Aalto University
School of Electrical
Engineering

Probabilistic Robotics
Introduction of SLAM in general
Extended Kalman Filter EKF-SLAM

ELEC-E8111 Autonomous Mobile Robots

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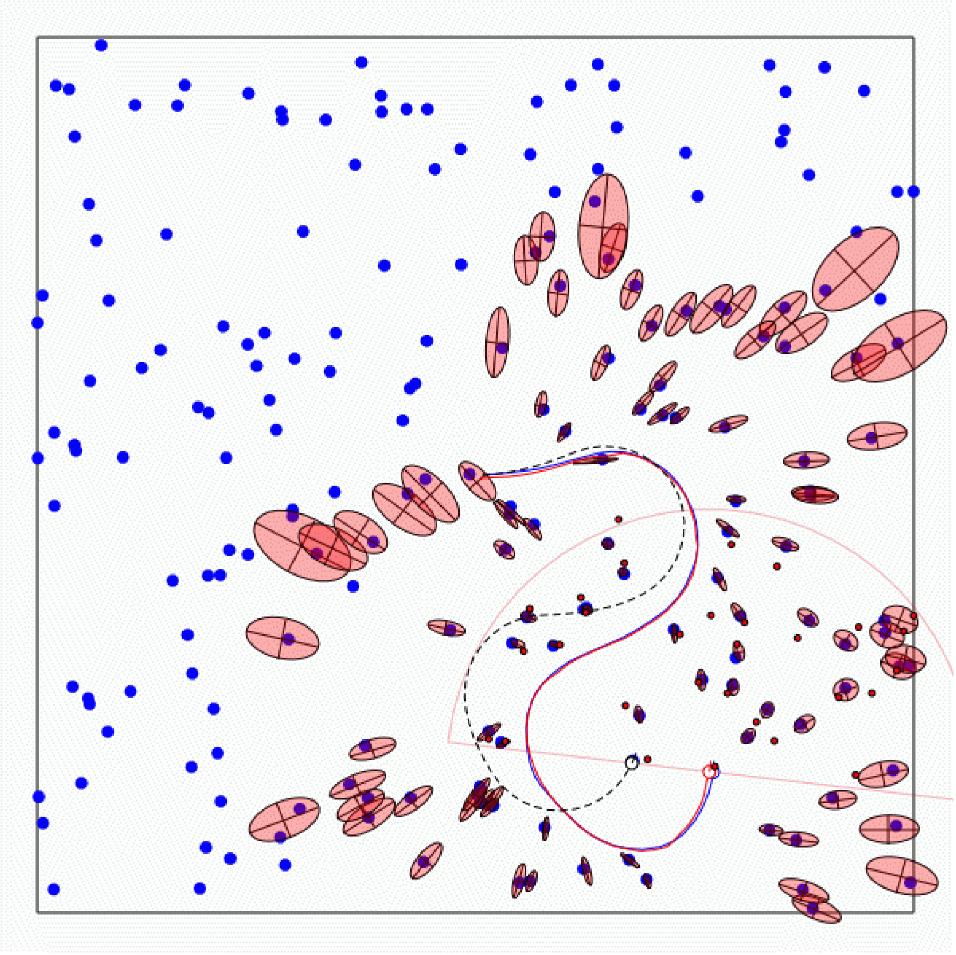
The SLAM Problem

A robot is exploring an unknown, static environment.



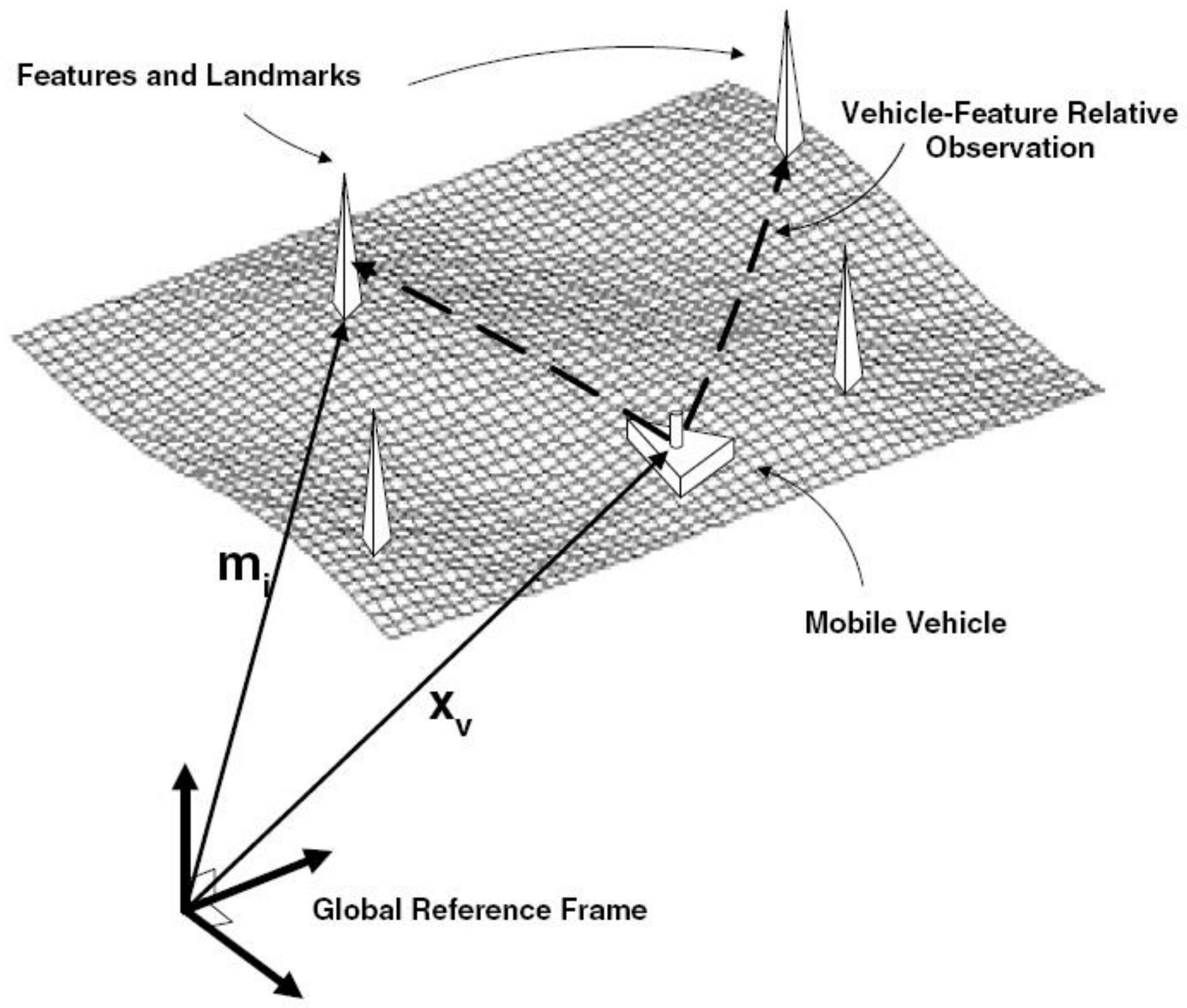
Given:

- The robot's controls
- Observations of nearby features

Estimate:

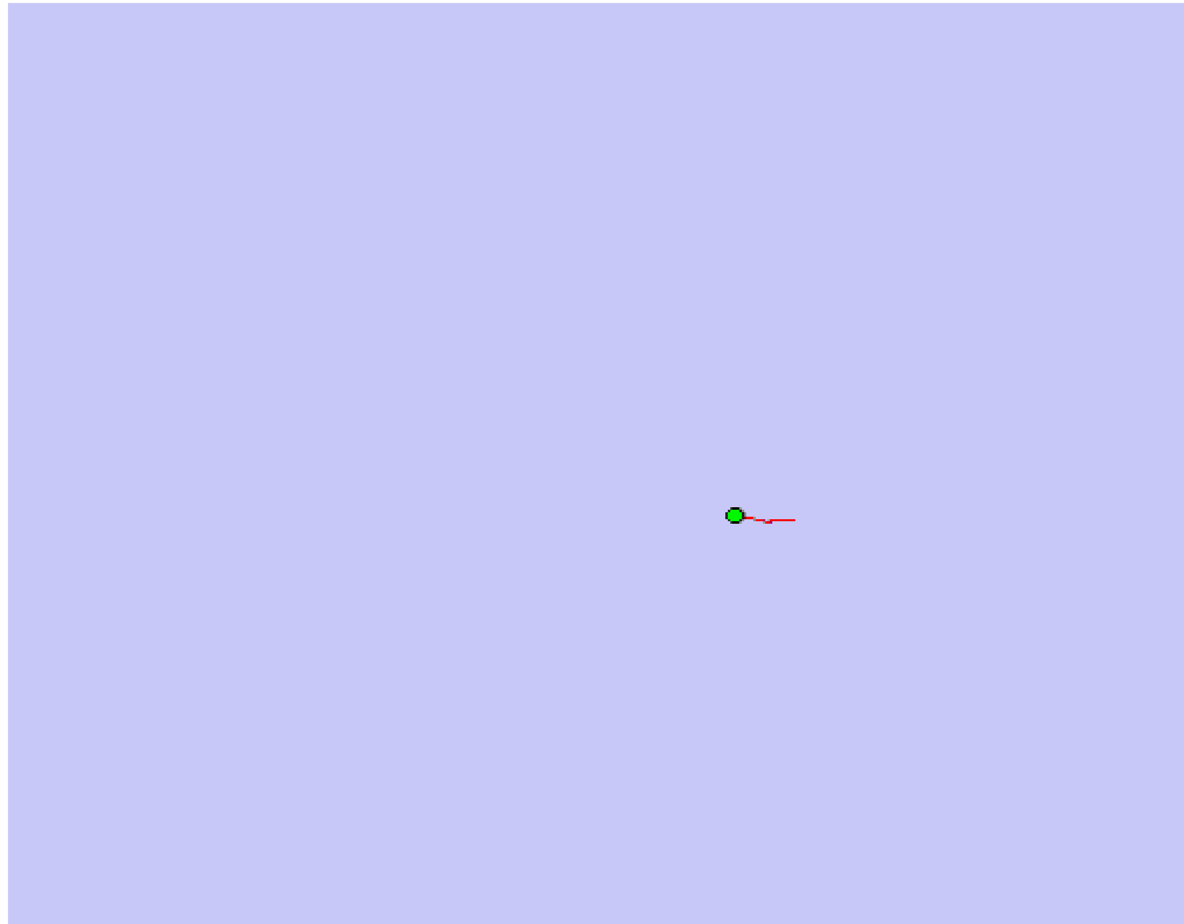
- Map of features
- Path of the robot

Structure of the Landmark-based SLAM-Problem

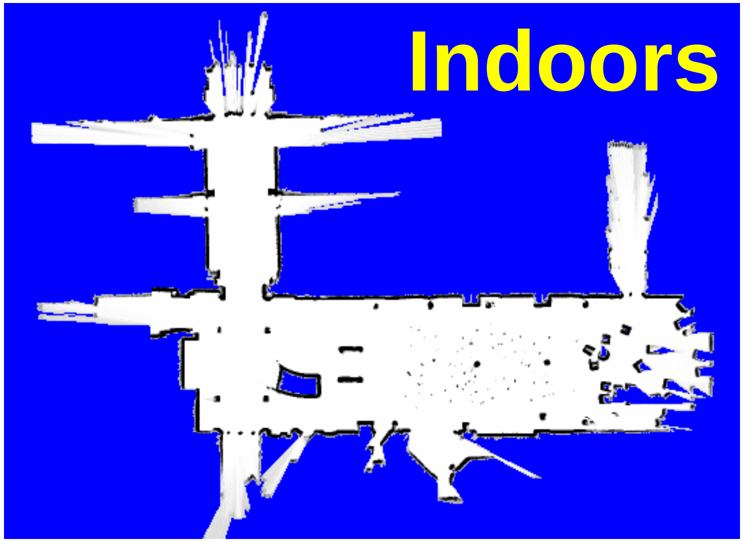


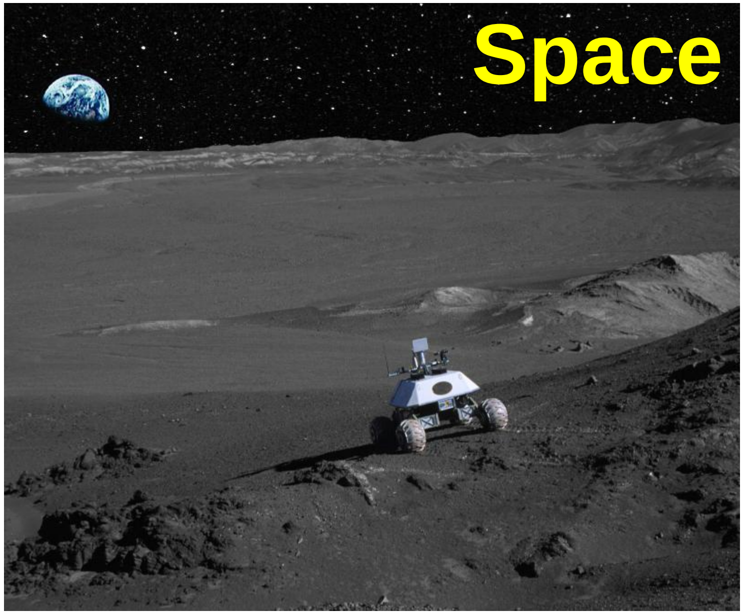
Mapping with Raw Odometry

- Run a separate video in Mycourses: haehnel-RawOdometry-anim.avi
- Not SLAM, just mapping on the basis of laser scans as perception and odometry as loalization method
- Error in pose accumulates quickly



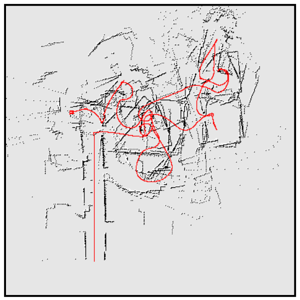
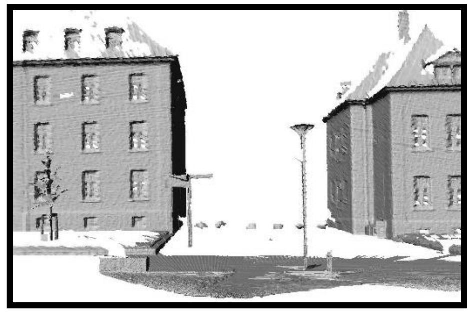
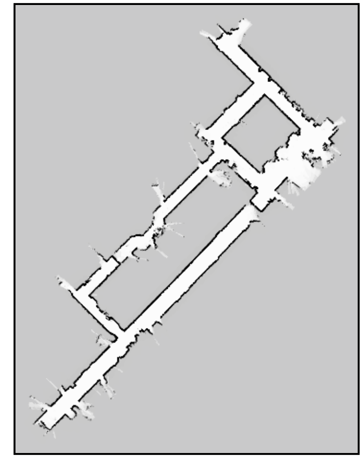
SLAM Applications, Forests





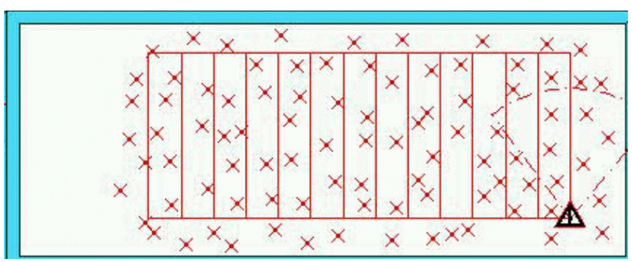
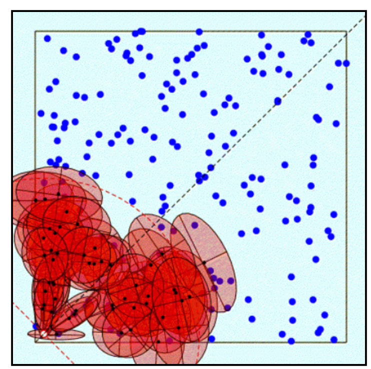
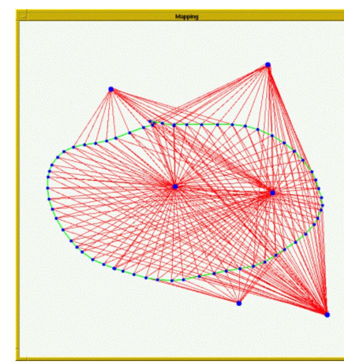
Representations

- Grid maps or scans

[Lu & Milios, 97; Gutmann, 98: Thrun 98; Burgard, 99; Konolige & Gutmann, 00; Thrun, 00; Arras, 99; Haehnel, 01;…]

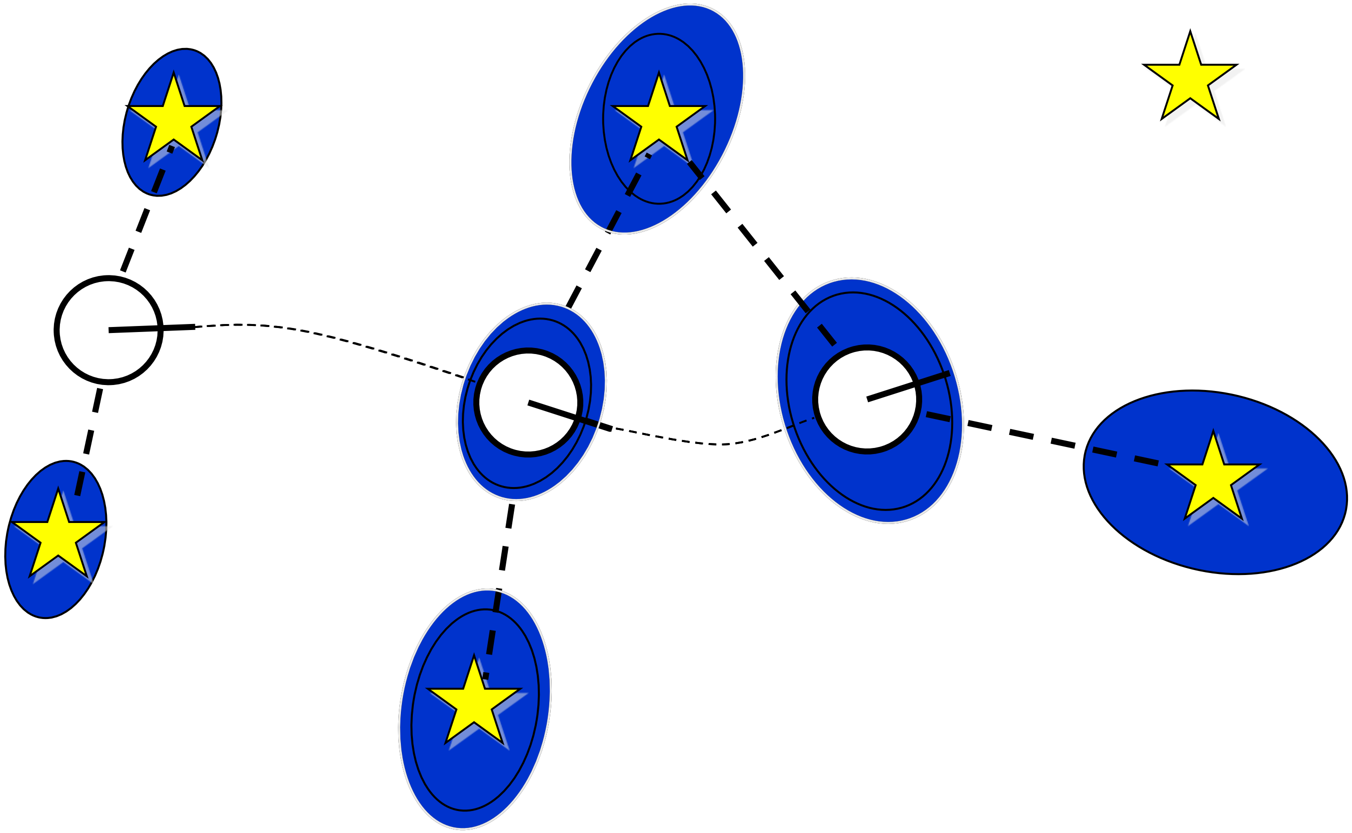
- Landmark-based

[Leonard et al., 98; Castelanos et al., 99: Dissanayake et al., 2001; Montemerlo et al., 2002;…

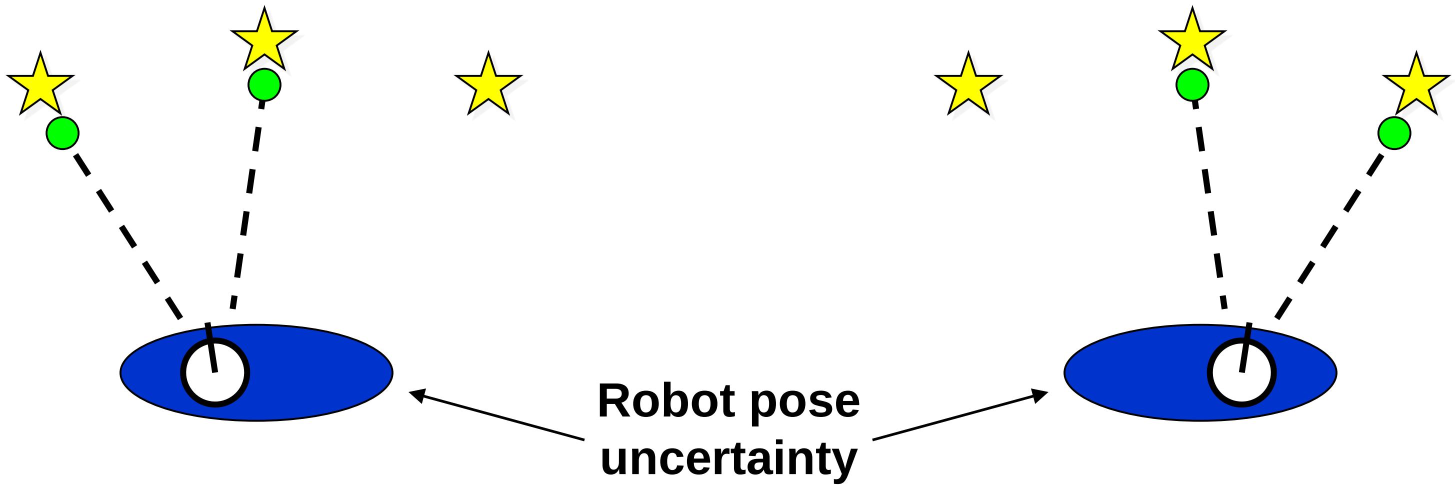
Why is SLAM a hard problem?

SLAM: robot path and map are both **unknown**



Robot path error correlates errors in the map

Why is SLAM a hard problem?



- In the real world, the mapping between observations and landmarks is unknown
- Picking wrong data associations can have catastrophic consequences
- Pose error correlates data associations

SLAM:

Simultaneous Localization and Mapping

- Full SLAM: Estimates entire path and map!

$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t})$$

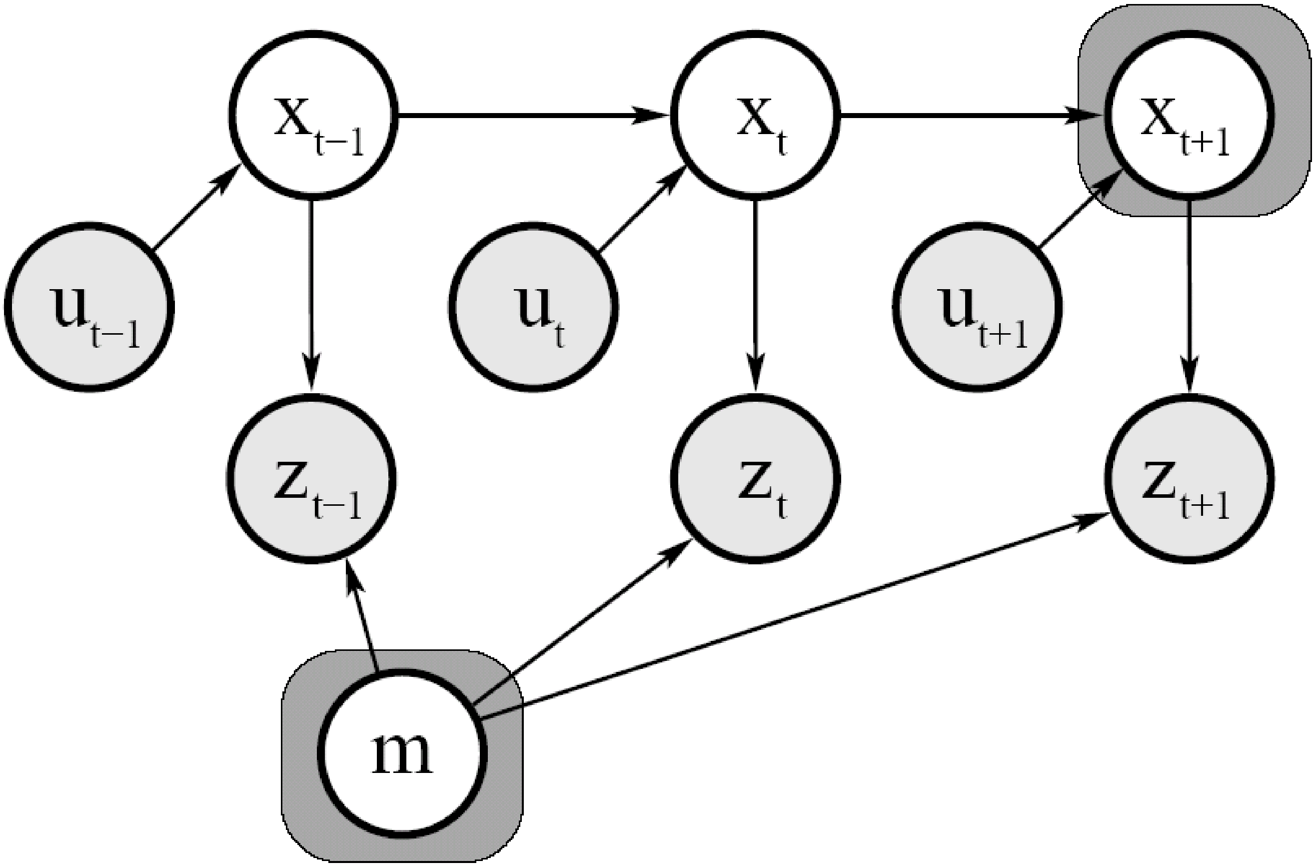
- Online SLAM:

$$p(x_t, m \mid z_{1:t}, u_{1:t}) = \int\int \cdots \int p(x_{1:t}, m \mid z_{1:t}, u_{1:t}) \, dx_1 dx_2 \ldots dx_{t-1}$$

Integrations typically done one at a time

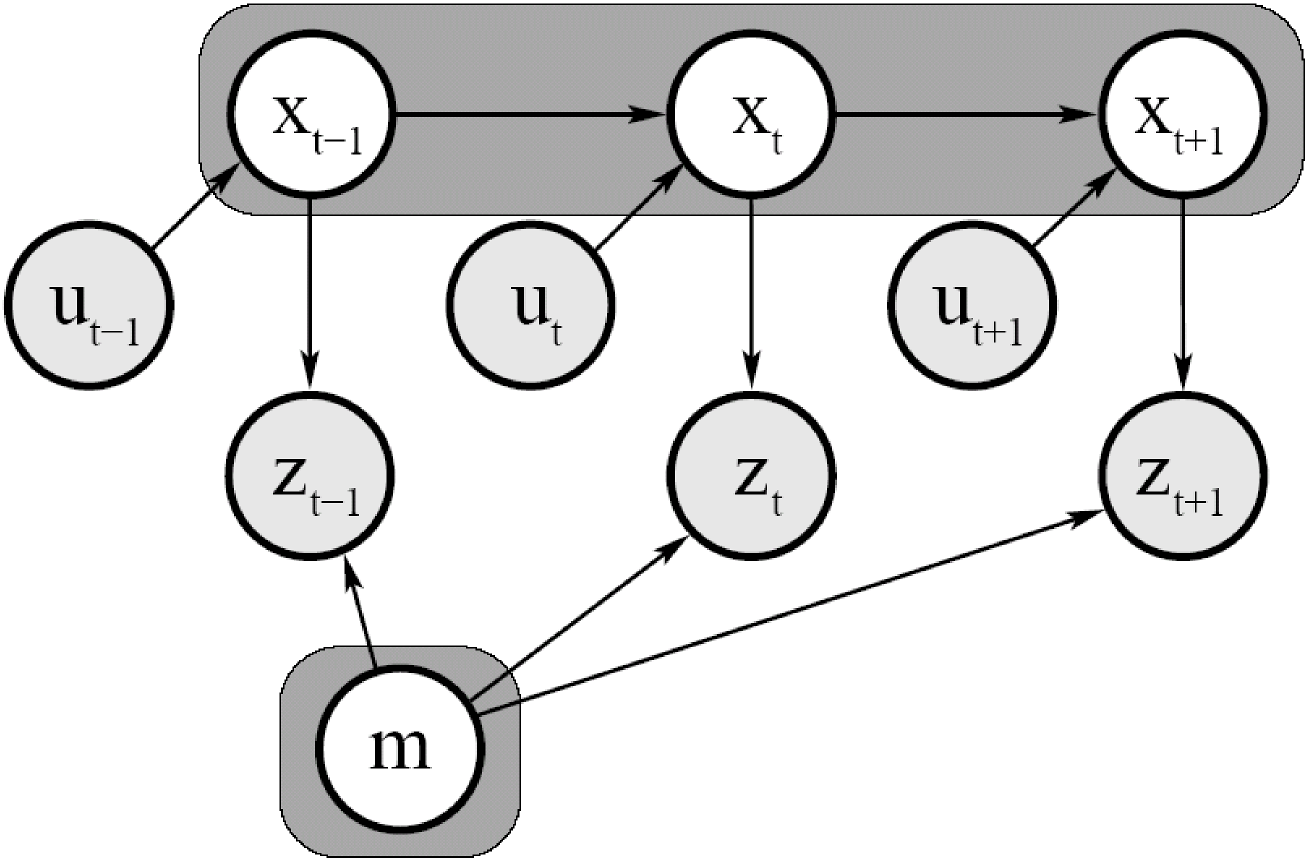
Estimates most recent pose and map!

Graphical Model of Online SLAM:



$$p(x_t, m \mid z_{1:t}, u_{1:t}) = \int\int \dots \int p(x_{1:t}, m \mid z_{1:t}, u_{1:t}) \, dx_1 \, dx_2 \dots dx_{t-1}$$

Graphical Model of Full SLAM:



$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t})$$

Techniques for Generating Consistent Maps

- Scan matching
- EKF SLAM
- Fast-SLAM
- Probabilistic mapping with a single map and a posterior about poses Mapping + Localization
- Graph-SLAM, SEIFs

Scan Matching

Maximize the likelihood of the i-th pose and map relative to the (i-1)-th pose and map.

$$\hat{x}_t = \underset{x_t}{\arg\max} \left\{ p(z_t \mid x_t, \hat{m}^{[t-1]}) \cdot p(x_t \mid u_{t-1}, \hat{x}_{t-1}) \right\}$$

current measurement

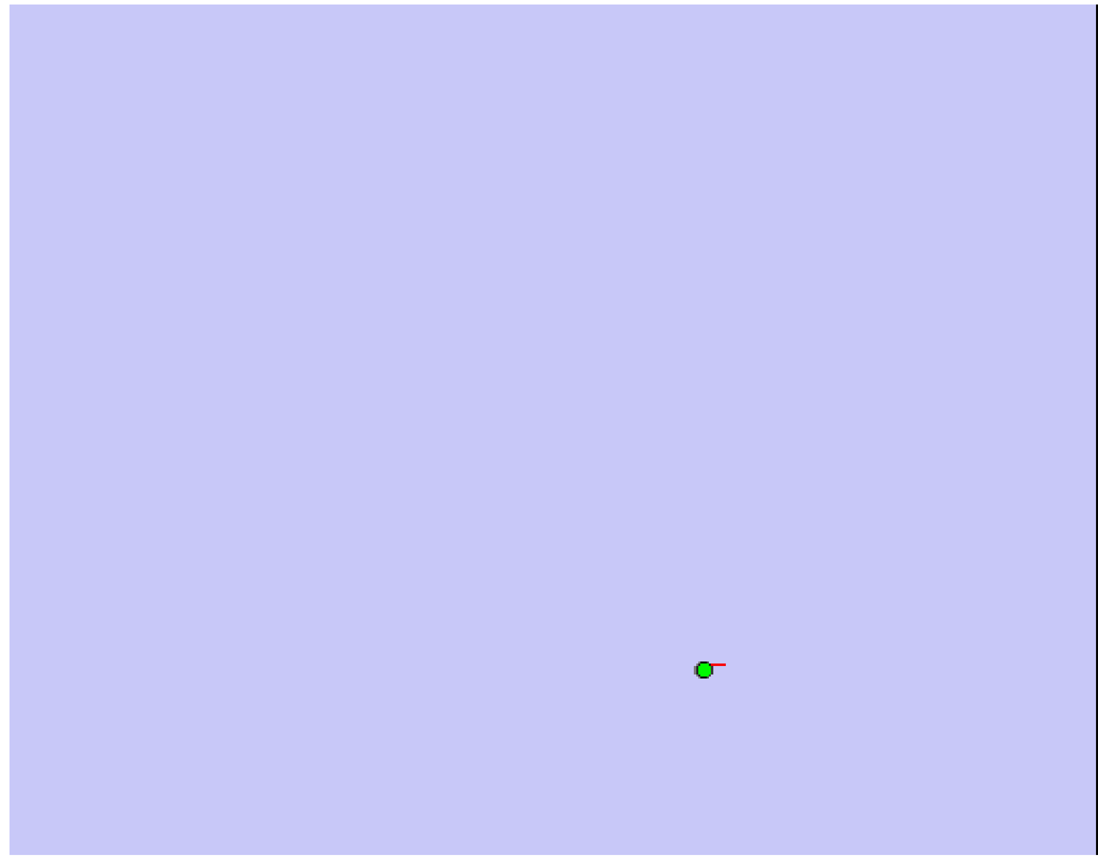
map constructed so far

robot motion

Calculate the map $\hat{m}^{[t]}$ according to "mapping with known poses" based on the poses and observations.

Scan Matching Example

- Run a separate video in Mycourses: haehnel-ScanMatching-anim.avi
- Real SLAM, localization on the basis of laser scan matching, only one estimate for pose
- Errors do not anymore accumulate too fast for mapping and closing the loop. Mapping works



SLAM with Extended Kalman Filter (EKF)

- Applies EKF to on-line SLAM using maximum likelihood data association
- Feature based SLAM
- Augmented state contains pose (x, y and heading) plus 2D coordinates for each landmark
- For pose the dynamic model is kinematics, for landmarks just driving noise.

Kalman Filter Algorithm

1. Algorithm **Kalman_filter**($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$):

2. Prediction:
3. $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$
4. $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$

5. Correction:
6. $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$
7. $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$
8. $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$

9. Return μ_t, Σ_t

(E)KF-SLAM

- Map with N landmarks:(3+2N)-dimensional Gaussian, (belief is the aposteriore estimate)

$$Bel(x_t, m_t) = \left\langle \begin{pmatrix} x \\ y \\ \theta \\ l_1 \\ l_2 \\ \vdots \\ l_N \end{pmatrix}, \begin{pmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xl_1} & \sigma_{xl_2} & \cdots & \sigma_{xl_N} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{y\theta} & \sigma_{yl_1} & \sigma_{yl_2} & \cdots & \sigma_{yl_N} \\ \sigma_{x\theta} & \sigma_{y\theta} & \sigma_\theta^2 & \sigma_{\theta l_1} & \sigma_{\theta l_2} & \cdots & \sigma_{\theta l_N} \\ \sigma_{xl_1} & \sigma_{yl_1} & \sigma_{\theta l_1} & \sigma_{l_1}^2 & \sigma_{l_1 l_2} & \cdots & \sigma_{l_1 l_N} \\ \sigma_{xl_2} & \sigma_{yl_2} & \sigma_{\theta l_2} & \sigma_{l_1 l_2} & \sigma_{l_2}^2 & \cdots & \sigma_{l_2 l_N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{xl_N} & \sigma_{yl_N} & \sigma_{\theta l_N} & \sigma_{l_1 l_N} & \sigma_{l_2 l_N} & \cdots & \sigma_{l_N}^2 \end{pmatrix} \right\rangle$$

- Can handle hundreds of dimensions

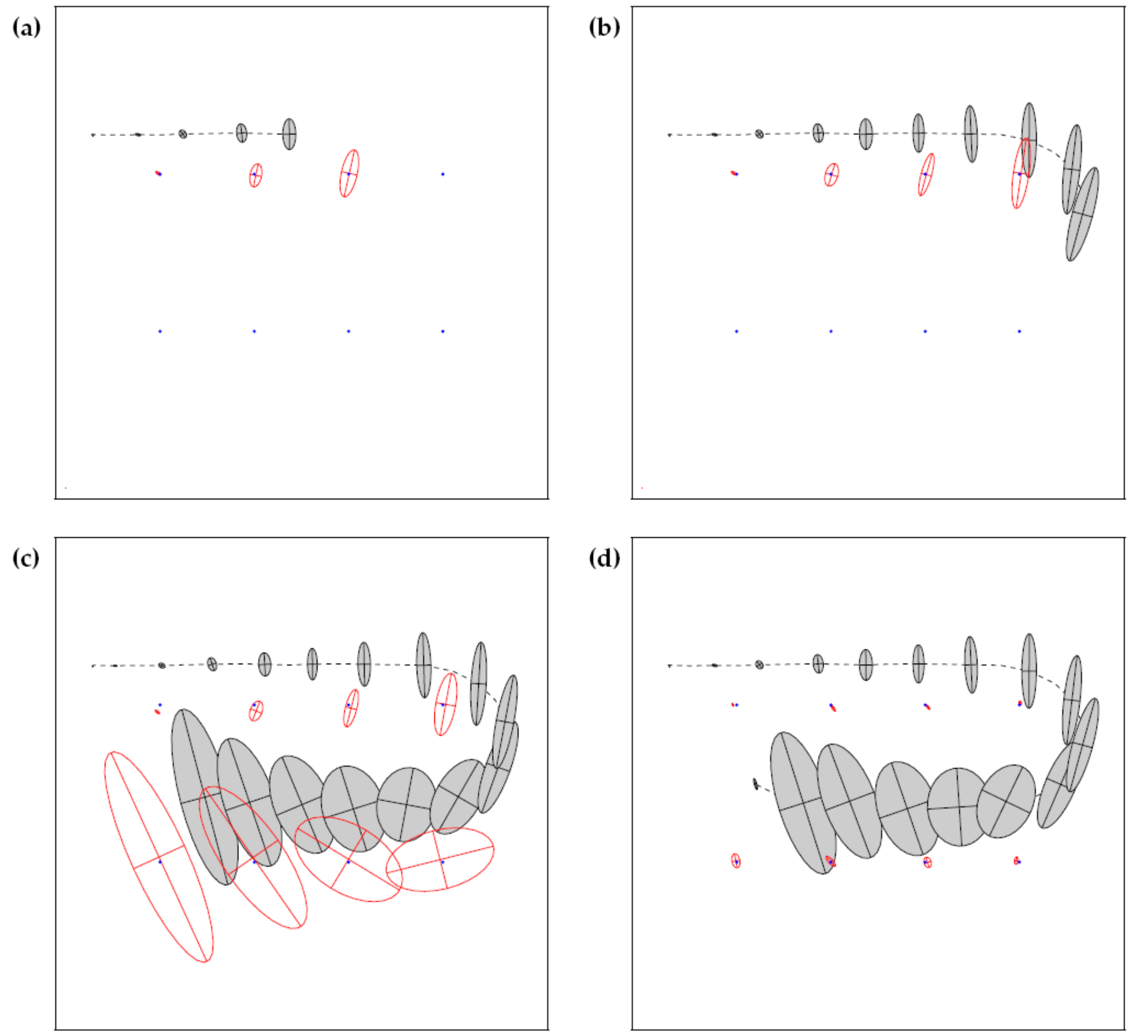
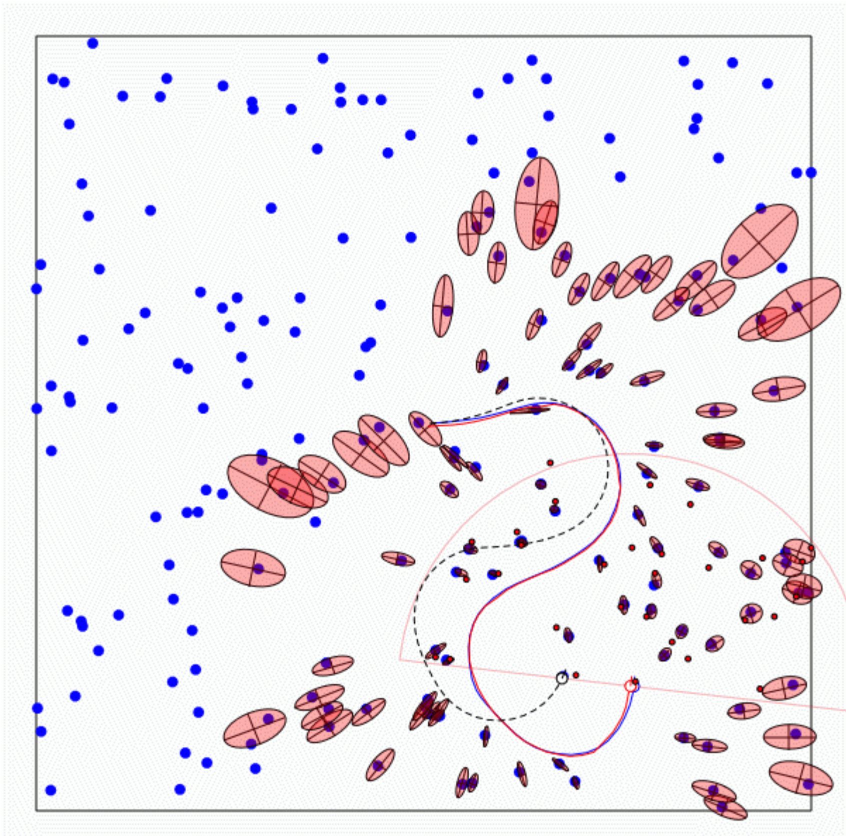
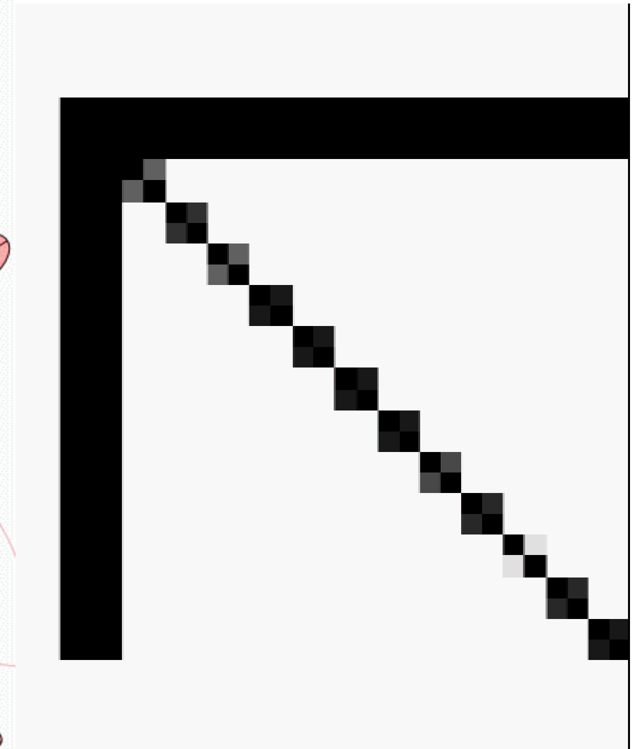


Figure 10.3 EKF applied to the online SLAM problem. The robot's path is a dotted line, and its estimates of its own position are shaded ellipses. Eight distinguishable landmarks of unknown location are shown as small dots, and their location estimates are shown as white ellipses. In (a)–(c) the robot's positional uncertainty is increasing, as is its uncertainty about the landmarks it encounters. In (d) the robot senses the first landmark again, and the uncertainty of *all* landmarks decreases, as does the uncertainty of its current pose. Image courtesy of Michael Montemerlo, Stanford University.

Classical Solution – The EKF

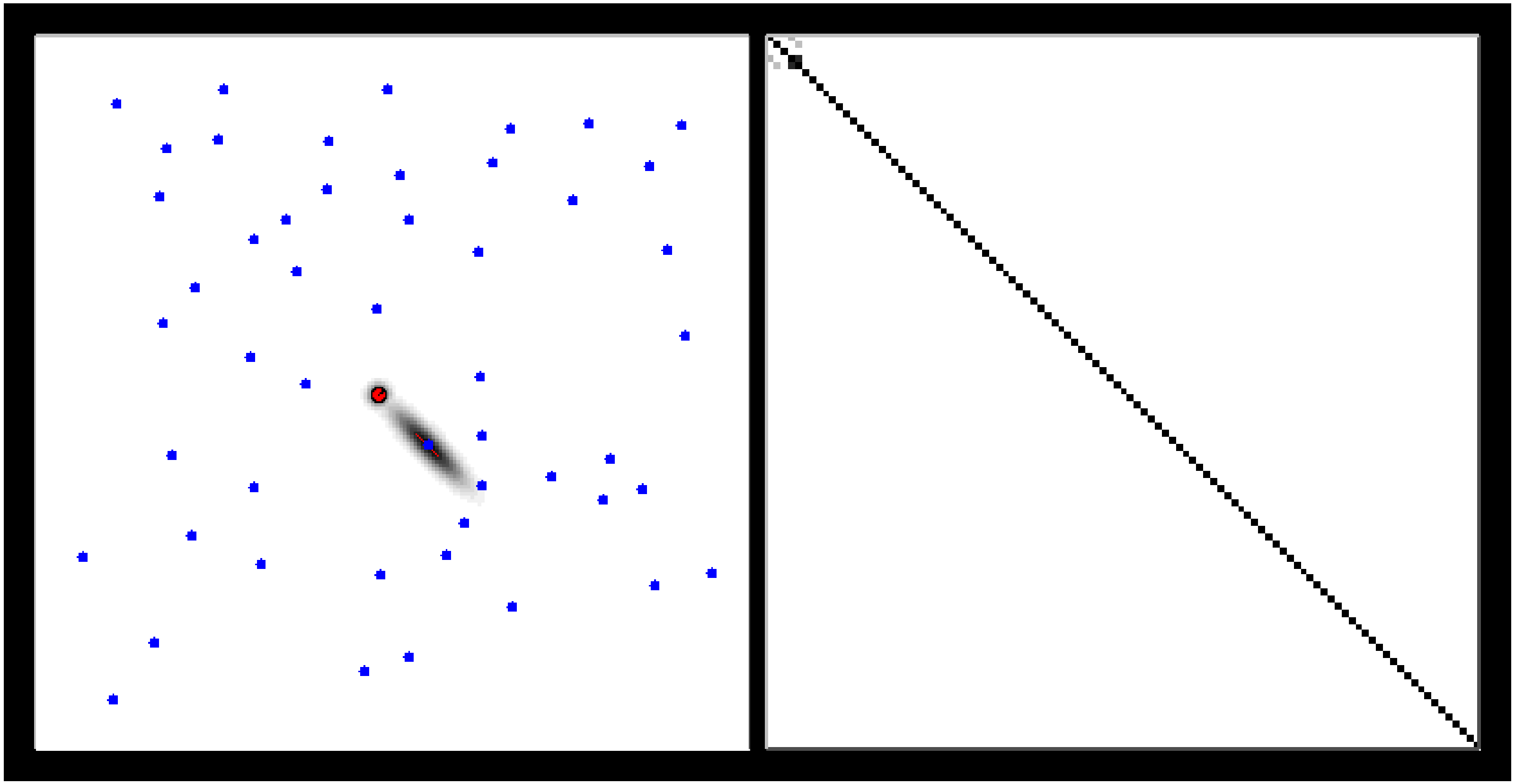




Blue path = true path **Red path** = estimated path **Black path** = odometry

- Approximate the SLAM posterior with a high-dimensional Gaussian [Smith & Cheesman, 1986] …
- Single hypothesis data association

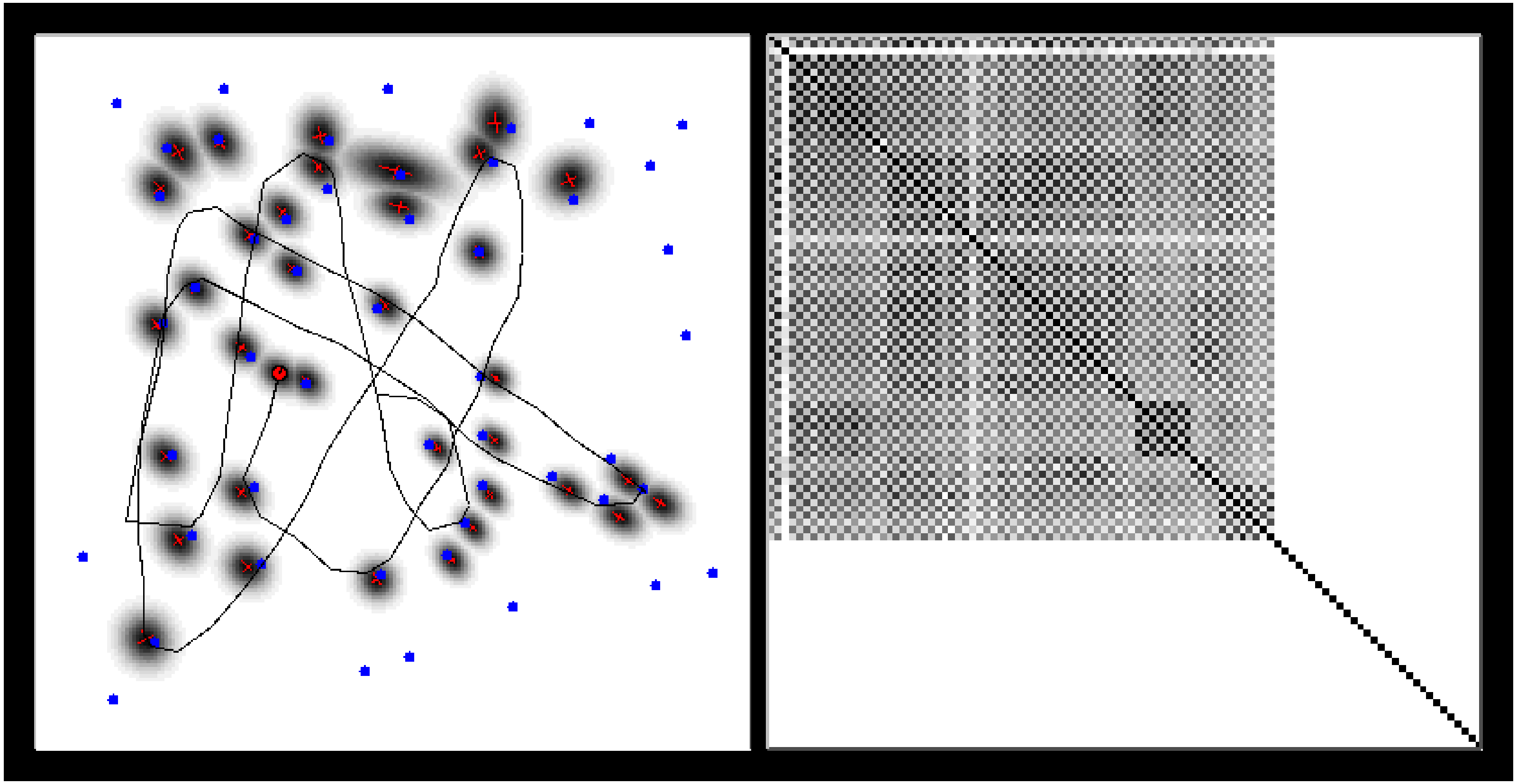
EKF-SLAM



Map

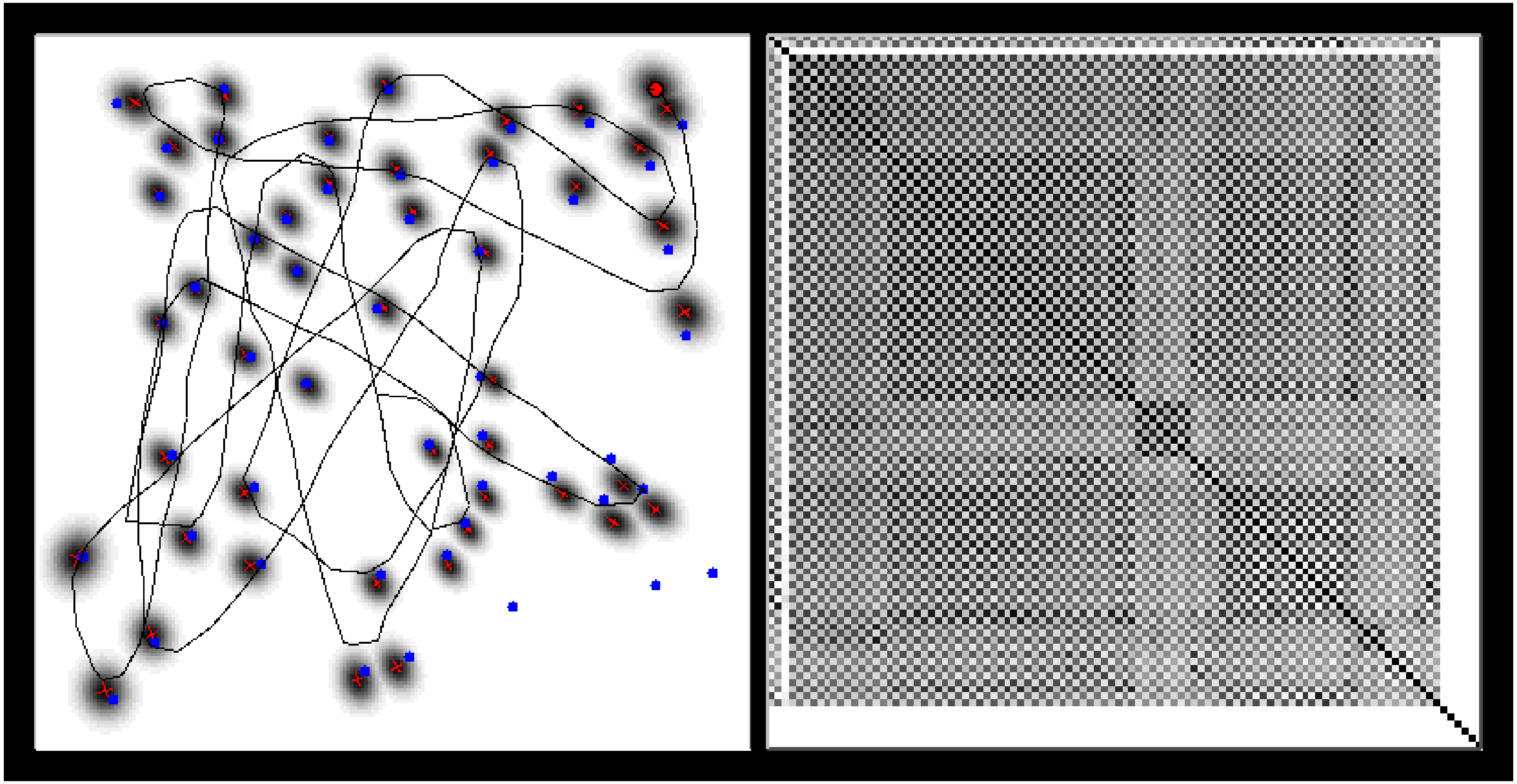
Correlation matrix

EKF-SLAM



Map Correlation matrix

EKF-SLAM



Map Correlation matrix

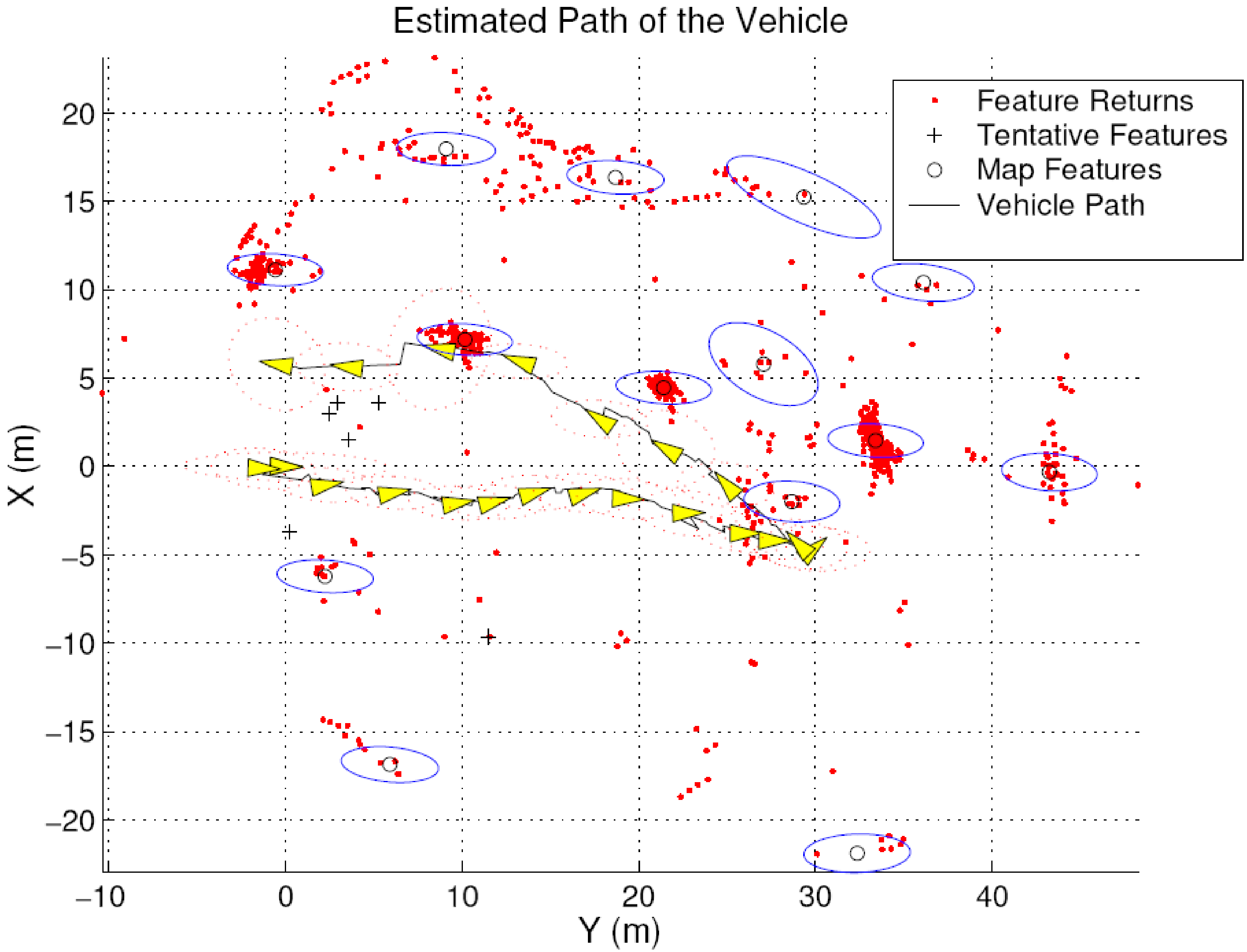


Figure 10.5 Example of Kalman filter estimation of the map and the vehicle pose. Image courtesy of Stefan Williams and Hugh Durrant-Whyte, Australian Centre for Field Robotics.

Properties of KF-SLAM (Linear Case) [Dissanayake et al., 2001]

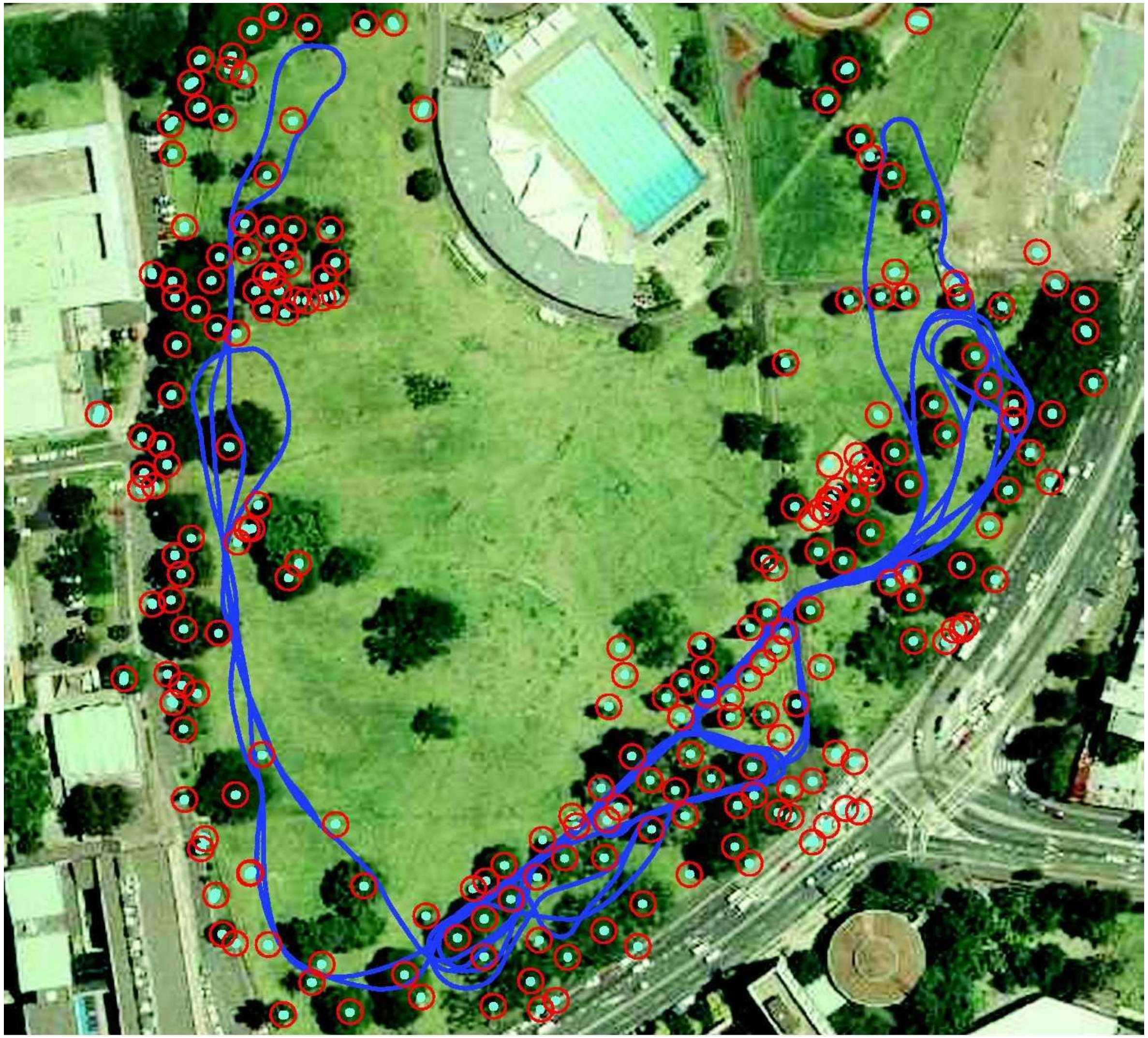
Theorem:

The determinant of any sub-matrix of the map covariance matrix decreases monotonically as successive observations are made.

Theorem:

In the limit the landmark estimates become fully correlated

Victoria Park Data Set



[courtesy by E. Nebot]

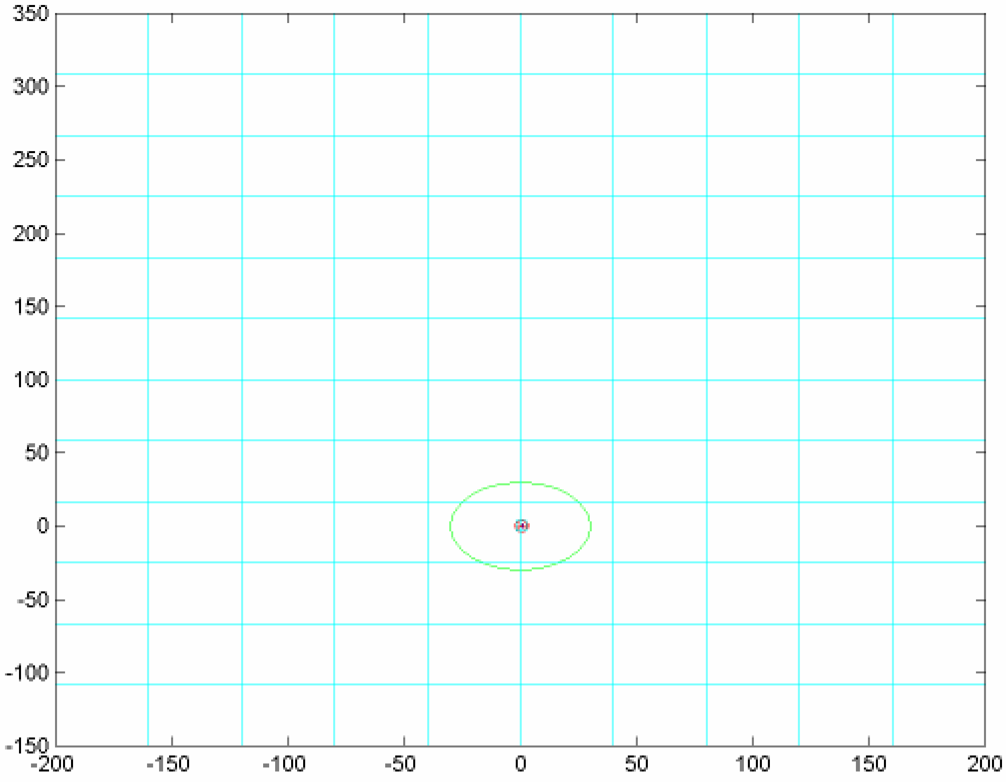
Victoria Park Data Set Vehicle

[courtesy by E. Nebot]

Data Acquisition

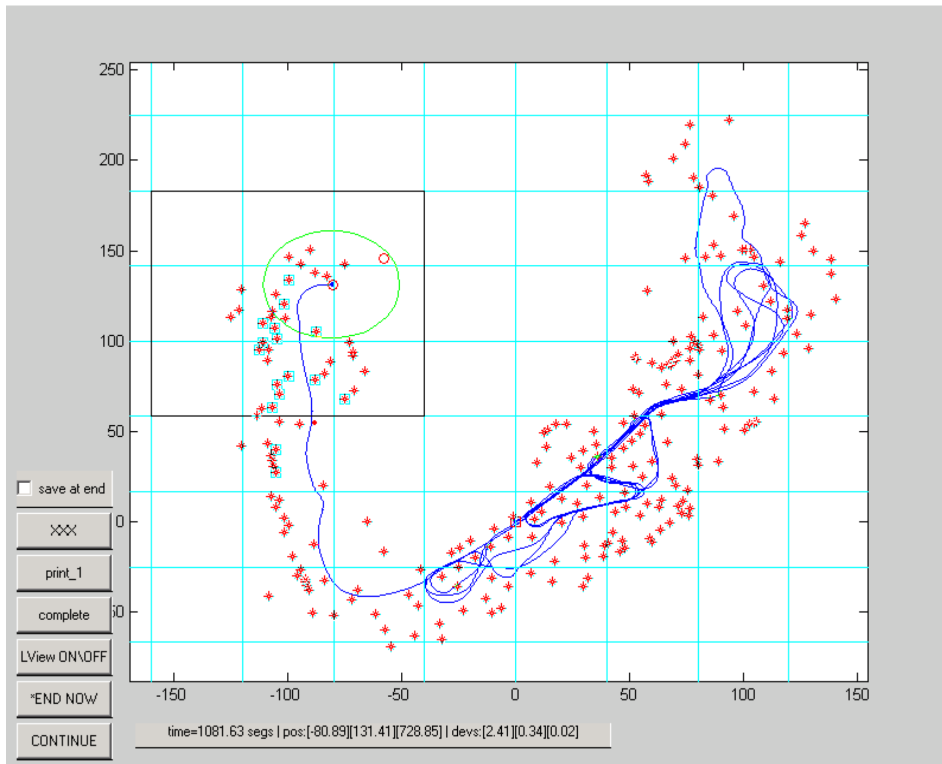
[courtesy by E. Nebot]

SLAM



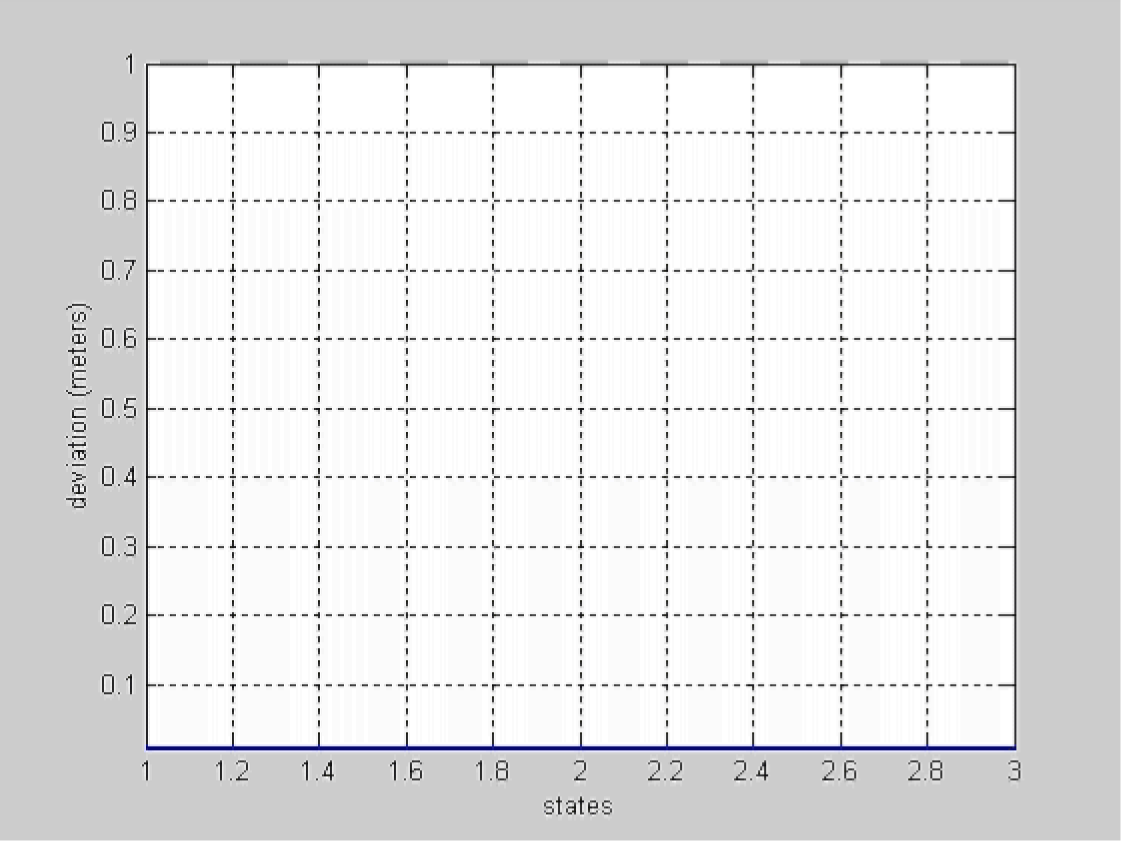
[courtesy by E. Nebot]

Map and Trajectory



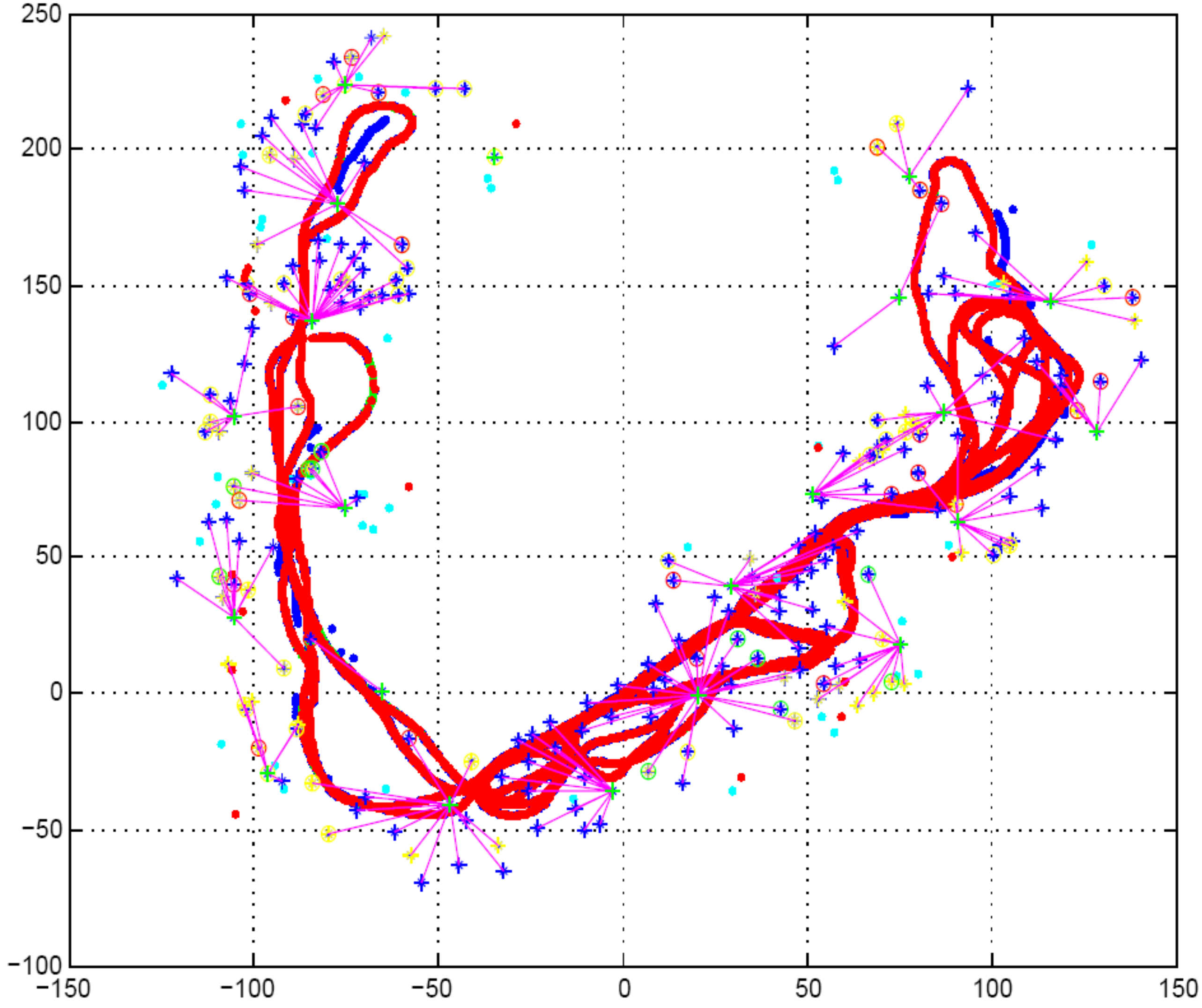
[courtesy by E. Nebot]

Landmark Covariance



[courtesy by E. Nebot]

Estimated Trajectory

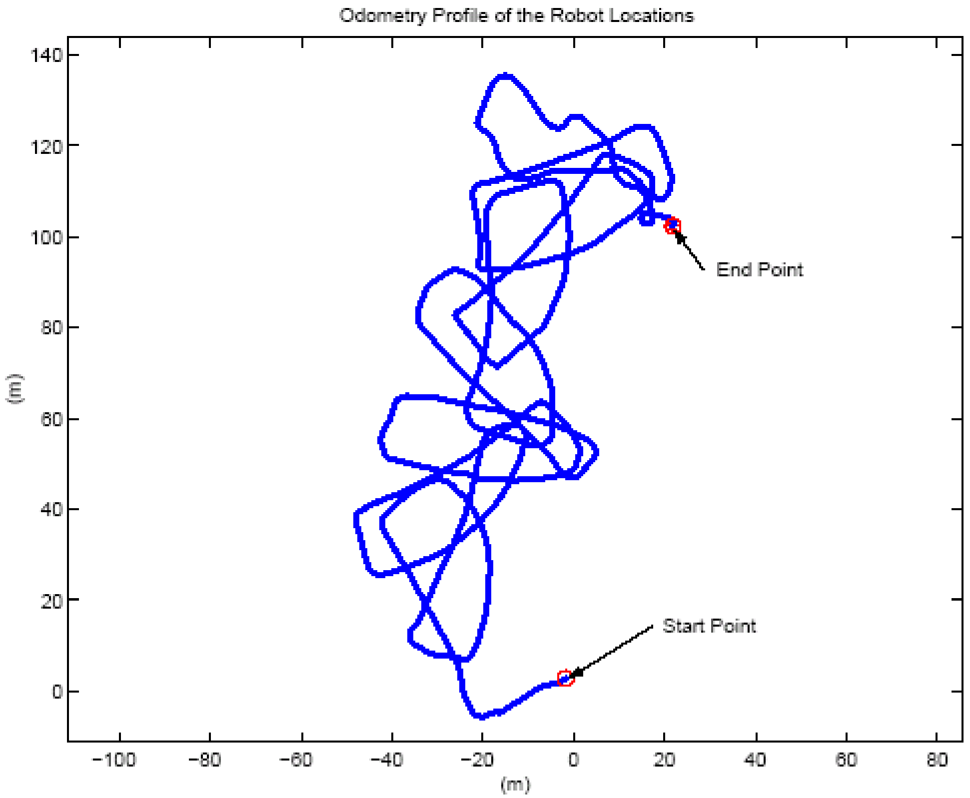


[courtesy by E. Nebot]

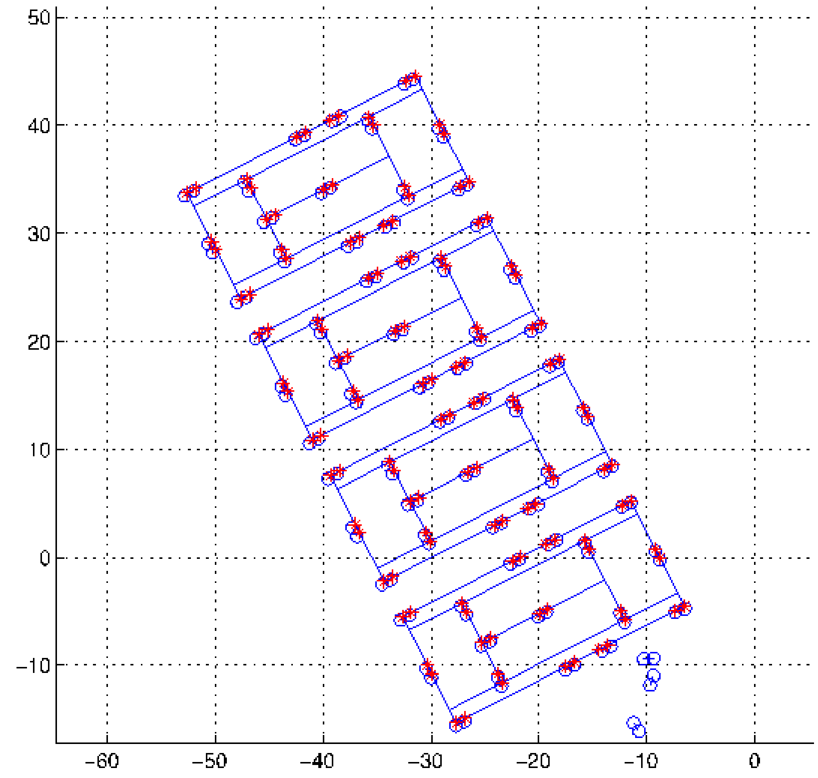
EKF SLAM Indoor Application

[courtesy by John Leonard]

EKF SLAM Application



Only odometry, errors accumulate



EKF SLAM

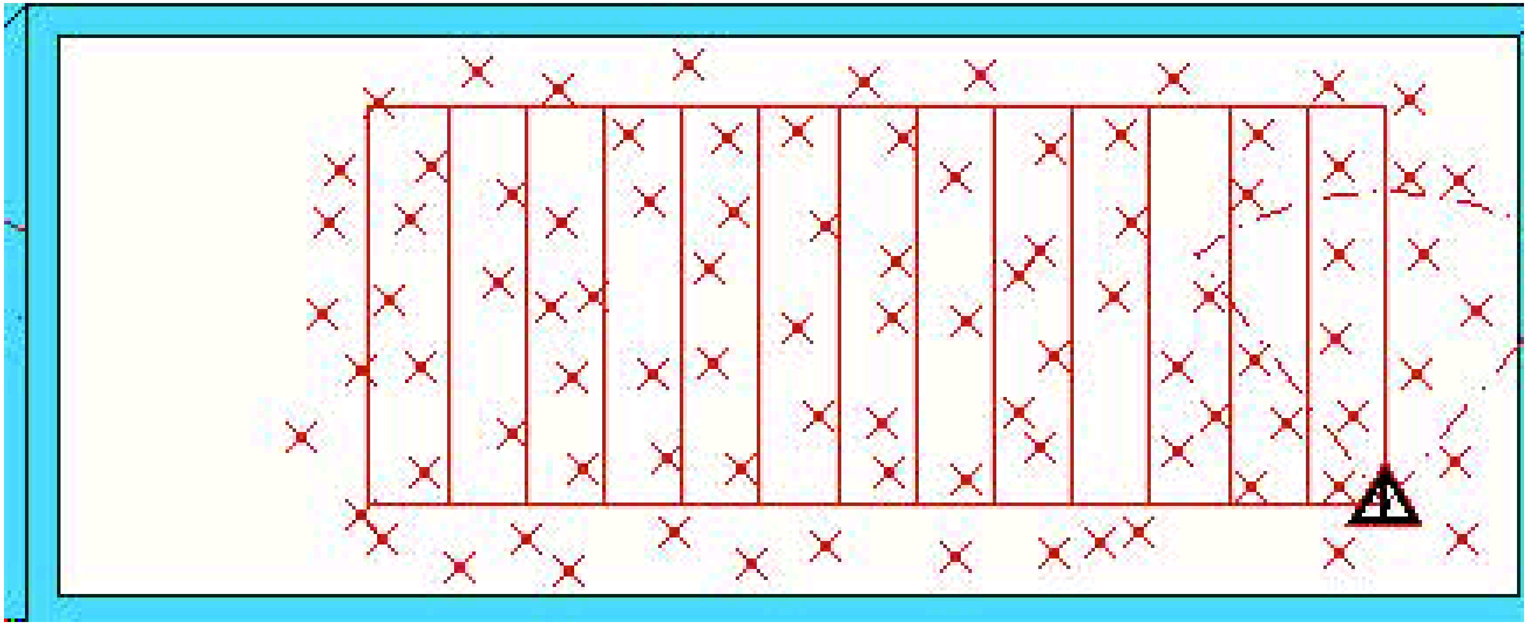
trajectory and map

[courtesy by John Leonard]

Approximations for SLAM

- Local submaps
  [Leonard et al.99, Bosse et al. 02, Newman et al. 03]
- Sparse links (correlations)
  [Lu & Milios 97, Guivant & Nebot 01]
- Sparse extended information filters
  [Frese et al. 01, Thrun et al. 02]
- Thin junction tree filters
  [Paskin 03]
- Rao-Blackwellisation (FastSLAM)
  [Murphy 99, Montemerlo et al. 02, Eliazar et al. 03, Haehnel et al. 03]

Sub-maps for EKF SLAM



[Leonard et al, 1998]

EKF-SLAM Summary

- Quadratic in the number of landmarks: $O(n^2)$
- Convergence results for the linear case.
- Can diverge if nonlinearities are large!
- Have been applied successfully in large-scale environments.
- Approximations reduce the computational complexity.