**

$$p(x_{k+1}|x_k, x_{k-1}, \dots u_k, u_{k-1}, \dots, z_{k+1}, z_k, \dots) = p(x_{k+1}|x_k, u_k, z_{k+1})$$

# Markov process



# Extended Kalman Filter

- **The Extended Kalman Filter model assumes that the state of the system at time *k* evolves to the state at *k+1* according to equation**

$$x_{k+1} = f(x_k, u_k) + v_k$$

$$v_k \sim N(0, Q_k)$$

- **A measurement of the state at an instant k is described by**

$$z_k = h(x_k) + w_k$$

$$w_k \sim N(0, R_k)$$

# Extended Kalman Filter

- **Assuming initial state is known...**

$$\hat{x}_{0|0} = x_0$$

- **...the state at an instant** k+1 **can be predicted.**

$$\hat{x}_{k+1|k} = f\left(\hat{x}_{k|k}, u_k\right)$$

- **Based on the estimated state, the prediction of the next measurement**

$$\hat{z}_{k+1|k} = h\left(\hat{x}_{k+1|k}\right)$$

- **...and the the difference between obtained measurement**

$$\nu_{k+1} = z_{k+1} - \hat{z}_{k+1|k}$$

# Extended Kalman Filter

- **Assuming also that the initial state covariance *P* is known...**

$$P_{0|0} = P_0$$

- **...the state prediction covariance can be calculated**

$$P_{k+1|k} = F_k P_{k|k} F_k^{'} + Q_k$$

$$F_k = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}_{k|k}}$$

- **The covariance update formula is based on linearization and assumption that the noise *v* is white and has zero mean**

# Extended Kalman Filter

- **Using the state prediction covariance, the measurement prediction covariance *S* is obtained**

$$S_{k+1} = H_{k+1} P_{k+1|k} H_{k+1}^{'} + R_{k+1}$$

$$H_{k+1} = \left.\frac{\partial h}{\partial x}\right|_{\hat{x}_{k+1|k}}$$

- **Again, the formula is based on linearization and assumption that the noise *w* is white and has zero mean**

# Extended Kalman Filter

- **The filter gain *W* is calculated from the state and measurement prediction covariances**

$$W_{k+1} = P_{k+1|k} H'_{k+1|k} S^{-1}_{k+1|k}$$

- **The updated state at the next instant of time is sum of the predicted state and the correction term**

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + W_{k+1} \nu_{k+1}$$

- **The updated covariance associated with the next state**

$$P_{k+1|k+1} = P_{k+1|k} - W_{k+1} S_{k+1} W'_{k+1}$$

# Extended Kalman Filter

- **The probability density function estimate of the Markov process state is**

$$p(x_k) \sim N\left(\hat{x}_{k|k}, P_{k|k}\right)$$

# Extended Kalman Filter - Summary

## State prediction

$$\hat{x}_{k+1|k} = f(\hat{x}_{k|k}, u_k)$$

$$F_k = \frac{\partial f}{\partial x}\bigg|_{\hat{x}_{k|k}}$$

$$P_{k+1|k} = F_k P_{k|k} F_k' + Q_k$$

## Measurement prediction

$$\hat{z}_{k+1|k} = h(\hat{x}_{k+1|k})$$

$$H_{k+1} = \frac{\partial h}{\partial x}\bigg|_{\hat{x}_{k+1|k}}$$

$$S_{k+1} = H_{k+1} P_{k+1|k} H_{k+1}' + R_{k+1}$$

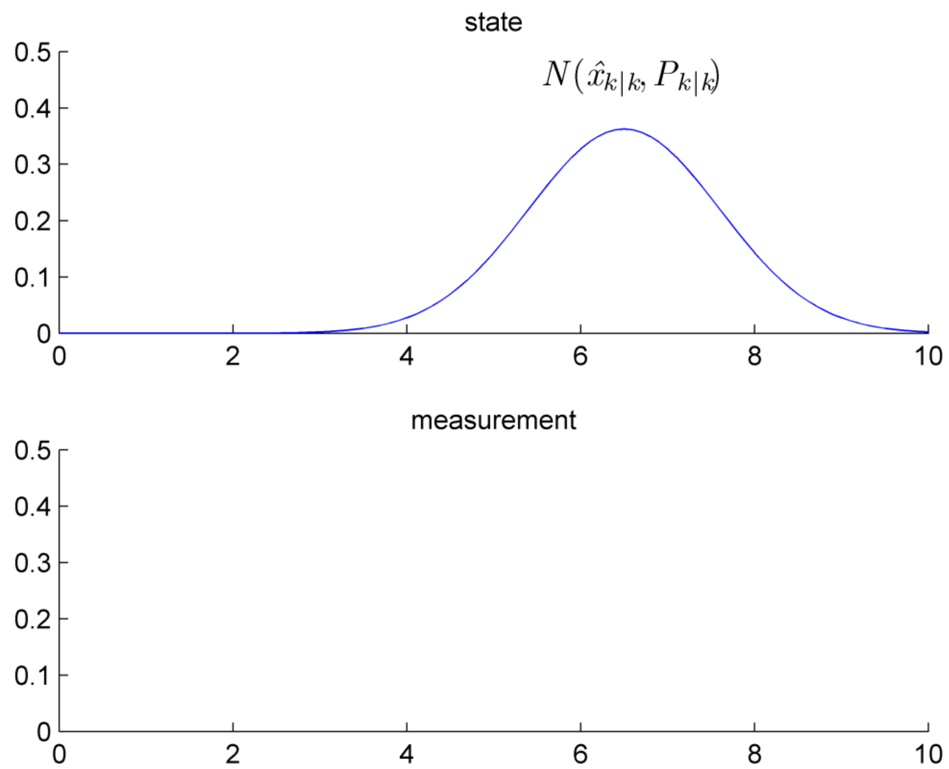
## Update

$$\nu_{k+1} = z_{k+1} - \hat{z}_{k+1|k}$$

$$W_{k+1} = P_{k+1|k} H_{k+1|k}' S_{k+1|k}^{-1}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + W_{k+1}\nu_{k+1}$$

$$P_{k+1|k+1} = P_{k+1|k} - W_{k+1} S_{k+1} W_{k+1}'$$



# Tractor-trailer state estimate

Aalto-yliopisto

Juha Backman
1.3.2017

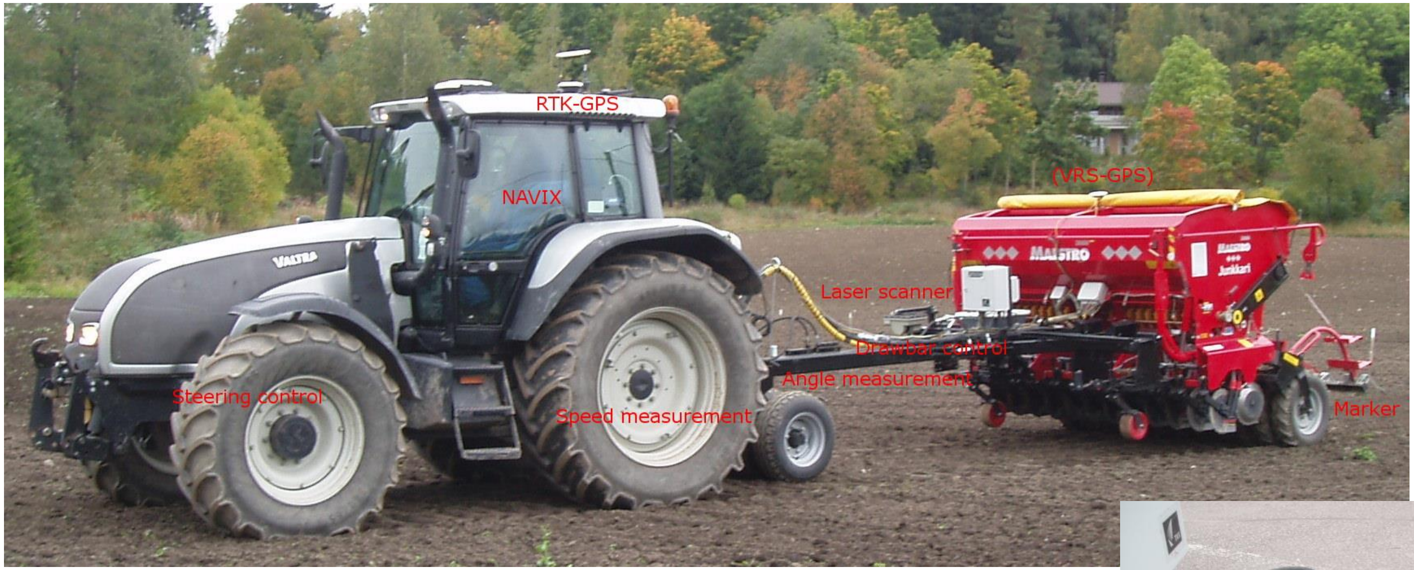
AGROMASSI

# Objective

- **Why?**
  - Automatic guidance needs an accurate estimate of the current state of the tractor and the trailer
- **Difficulties?**
  - The GPS measurement includes always noise and it is delayed
  - Actuators are slow and inaccurate (include dynamics)
  - Slipping is always present in field (odometry is inaccurate)
- **How?**
  - Local measurement can be used to improve the positioning accuracy
  - IMU can be used to improve the heading estimate

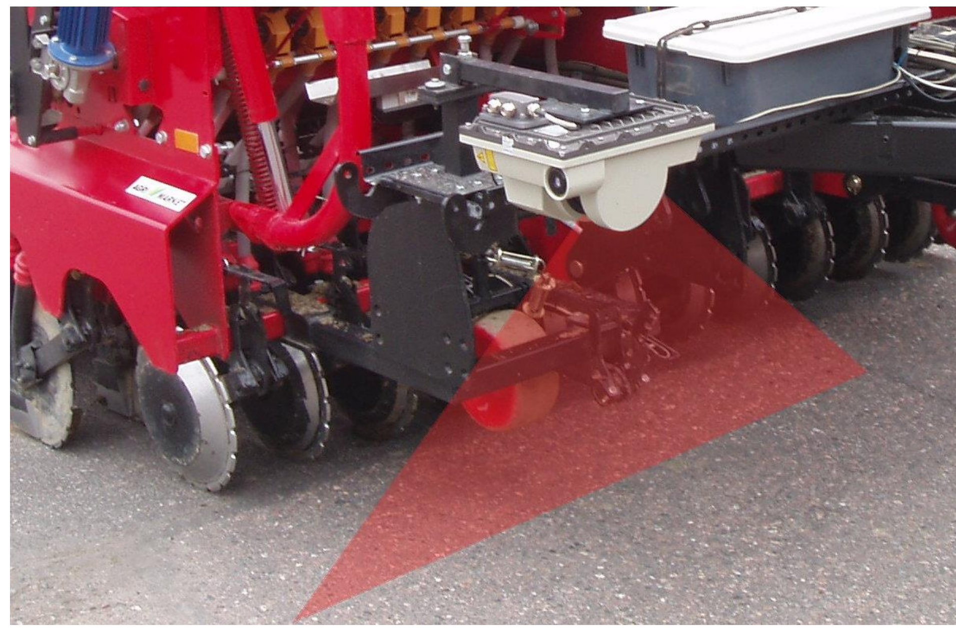
  → A realistic model of the system together with Kalman filter is needed

# Equipments

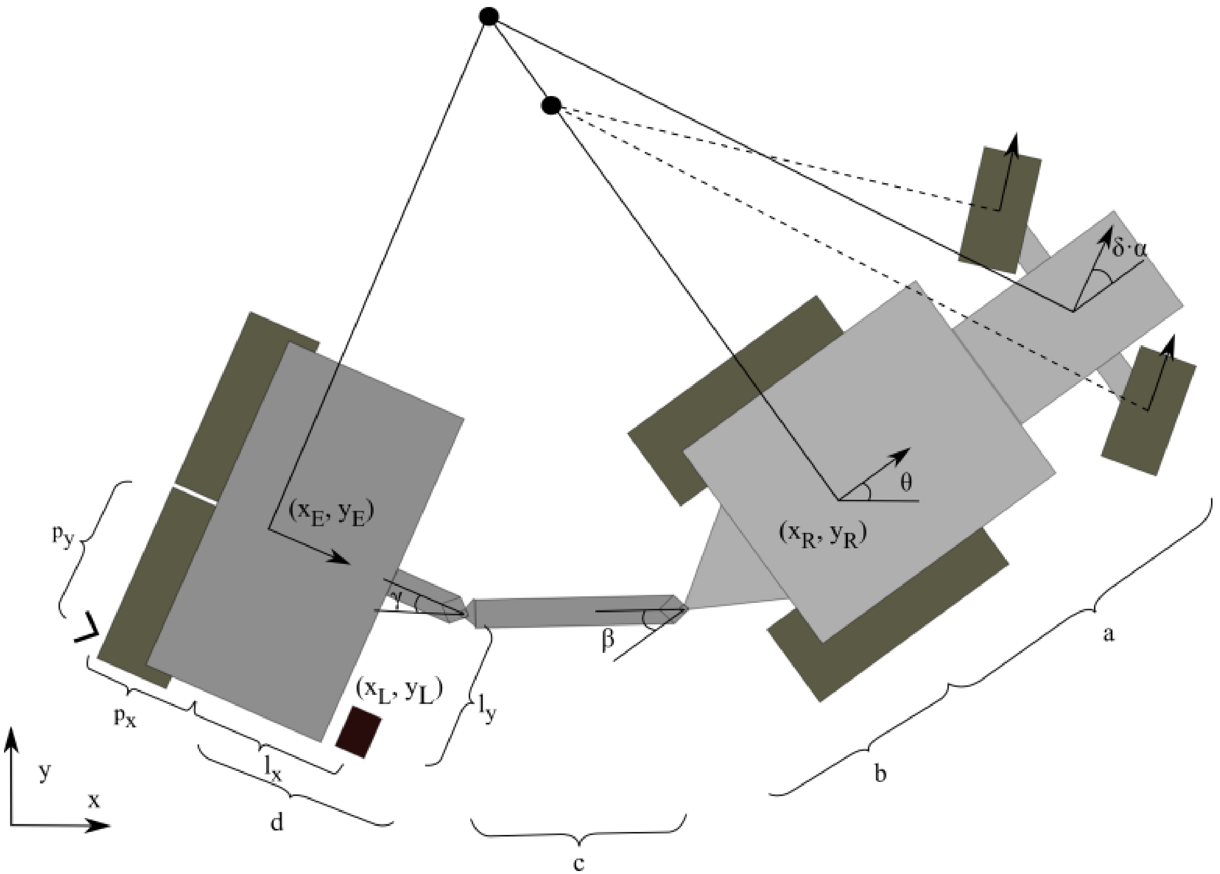

RTK-GPS
NAVIX
(VRS-GPS)
Laser scanner
Drawbar control
Angle measurement
Steering control
Speed measurement
Marker

# Equipments

- The edge of the sown are is marked with a small plough
- The mark is recoqnized with the help of laser scanner



# Model of the tractor-trailer system



# Model of the tractor-trailer system

- **Assumption: Ground is ideal and slipping affects only the front steering wheels sideways**
- **Kinematic model of the tractor**

$$\begin{bmatrix} x_R(t_{k+1}) \\ y_R(t_{k+1}) \\ \theta(t_{k+1}) \\ \delta(t_{k+1}) \end{bmatrix} = \begin{bmatrix} x_R(t_k) + v_t(t_k)\cos\theta(t_k)\,T \\ y_R(t_k) + v_t(t_k)\sin\theta(t_k)\,T \\ \theta(t_k) + v_t(t_k)\dfrac{\tan\delta(t_k)\alpha_t(t_k)}{a}T \\ \delta(t_k) \end{bmatrix}$$

- **Dynamics of the tractor actuators**

$$\begin{bmatrix} v_t(t_{k+1}) \\ \alpha_t(t_{k+1}) \end{bmatrix} = \begin{bmatrix} k_v v_t(t_k) + (1 - k_v)v_d(t_k) \\ k_\alpha \alpha_t(t_k) + (1 - k_\alpha)\alpha_d(t_k) \end{bmatrix}$$

# Model of the tractor-trailer system

- **Assumption: trailer does not slide sideways**
- **Kinematic behaviour of the trailer (continuous-time model)**

$$\dot{\beta} = \frac{-av_t \sin(\beta + \gamma_t) + v_t(d + c\cos\beta + b\cos(\beta + \gamma_t))\tan\alpha_t - ad\dot{\gamma}_t}{a(d + c\cos\gamma_t)}$$

- **Dynamics of the trailer actuator**

$$\gamma_t(t_{k+1}) = k_\gamma \gamma_t(t_k) + \left(1 - k_\gamma\right)\gamma_d(t_k)$$

# Model of the tractor-trailer system

- **Auxiliary states:**
  - Trailer position

$$\begin{bmatrix} x_E \\ y_E \end{bmatrix} = \begin{bmatrix} x_R - b\cos\theta - c\cos(\beta - \theta) - d\cos(\beta + \gamma_{actual} - \theta) \\ y_R - b\sin\theta + c\sin(\beta - \theta) + d\sin(\beta + \gamma_{actual} - \theta) \end{bmatrix}$$

  - Laser scanner position

$$\begin{bmatrix} x_L \\ y_L \end{bmatrix} = \begin{bmatrix} x_E + l_x\cos(\beta + \gamma_{actual} - \theta) - l_y\sin(\beta + \gamma_{actual} - \theta) \\ y_E - l_x\sin(\beta + \gamma_{actual} - \theta) - l_y\cos(\beta + \gamma_{actual} - \theta) \end{bmatrix}$$

  - Plough position

$$\begin{bmatrix} x_P \\ y_P \end{bmatrix} = \begin{bmatrix} x_E + p_x\cos(\beta + \gamma_{actual} - \theta) - p_y\sin(\beta + \gamma_{actual} - \theta) \\ y_E - p_x\sin(\beta + \gamma_{actual} - \theta) - p_y\cos(\beta + \gamma_{actual} - \theta) \end{bmatrix}$$

# Model of the tractor-trailer system

- **Resulting state vector**

$$x = [x_R \quad y_R \quad \theta \quad \delta \quad v_t \quad \alpha_t \quad \beta \quad \gamma_t \quad \dot{\gamma}_t \quad x_E \quad y_E]^T$$

- **Resulting control vector**

$$u = [v_d \quad \alpha_d \quad \mu_d]^T$$

- **System model**
  - Includes equations from three previous slides

$$x(t_{k+1}) = f(x(t_k), u(t_k))$$

# Estimation model

- **Standard estimation model with mutuallu independent white noise**

$$\hat{x}(t_{k+1}) = f_{est}(\hat{x}(t_k), u(t_k)) + \mathrm{w}(t_k)$$

$$\hat{y}(t_k) = h(\hat{x}(t_k)) + v(t_k)$$

- **Measurements are delayed**
  → Estimated state vector includes delayed states
  → Totally 44 variables in state vector

$$\hat{x}(t_k) = [x(t_k) \quad x(t_{k-1}) \quad x(t_{k-2}) \quad \ldots \quad x(t_{k-n})]^T$$

$$f_{est}\left(\hat{x}(t_k), u(t_k)\right) = [f(x(t_k), u(t_k)) \quad x(t_k) \quad x(t_{k-1}) \quad \ldots \quad x(t_{k-n+1})]^{\mathrm{T}}$$

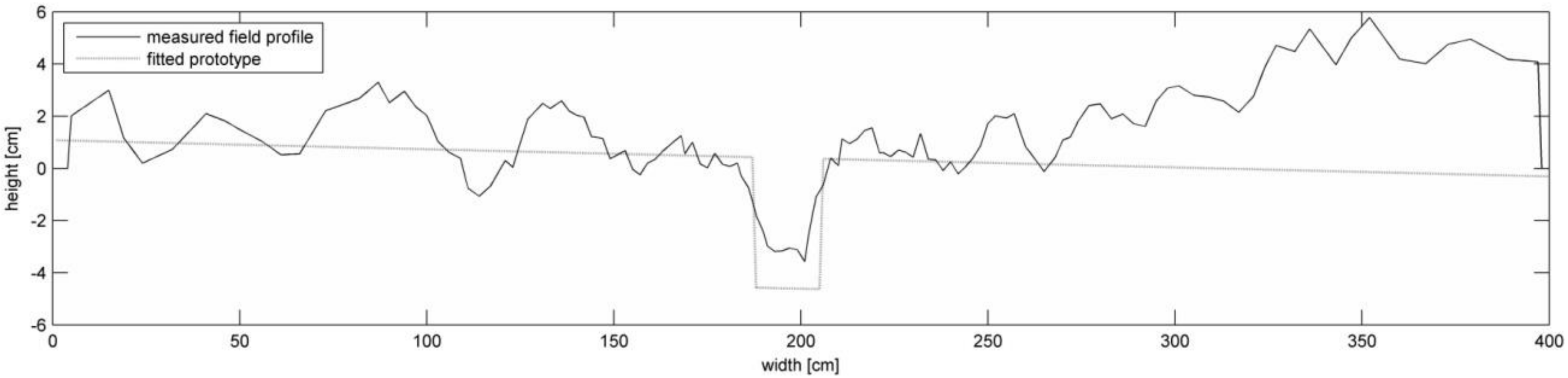
# Estimation model

- **Measurements are delayed**
  → Measurement function "picks" the right state variables

$$h(\hat{x}(t_k)) = \begin{bmatrix} x_R\left(t_{k-\tau(x_R)}\right) \\ y_R\left(t_{k-\tau(y_R)}\right) \\ \theta\left(t_{k-\tau(\theta)}\right) \\ v_t\left(t_{k-\tau(v_t)}\right) \\ \alpha_t\left(t_{k-\tau(\alpha_t)}\right) \\ \beta\left(t_{k-\tau(\beta)}\right) \\ \gamma_t\left(t_{k-\tau(\gamma_t)}\right) \\ x_E\left(t_{k-\tau(x_E)}\right) \\ y_E\left(t_{k-\tau(y_E)}\right) \end{bmatrix}$$

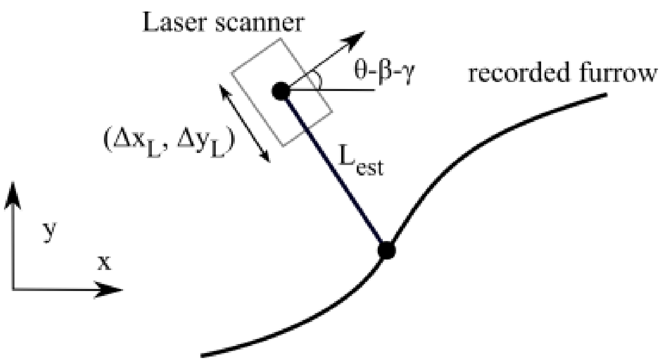
# Merging the local measurement

- **The laser scanner measures only field profile**
- **The edge of the previous swath is localized from the field profile**
  - Edge is marked with small plough
  - The edge is recognized from the profile by minimizing the MSE continuously



# Merging the local measurement

- **The estimate of the previous swath is also recorded into a memory**
- **The estimate of the laser scanner measurement can be calculated from the current estimate of the laser scanner position and the estimate of the previous swath edge**

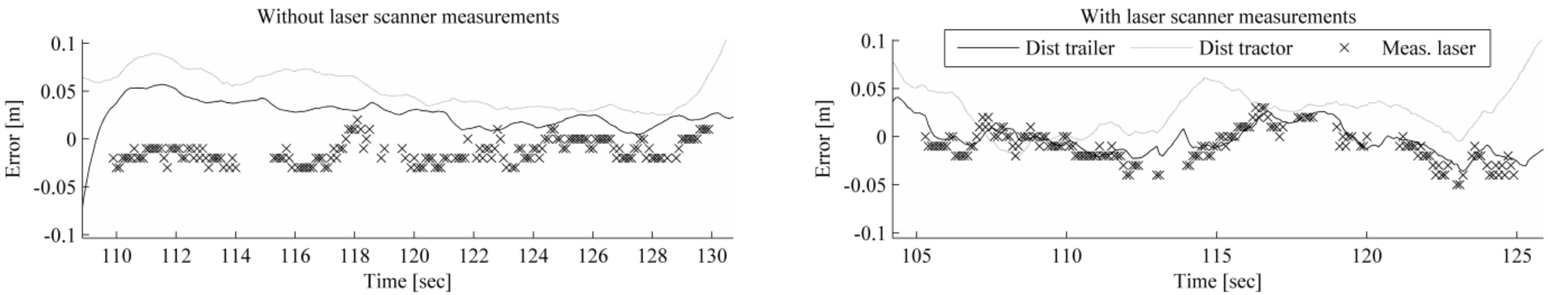



- **The corresponding innovation terms in the Extended Kalman Filter**

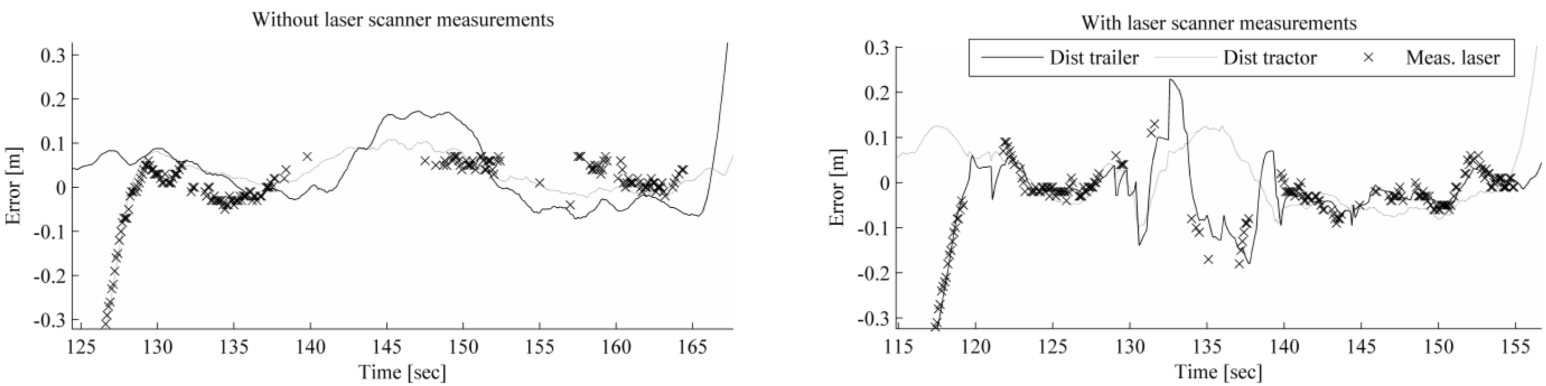
$$\Delta x_E = (L_{meas} - L_{est}) \cos(\theta - \beta - \gamma_t)$$
$$\Delta y_E = (L_{meas} - L_{est}) \sin(\theta - \beta - \gamma_t)$$

# Results

- **Error estimate in straight driving line**




- **Error estimate in curved driving line**

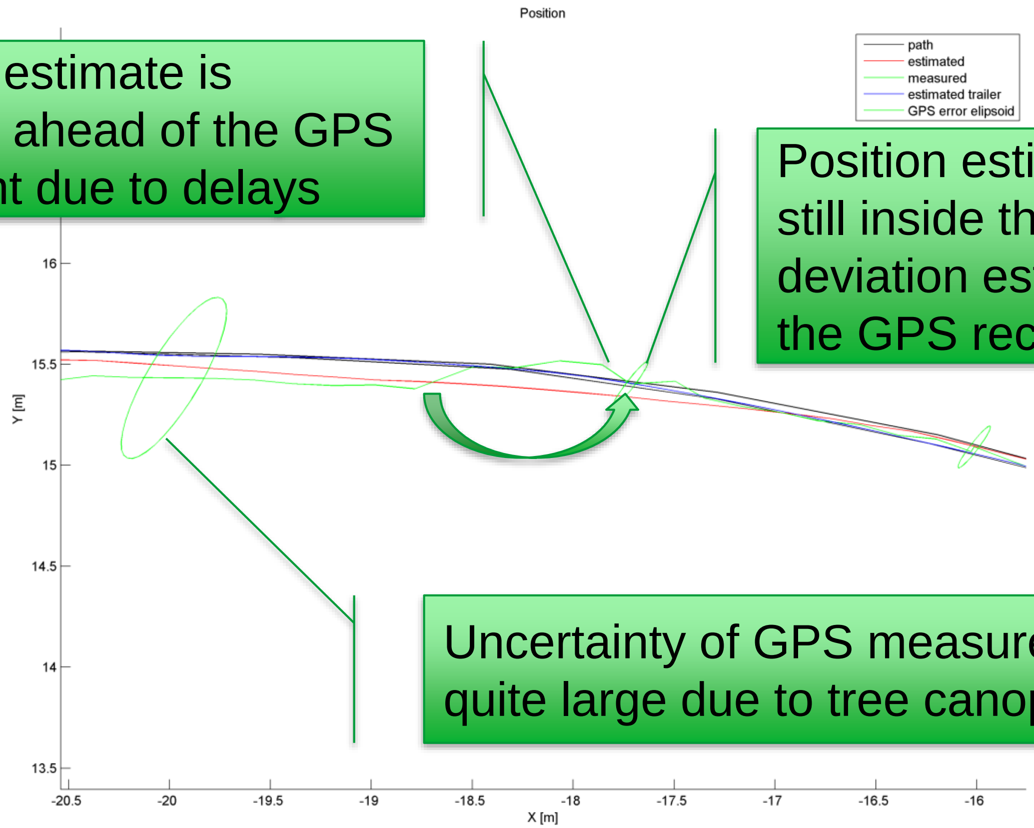


# Results – position estimate

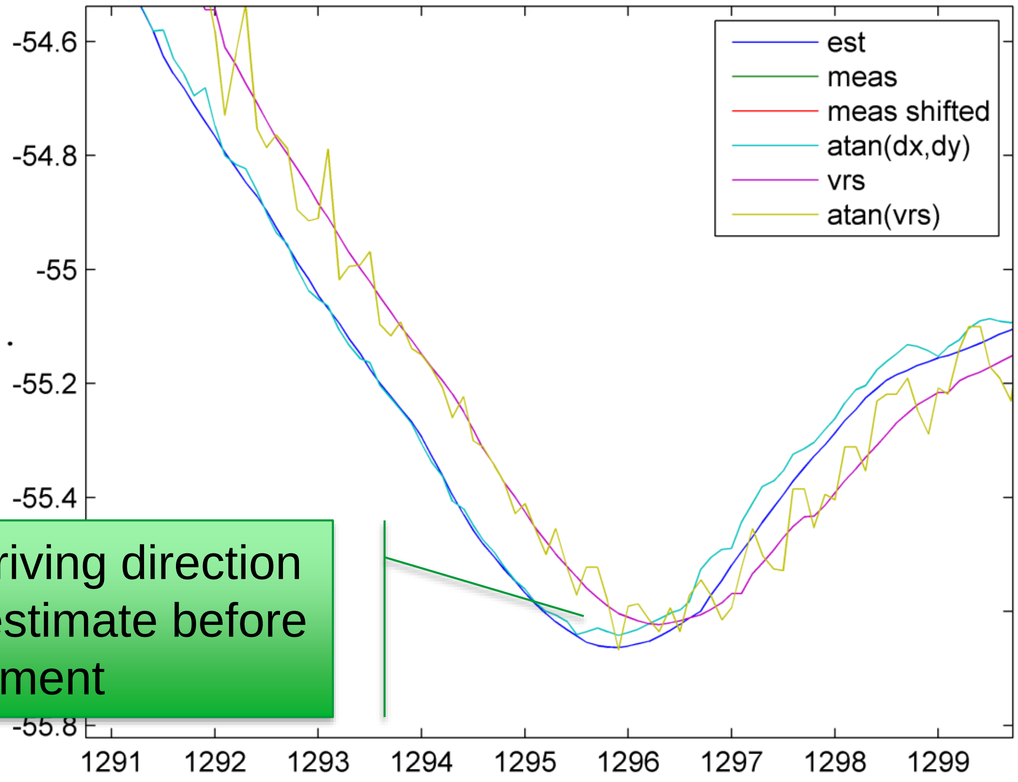
The position estimate is actually ~1m ahead of the GPS measurement due to delays

Position estimate is still inside the deviation estimate of the GPS receiver

Uncertainty of GPS measurement is quite large due to tree canopy



# Results – Heading estimate



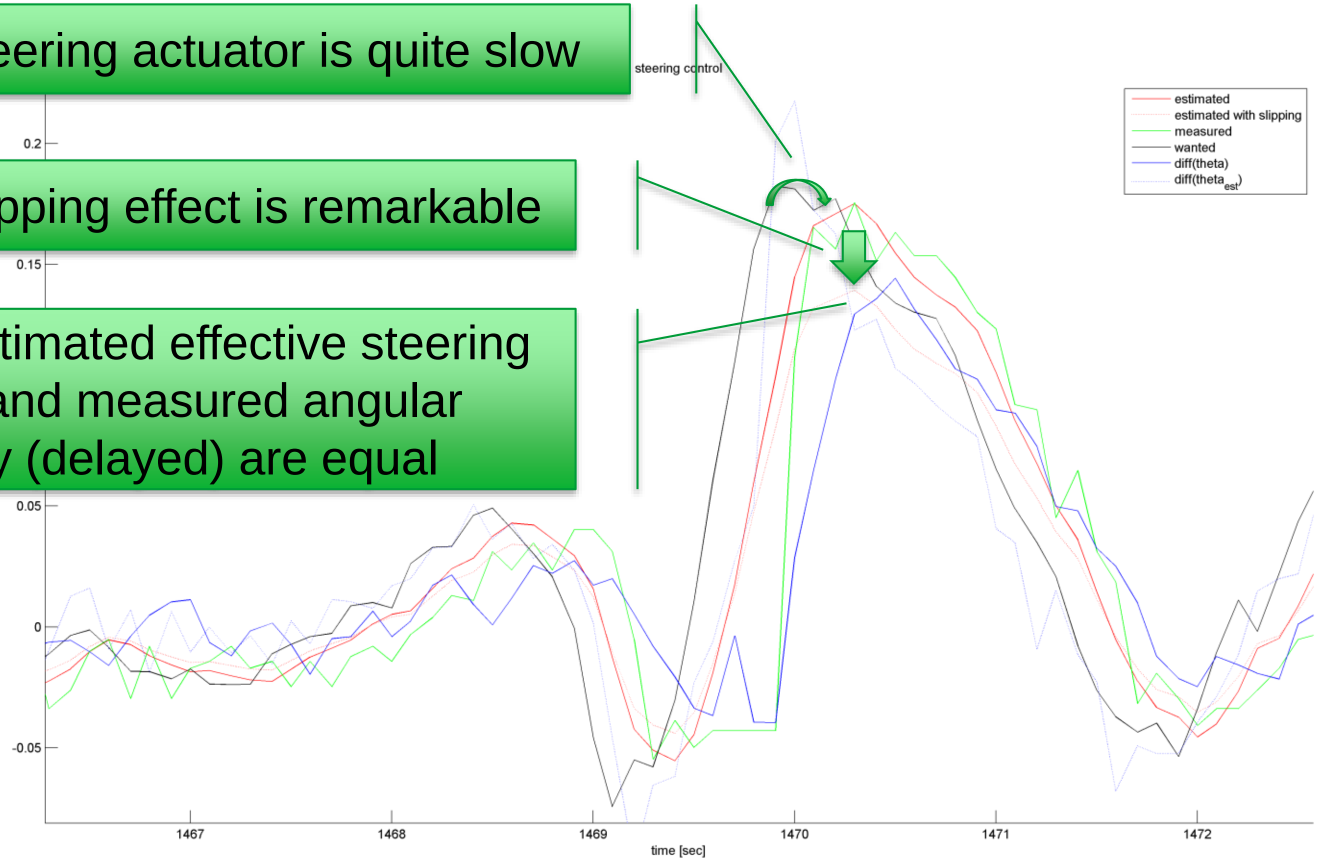

Change of driving direction is visible in estimate before the measurement

# Results – Steering estimate

The steering actuator is quite slow

The slipping effect is remarkable

The estimated effective steering angle and measured angular velocity (delayed) are equal



# Summary

- **Local measurement improves position estimate**
- **Forward prediction does not reduce the accuracy**

- **Overall accuracy of the system (including steering control) is well below 10 cm (below 5 cm in straight driving line)**