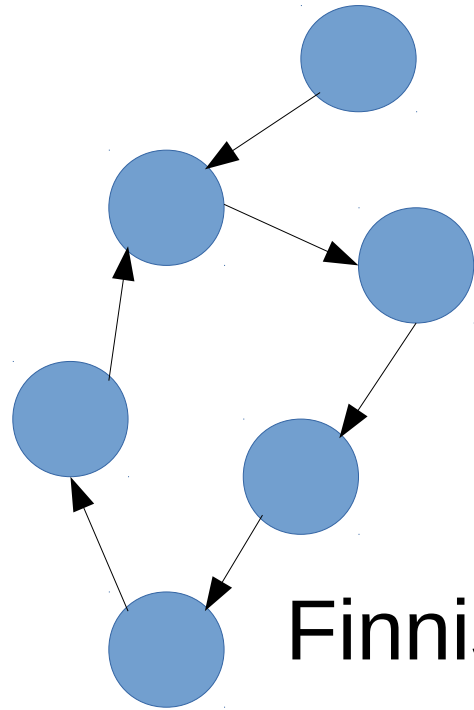


Graph SLAM



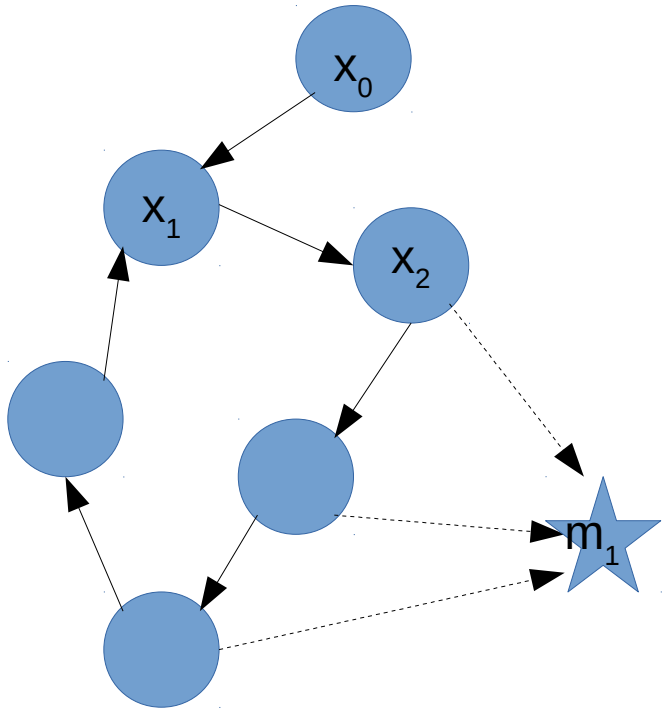
Ville Lehtola, Dr.Sc.
Senior Scientist

Finnish Geospatial Research Institute
12.3.2018

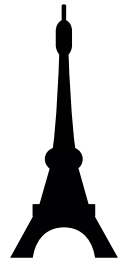
Formalism adapted to the book
"Probabilistic robotics", Thrun, Burgard, Fox (2005)

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Graph



- Directed or undirected
- Nodes are robot poses
- Links are either
 - consecutive poses OR
 - features sensed through measurement



Icons made by Freepik
www.flaticon.com CC 3.0 BY

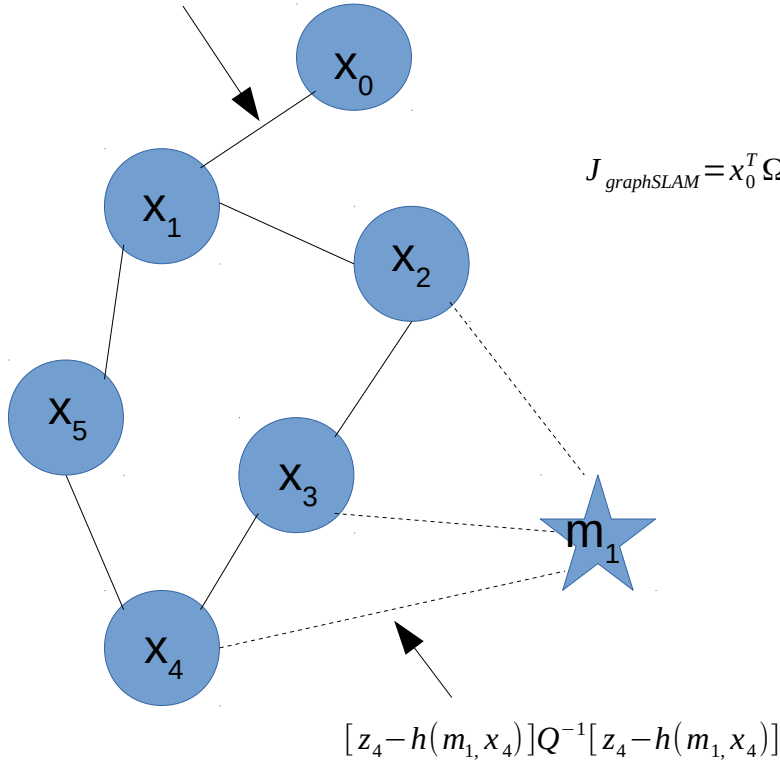
Learning goal

$$J_{\text{graphSLAM}} = x_0^T \Omega_0 x_0 + \sum_t [x_t - g(u_t, x_{t-1})] R^{-1} [x_t - g(u_t, x_{t-1})] + \sum_t [z_t - h(m_c, x_t)] Q^{-1} [z_t - h(m_c, x_t)]$$



Graph SLAM

$$[x_1 - g(u_1, x_0)] R^{-1} [x_1 - g(u_1, x_0)]$$



$$[z_4 - h(m_1, x_4)] Q^{-1} [z_4 - h(m_1, x_4)]$$

- Learning goals

$$J_{\text{graphSLAM}} = x_0^T \Omega_0 x_0^T + \sum_t [x_t - g(u_t, x_{t-1})] R^{-1} [x_t - g(u_t, x_{t-1})] + \sum_t [z_t - h(m_c, x_t)] Q^{-1} [z_t - h(m_c, x_t)]$$

- Graph construction and optimization
- Case forest SLAM
- Advanced topics

Why Graph SLAM?

- The use and formulation of **constraints**
 - **Measurement** constraints integrate the meas. model
 - **Motion** constraints integrate the motion model
 - e.g. post-processing noisy data in laboratory
- Ability to build large scale global maps
 - Sparse graph, motion constraints build linearly in time
 - Eased loop closure
 - Amount of landmarks may be very large, > 1000

$$J_{graphSLAM} = x_0^T \Omega_0 x_0^T + \sum_t [x_t - g(u_t, x_{t-1})] R^{-1} [x_t - g(u_t, x_{t-1})] + \sum_t [z_t - h(m_c, x_t)] Q^{-1} [z_t - h(m_c, x_t)]$$

Initial or anchor constraint

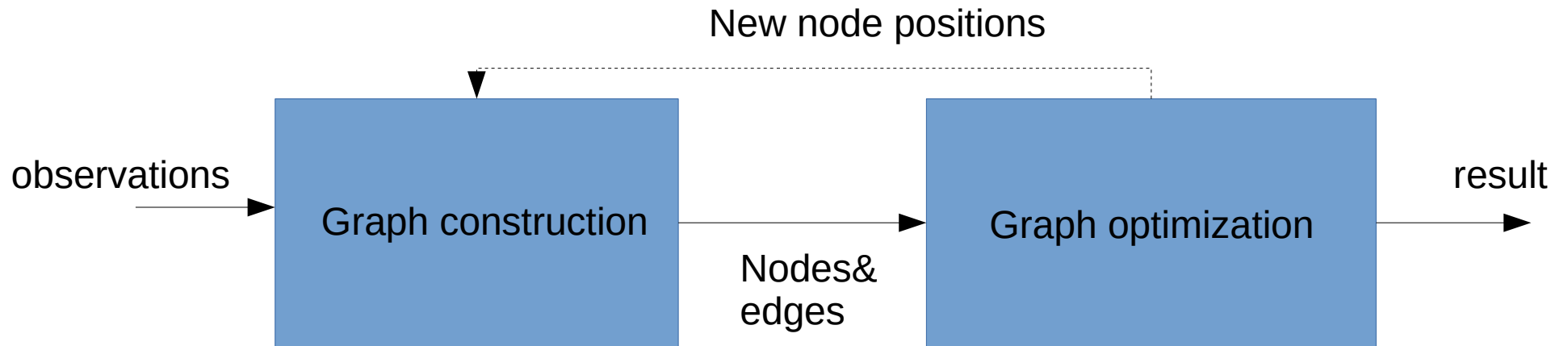
Graph SLAM vs other SLAM

- GraphSLAM is in post-processing phase
 - After the data is captured, **offline / batch algorithm**
 - Full access to **all data**
 - Solve the **full SLAM problem**
- Other SLAM
 - Online / real-time / continuous-time
 - Need fast processing, cannot access all data
 - Extended Kalman Filter, EKF SLAM

Over all time steps

$$J_{\text{graphSLAM}} = x_0^T \Omega_0 x_0^T + \sum_t [x_t - g(u_t, x_{t-1})] R^{-1} [x_t - g(u_t, x_{t-1})] + \sum_t [z_t - h(m_c, x_t)] Q^{-1} [z_t - h(m_c, x_t)]$$

Graph-SLAM idea (1)

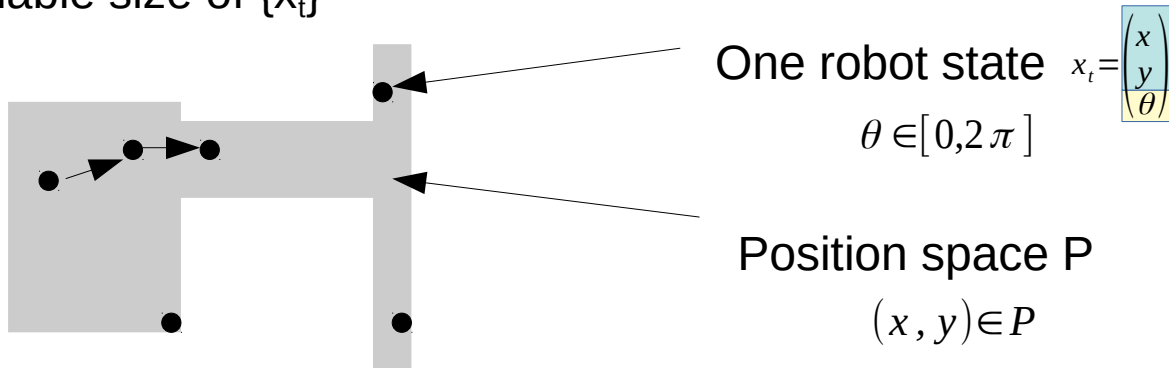


Graph-SLAM idea (2)

- Build a graph to represent the problem.
 - Every node in the graph corresponds to a pose of the robot during mapping
 - Every edge between two nodes corresponds to a spatial constraint between them
- Once we have the graph, we optimize the most likely map of the environment by correcting the nodes
 - minimize the error introduced by the constraints
 - Motion, measurement

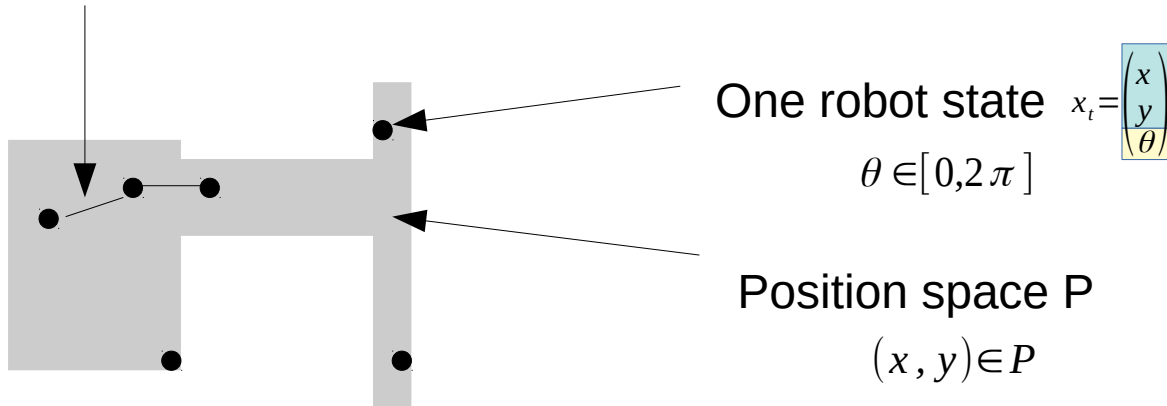
State space S

- Robot state: **position** and **orientation** $x_t \in S$
- Cf. Physical particle (state: position and momentum)
- Thought experiment: Robot so small it would be an aerosol, a particle in the air, scout the space with diffusion
- How to discretize S so that the problem is computationally tractable?
 - Reasonable size of $\{x_t\}$



Motion constraint (1)

- We **control** the robot
- Discretize the state space w.r.t. time Δt , or by saying that there can be only one state per traveled distance D_0
 - e.g. $D_0 = 1$ m

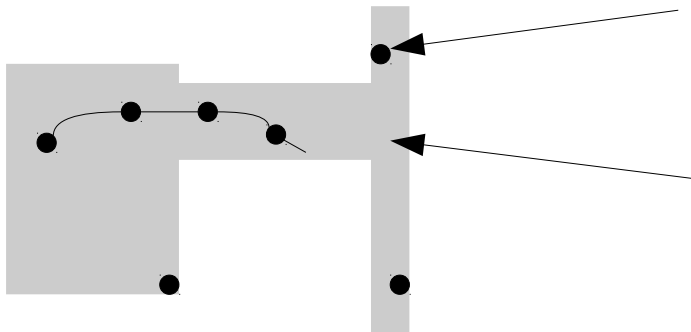
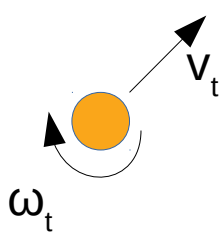


Motion constraint (2)

- Used **controls** tell us how the robot should move
 - Translational velocity v_t
 - Rotational velocity ω_t
 - Turn radius $r = |v_t / \omega_t|$

Indexing notation different here:

$$\begin{pmatrix} x_i \\ y_i \\ \theta_i \end{pmatrix} = \begin{pmatrix} x_{i-1} \\ y_{i-1} \\ \theta_{i-1} \end{pmatrix} + \begin{pmatrix} -\frac{v_i}{\omega_i} \sin \theta_{i-1} + \frac{v_i}{\omega_i} \sin(\theta_{i-1} + \omega_i \Delta t) \\ \frac{v_i}{\omega_i} \cos \theta_{i-1} - \frac{v_i}{\omega_i} \cos(\theta_{i-1} + \omega_i \Delta t) \\ \omega_i \Delta t \end{pmatrix} \rightarrow g(u_i, x_{i-1})$$



One robot state $x_t = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$

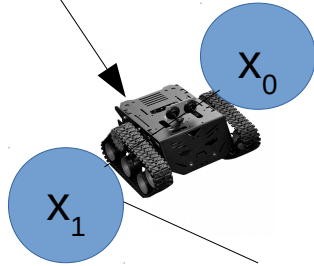
$$\theta \in [0, 2\pi]$$

Position space P

$$(x, y) \in P$$

Motion constraint (3)

$$[x_1 - g(u_1, x_0)] R^{-1} [x_1 - g(u_1, x_0)]$$



Current state estimate y_t

Jacobian of g

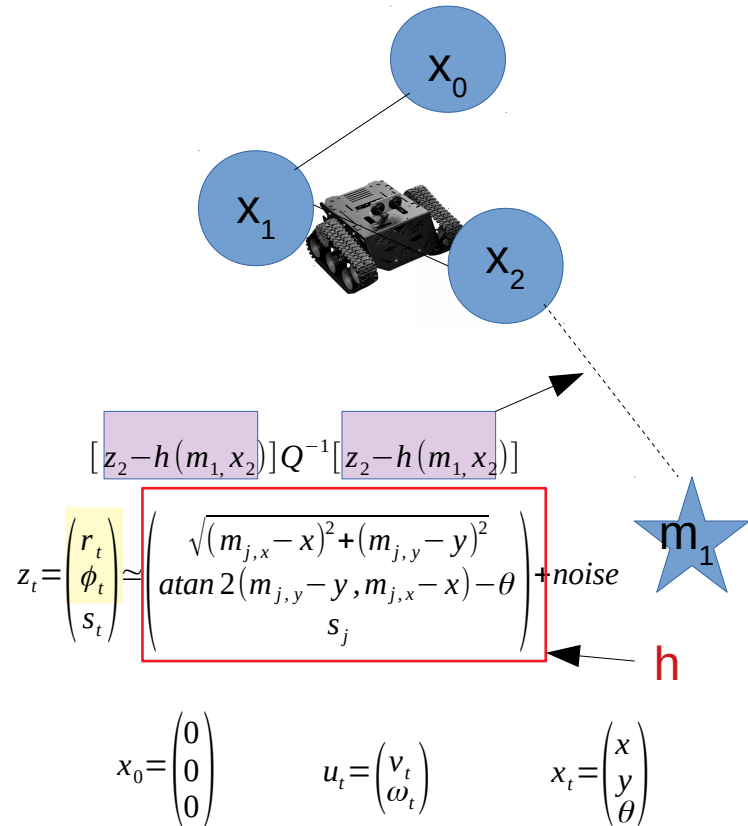
$$g(u_t, x_{t-1}) \approx g(u_t, \mu_{t-1}) + G_t(x_{t-1} - \mu_{t-1})$$

$$x_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad u_t = \begin{pmatrix} v_t \\ \omega_t \end{pmatrix} \quad x_t = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$$

Initial conditions (anchor)

- When the robot changes its state from x_0 to x_1
- Does the state change satisfy the motion constraint?
- Motion model: g
 - Models updates to the state vector
 - Translational velocity v_1
 - Angular velocity ω_1
 - Linearized with Taylor expansion so that current state estimate μ_t may be used

Measurement constraint (1)

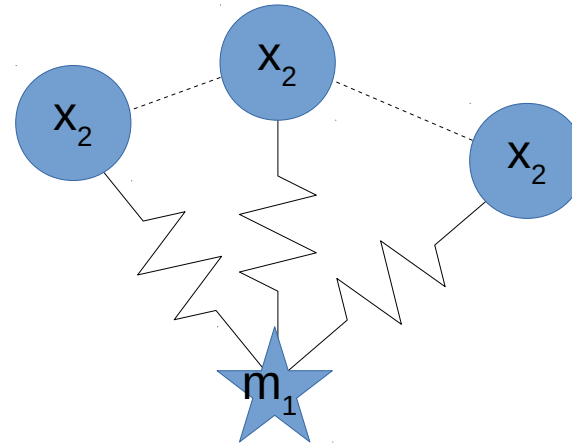
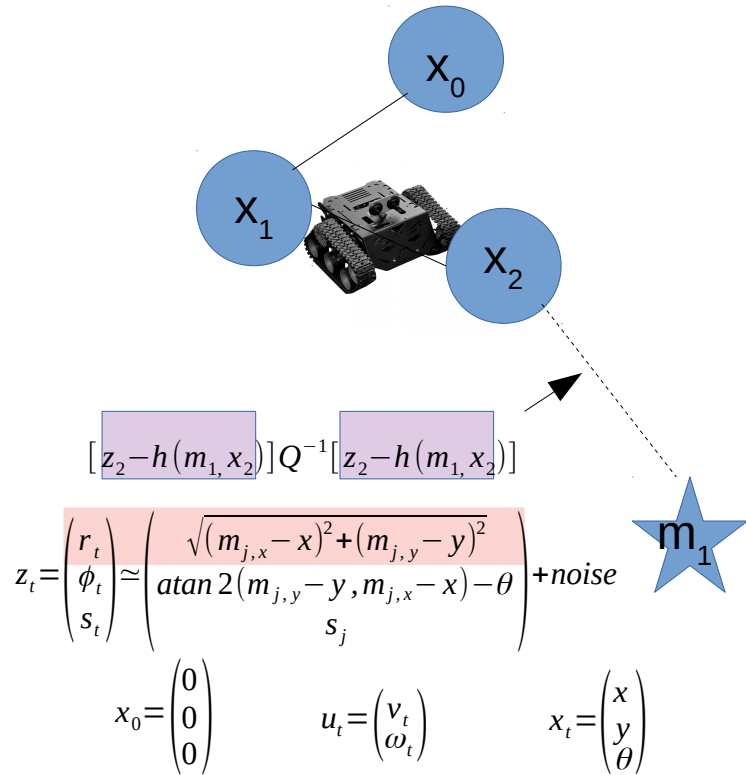


- When the robot sees the landmark m_1 from x_2
 - Measurement model: h
 - Landmark m_1 with signature s_1
 - Observer position x_2
- Compare the measurement z_2 against the previously known position of m_1
 - Is m_1 at the same range r and at the same viewing angle ϕ ?
 - But is it the same landmark than before?
 - correspondence



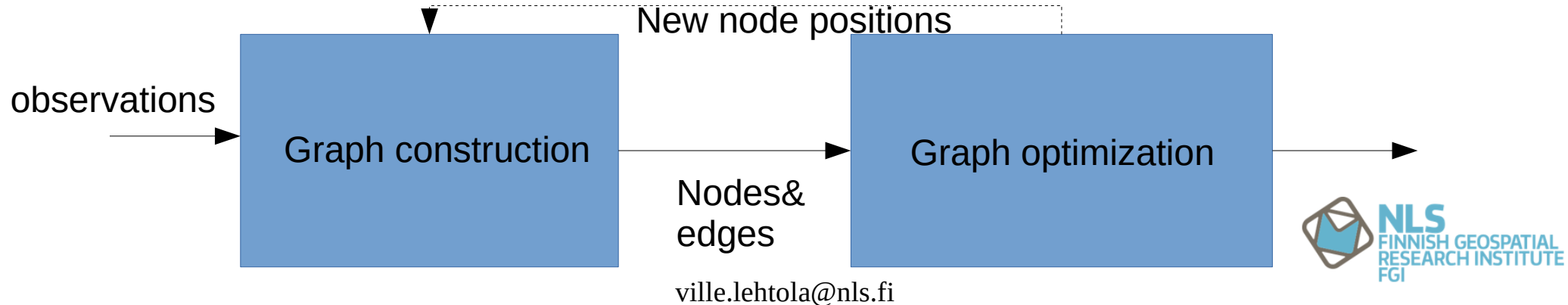
Measurement constraint (2)

- **Range:** Landmark observation is a bit like a 1D spring in a spring-mass model



Uncertainties

- At beginning, state uncertainties are at the highest level
- Uncertainties are reduced through constraints, and iteration
- Graph building, trade-off
 - If the state space is sufficiently dense, the motion model linearization is more likely to work
 - If the state space is sufficiently dense, the measurement model is more likely to map, and correspond, the landmarks correctly
 - If the state space is too dense, CPUtime cost becomes high



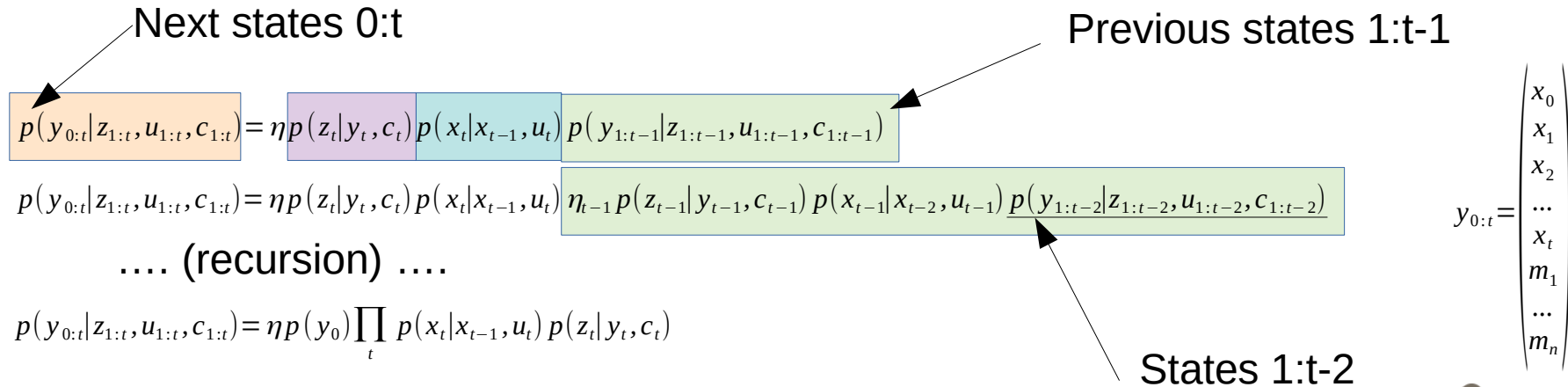
Bayesian scheme (1)

- Revision of basics
- Bayes' theorem
 - P(A|B) is a conditional probability, given B
 - The next state
 - P(A) is the prior (marginal probability)
 - The previous state
 - P(B|A) is the likelihood given A
 - Constraints
 - P(B) is the normalization factor
 - constant

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)}$$

Bayesian scheme (2)

- Updating the previous states 1:t-2 with **measurement** and **motion** constraints gives the next states 1:t-1
- Obtain states 1:t as a posterior distribution



Bayesian scheme (3)

- Obtain the final states 1:t as a recursive posterior of all the previous states 1,2,3... t-1
- Stack all **measurement** and **motion** constraints

$$p(y_{0:t}|z_{1:t}, u_{1:t}, c_{1:t}) = \eta p(y_0) \prod_t p(x_t|x_{t-1}, u_t) \prod_i p(z_t^i|y_t, c_t^i) \longrightarrow \text{Also Gaussian}$$

$$p(x_t|x_{t-1}, u_t) = \eta \exp\left(\frac{-1}{2}(x_t - g(u_t, x_{t-1}))^T R_t^{-1}(x_t - g(u_t, x_{t-1}))\right)$$

- Use convex $\log()$ function to get rid of Gaussian $\exp()$
 - (Also change signs, since $\log(p) < 0$, because $0 < p < 1$)

$$\log p(y_{0:t}|z_{1:t}, u_{1:t}, c_{1:t}) = \text{const.} + \log p(y_0) + \sum_t \log p(x_t|x_{t-1}, u_t) + \sum_i \log p(z_t^i|y_t, c_t^i)$$

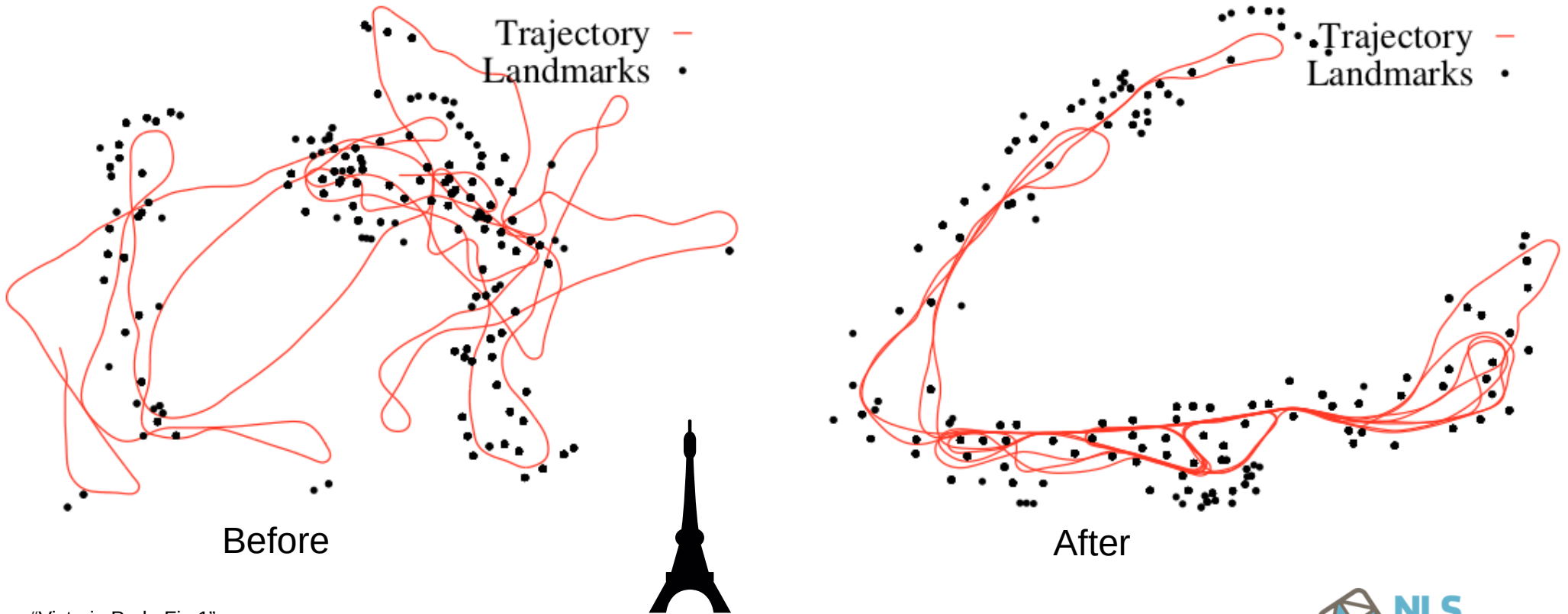
$$y_{0:t} = \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ \dots \\ x_t \\ m_1 \\ \dots \\ m_n \end{pmatrix}$$

$$\dots$$

$$J_{\text{graphSLAM}} = x_0^T \Omega_0 x_0^T + \sum_t [x_t - g(u_t, x_{t-1})]^T R^{-1} [x_t - g(u_t, x_{t-1})] + \sum_t [z_t - h(m_{c_t}, x_t)]^T Q^{-1} [z_t - h(m_{c_t}, x_t)]$$

Our goal \nearrow

2D odometry & 2D landmarks



“Victoria Park, Fig 1”

Kümmerle, Rainer, et al. “g 2 o: A general framework for graph optimization.” Robotics and Automation (ICRA), 2011 IEEE International Conference on. IEEE, 2011.

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Spherical environment constraint (1)

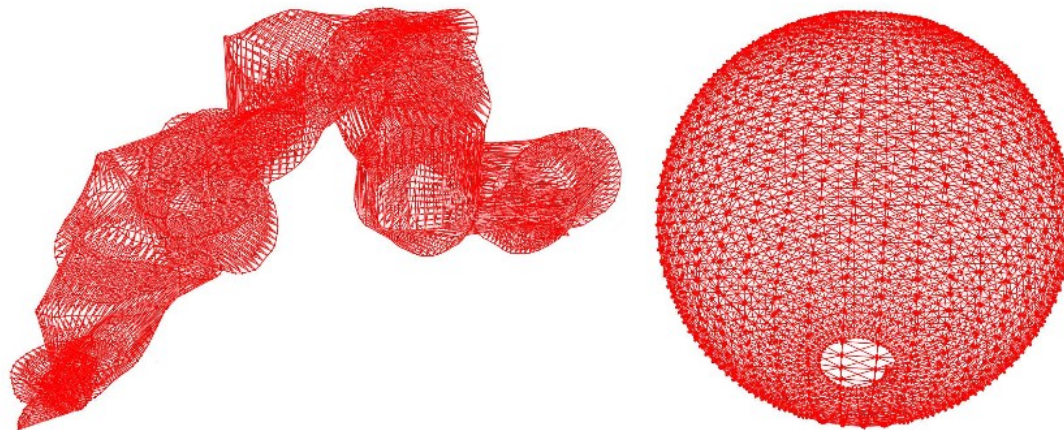


Fig. 11. Pose-graph obtained by simulating a robot moving on a sphere. Left: Initial configuration. Right: After optimizing the pose graph the sphere has accurately been recovered by Algorithm 2.

Image: A Tutorial on Graph-Based SLAM, Grisetti et al.

Spherical environment constraint (2)

- Simulated SLAM
- Measurement model:
 - Laser observations form a spherical surface

Graph SLAM with forest data (1)



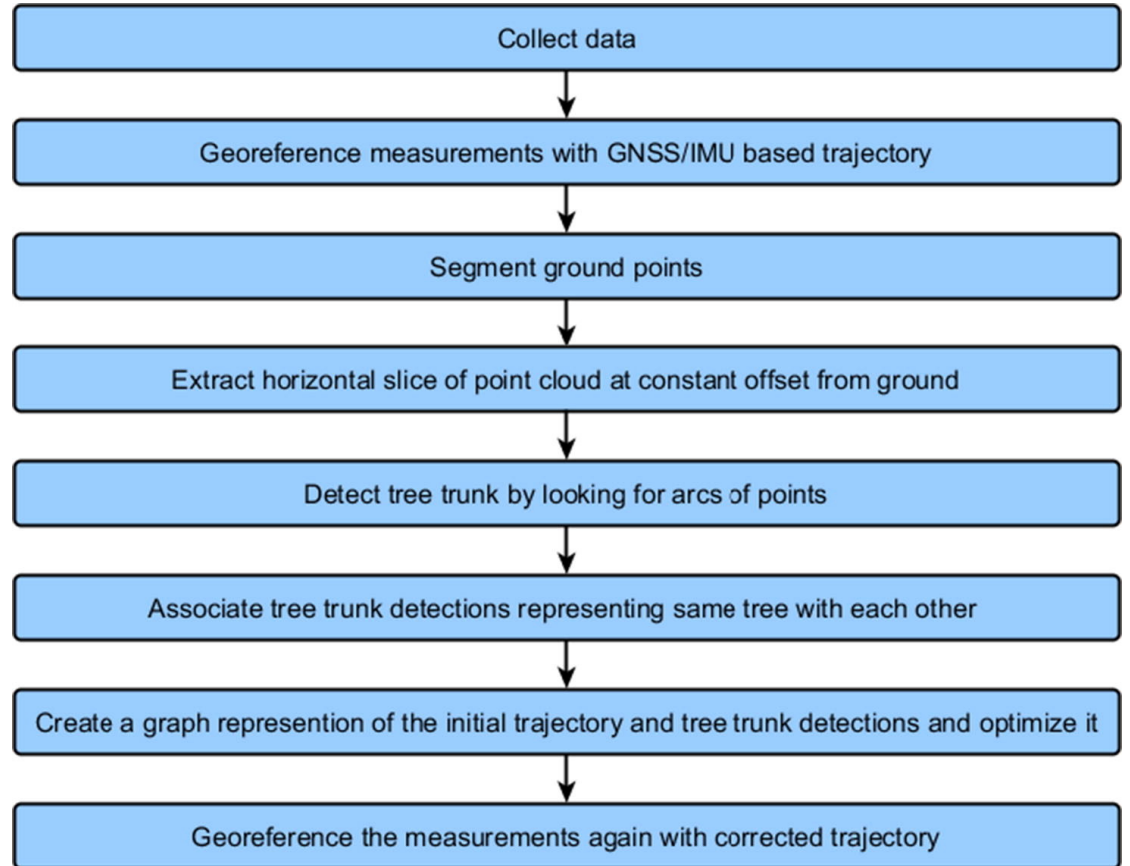
- Problem: Tree foliage causes GNSS errors
 - Acquired trajectory & 3D point cloud is noisy
- Solution: GraphSLAM
 - Correct trajectory

Kukko, A., Kaijaluoto, R., Kaartinen, H., Lehtola, V. V., Jaakkola, A., & Hyypä, J. (2017). Graph SLAM correction for single scanner MLS forest data under boreal forest canopy. *ISPRS Journal of Photogrammetry and Remote Sensing*, 132, 199-209.

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Graph SLAM with forest data (2)

- Nodes: the trajectory is formulated as a graph where the poses at consecutive timestamps (200 Hz in our case) and the detected features at certain time instance (captured as a mean of the feature points' timestamps) form the nodes
- Edges (or constraints) between the nodes are formed from the measured relative transformations between them.



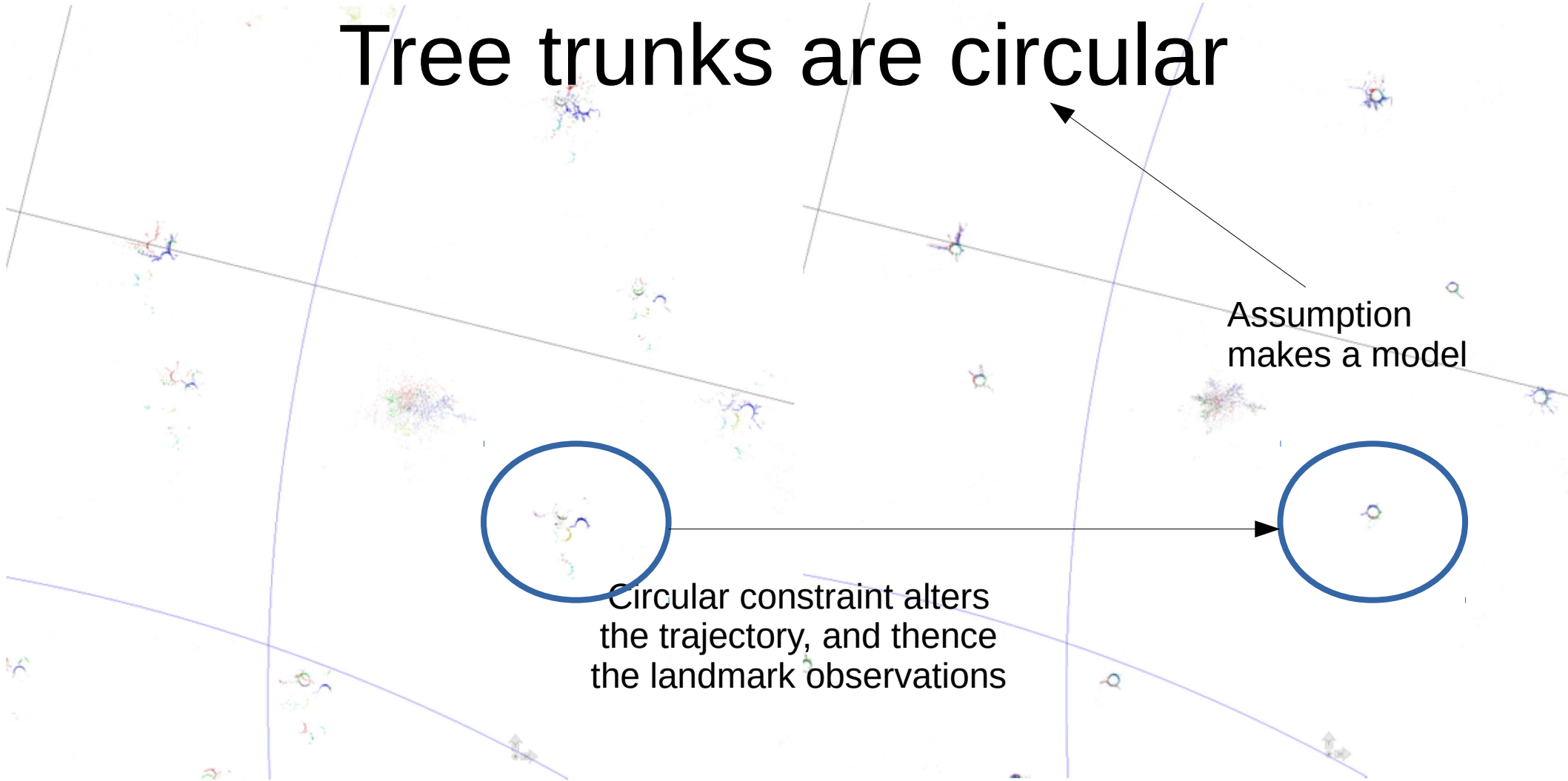
Tree trunks are circular

Assumption
makes a model

Circular constraint alters
the trajectory, and thence
the landmark observations

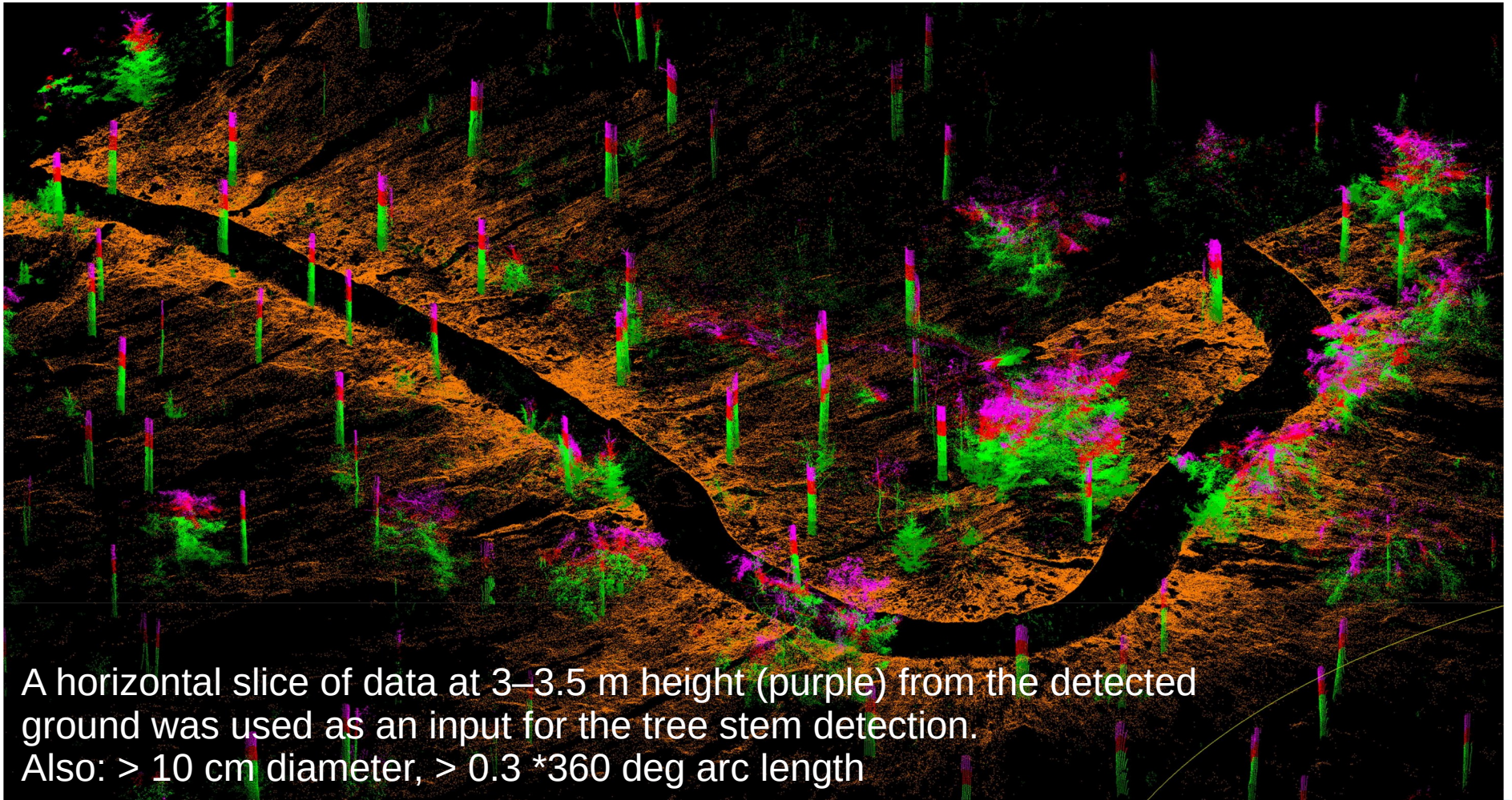
a

b



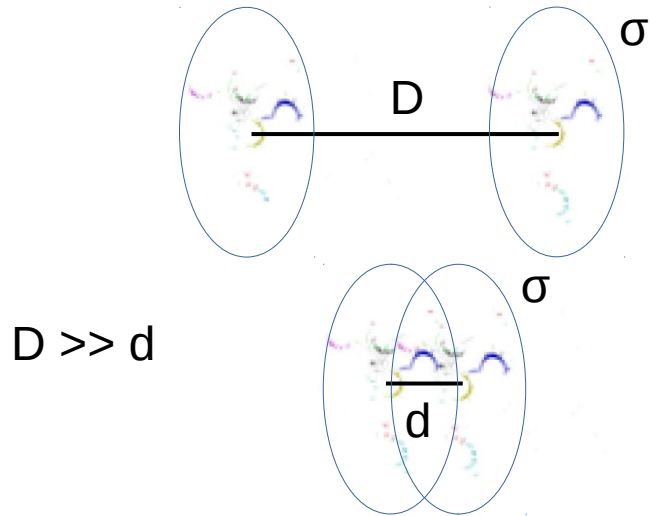
Graph SLAM with forest data (3)

- Construct the graph: the trajectory is expressed as poses.
 - Optimize the graph, i.e. correct the trajectory, iterate until convergence
 - **Measurement** model constraint: See if tree stems appear as circles from above
 - **Motion** model constraint: consecutive timestamps (200 Hz), no control signals
 - Result: 6 cm mean error in absolute tree stem locations



A horizontal slice of data at 3–3.5 m height (purple) from the detected ground was used as an input for the tree stem detection.
Also: > 10 cm diameter, $> 0.3 \cdot 360$ deg arc length

Graph SLAM with forest data (4)



- Method limitations
- Correspondence of landmarks
 - Which points belong to which trees?
 - What happens
 - if $D \rightarrow d$?

Graph SLAM with forest data (5)

- Method limitations:
 - observations from different trees need to be separable
 - i.e. initial correspondences must be good
 - Ergo: method fails in a dense forest where the distances between trees are smaller and the trajectory noise is larger

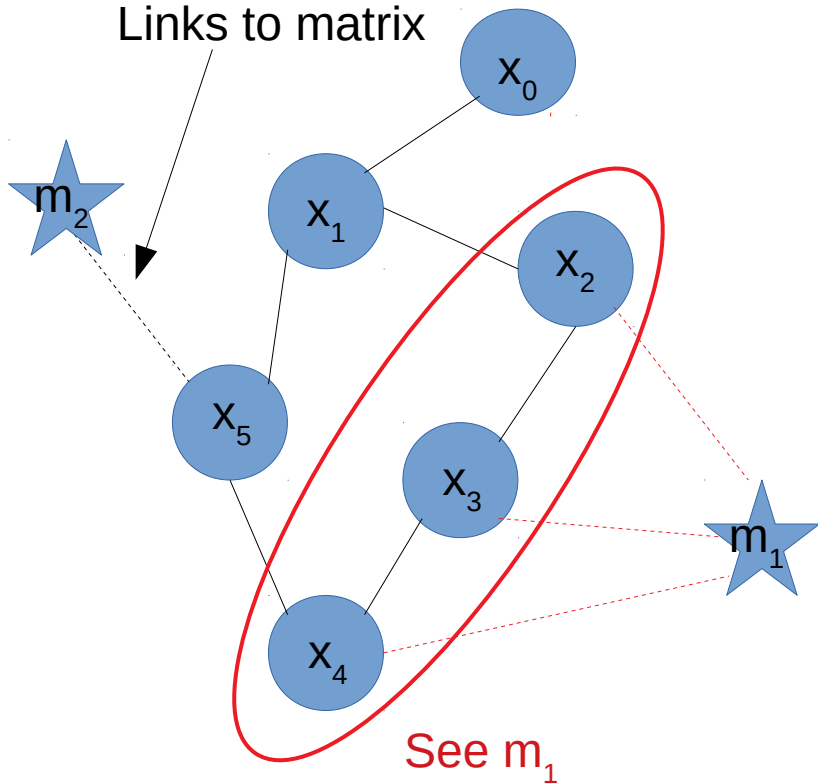
Information matrix Ω (1)

- Inverse of the covariance matrix
- Connected to the information vector

$$\Omega = \Sigma^{-1}$$

$$\xi = \Sigma^{-1} \mu = \Omega \mu$$

Information matrix Ω (2)



- Graph links are represented in matrix

	x_1	x_2	x_3	x_4	x_5	m_1	m_2
x_1	■	■	□	□	■	□	□
x_2	■	■	■	□	□	■	□
x_3	□	■	■	■	□	■	□
x_4	□	□	□	■	■	■	□
x_5	■	□	□	■	■	□	■
m_1	□	■	■	■	□	■	□
m_2	□	□	□	□	■	□	■

Ω

Information matrix Ω (3)

- Linearize:

- $$J_{graphSLAM} = x_0^T \Omega_0 x_0^T + \sum_t \underbrace{[x_t - g(u_t, x_{t-1})] R^{-1} [x_t - g(u_t, x_{t-1})]}_{\text{Quadratic and linear in } x_t} + \sum_t \underbrace{[z_t - h(y_t, c_t^i)] Q^{-1} [z_t - h(y_t, c_t^i)]}_{\text{Quadratic and linear in } y_t}$$

Landmark
correspondencies

- Collect quadratic terms to Ω and linear to ξ

$$J_{graphSLAM} = const - \frac{1}{2} y_{0:t}^T \Omega y_{0:t} + y_{0:t}^T \xi$$

Voilà!

Solving GraphSLAM

- GraphSLAM graph

$$J_{graphSLAM} = x_0^T \Omega_0 x_0^T + \sum_t [x_t - g(u_t, x_{t-1})] R^{-1} [x_t - g(u_t, x_{t-1})] + \sum_t [z_t - h(y_t, c_t^i)] Q^{-1} [z_t - h(y_t, c_t^i)]$$

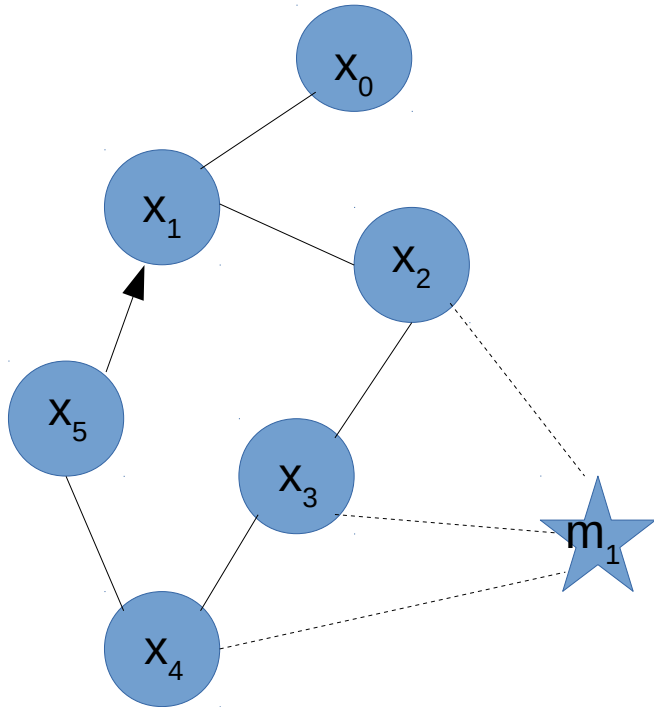
- (or use conjugate gradient or gradient descent)

- Levenberg-Marquardt

$$F = \underset{\alpha}{\operatorname{argmin}} \sum_{i=1}^m [y_i - f(x_i, \alpha)]^2$$

- Parameter vector α
- Damped least squares

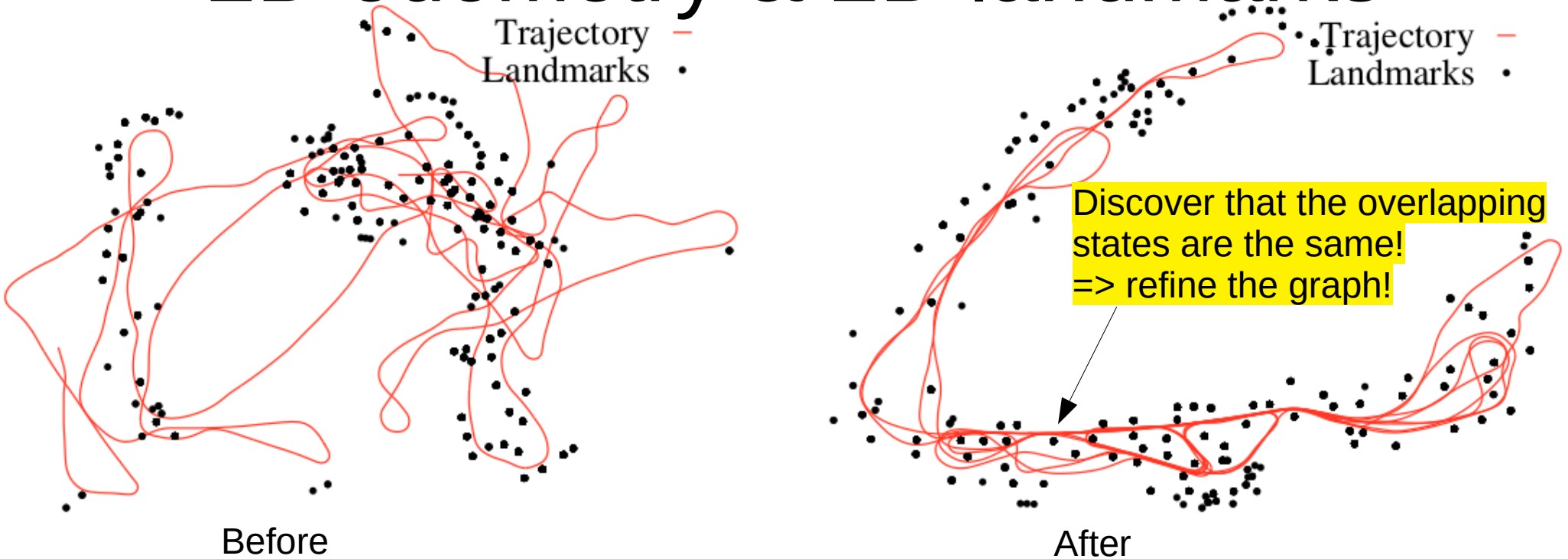
Loop closures (1)



- We see that $x_6 = x_1$!
- Hence, there is a loop in the graph
- But how do we actually recognize that $x_6 = x_1$?

Loop closures (2)

2D odometry & 2D landmarks



"Victoria Park, Fig 1"

Kümmerle, Rainer, et al. "g 2 o: A general framework for graph optimization." Robotics and Automation (ICRA), 2011 IEEE International Conference on. IEEE, 2011.

ville.lehtola@nls.fi

Loop closures (3)

$$\begin{aligned}
 X^*, S^* = \operatorname{argmin}_{X, S} & \sum_i \underbrace{\|f(\mathbf{x}_i, \mathbf{u}_i) - \mathbf{x}_{i+1}\|_{\Sigma_i}^2}_{\text{Odometry Constraints}} \\
 & + \sum_{i,j} \underbrace{\|\operatorname{sig}(s_{ij}) \cdot (f(\mathbf{x}_i, \mathbf{u}_{ij}) - \mathbf{x}_j)\|_{\Lambda_{ij}}^2}_{\text{Switched Loop Closure Constraints}} \\
 & + \sum_{i,j} \underbrace{\|\gamma_{ij} - s_{ij}\|_{\Xi_{ij}}^2}_{\text{Switch Prior Constraints}}
 \end{aligned}$$

On/off switch [0,1]

Note the similarity

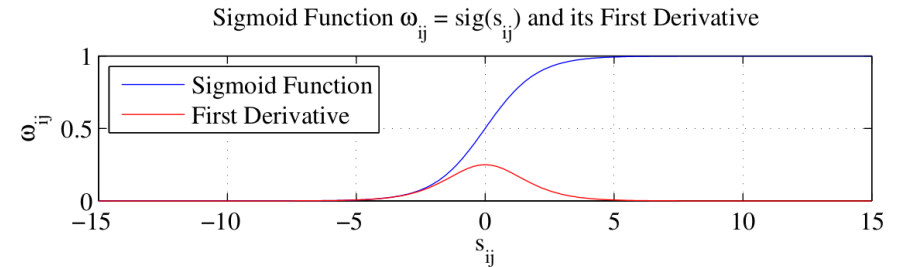
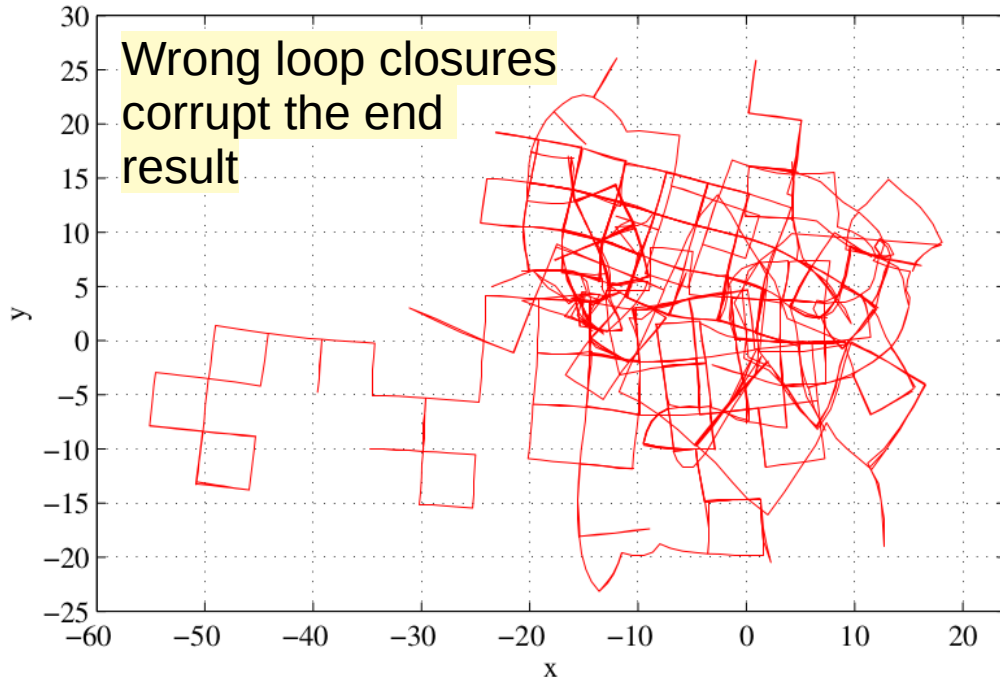


Fig. 4: The sigmoid function $\operatorname{sig}(s_{ij}) = 1/(1 + e^{-s_{ij}})$ and its derivative $\operatorname{sig}'(s_{ij}) = \operatorname{sig}(s_{ij}) \cdot (1 - \operatorname{sig}(s_{ij}))$

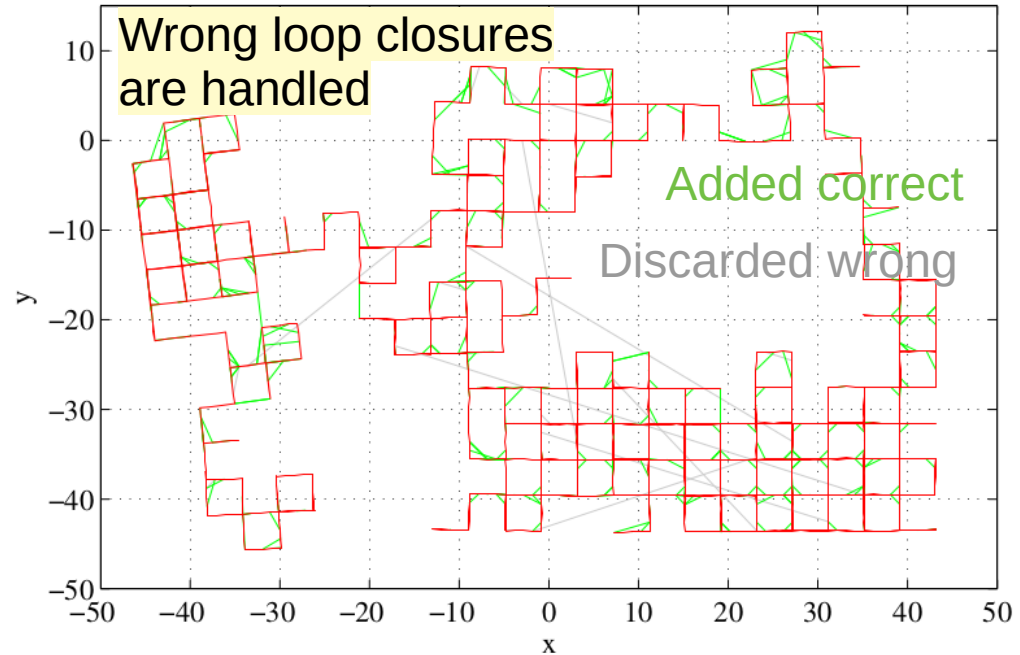
Exclude trivial solutions

Loop closures (4)

Manhattan World Dataset – iSAM v1.5



Manhattan World Dataset – Robust Back-End



Sünderhauf, Niko, and Peter Protzel. "Switchable constraints for robust pose graph SLAM." Intelligent Robots and Systems (IROS), 2012 IEEE/RSJ International Conference on. IEEE, 2012.

Pose matching

$$J_{graphSLAM} = x_0^T \Omega_0 x_0^T + \sum_t [x_t - g(u_t, x_{t-1})] R^{-1} [x_t - g(u_t, x_{t-1})] + \sum_t [z_t - h(m_{c_t}, x_t)] Q^{-1} [z_t - h(m_{c_t}, x_t)]$$

$$[z_t - h(y_t, c_t^i)] \rightarrow [x_i - f(u_{ij}, x_j)]$$

We can match any 2 poses:

$$x_i = f(u_{ij}, x_j) + \lambda_{ij}$$

Gaussian noise

=> Pose match check for x_i and x_j

- We can replace the landmark function $h()$ with a pose-match function $f()$
 - e.g. occupancy grid-based, vision

Pose matching (2)

Graph of key maps

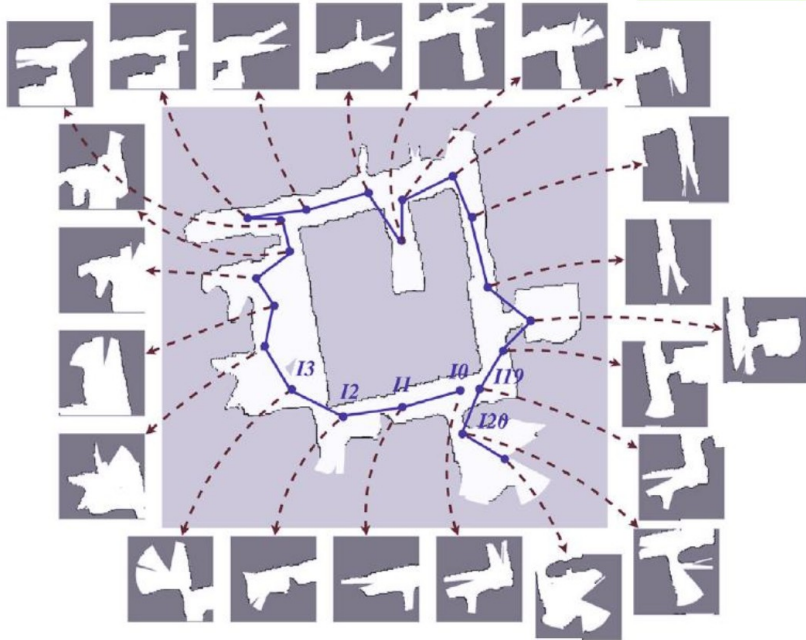


Fig. 3. Graph of key-maps along the route of the robot.

- Laser system
- Global map and local maps
- Each state is represented by a local 2D occupancy grid
- Visual feature techniques used to search for correspondences

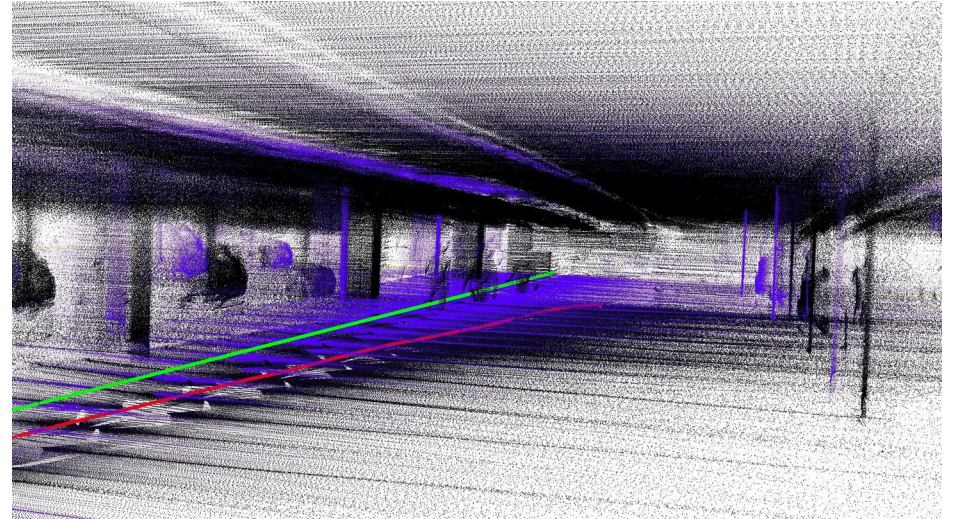
Gil, A., Juliá, M., & Reinoso, Ó. (2015). Occupancy grid based graph-SLAM using the distance transform, SURF features and SGD. *Engineering Applications of Artificial Intelligence*, 40, 1-10.

3D pointclouds (1)

- “Each point is a landmark”

$$E(\mathbf{R}, \mathbf{t}) = \frac{1}{N} \sum_{i=1}^N \|\mathbf{m}_i - (\mathbf{R}\mathbf{d}_i + \mathbf{t})\|^2$$

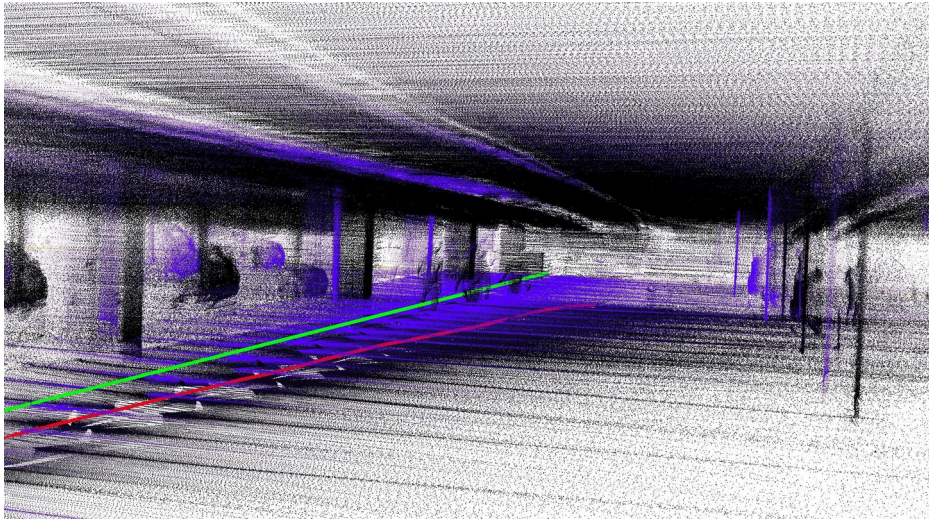
- Forget correspondence, use iterative closest point (ICP)
 - rotation&translation
- Correct the trajectory



Lehtola, V. V., et al. "Localization corrections for mobile laser scanner using local support-based outlier filtering." ISPRS Annals of the Photogrammetry, Remote Sensing and Spatial Information Sciences 3 (2016): 81.

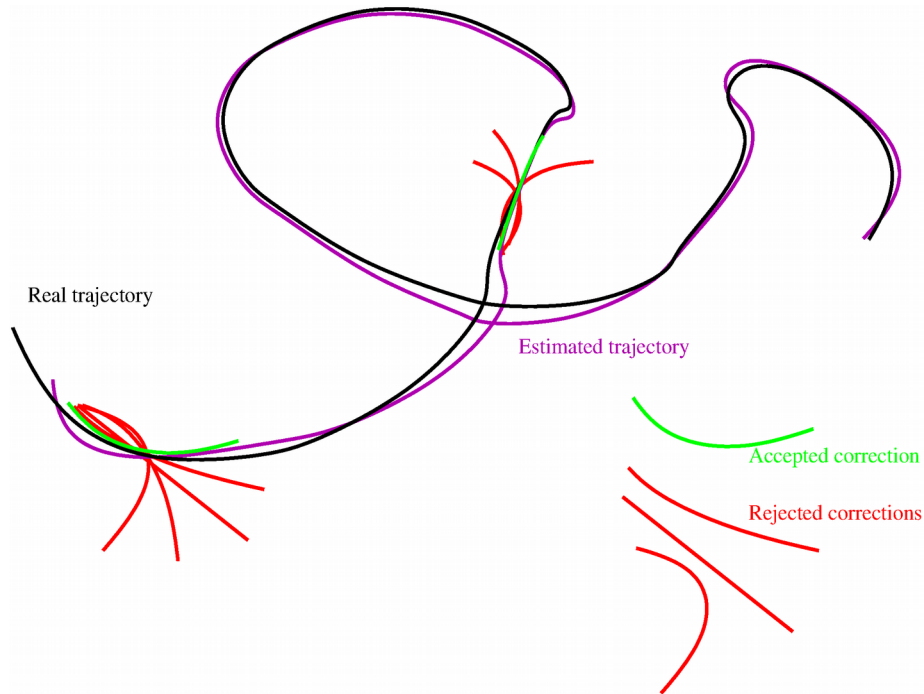
Nüchter, A., Lingemann, K., Hertzberg, J., & Surmann, H. (2007). 6D SLAM—3D mapping outdoor environments. *Journal of Field Robotics*, 24(8-9), 699-722.

3D pointclouds (2)



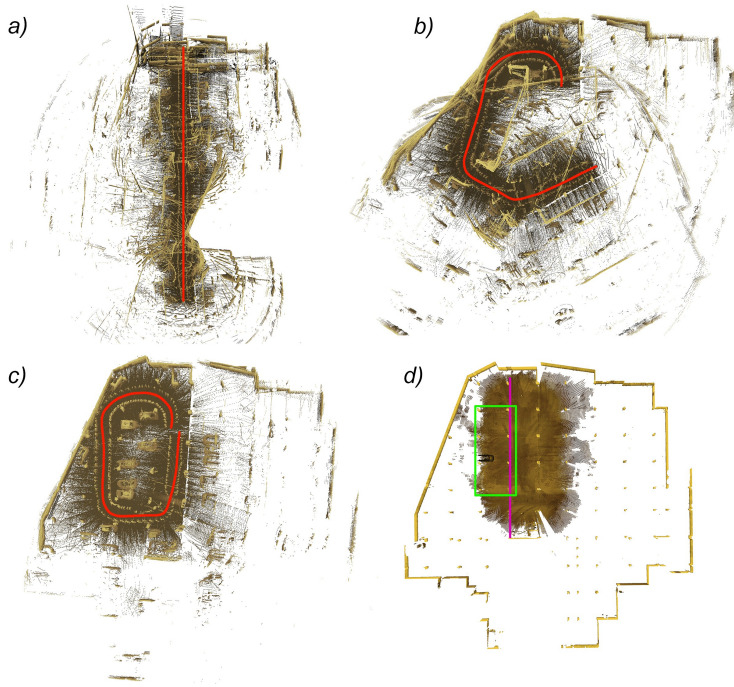
- “Each point is a landmark”
 - Need to downsample!
 - But how to overcome local minima in energy function minimization?
 - ICP does not work, it is greedy

3D pointclouds (3)



- Build an error function to evaluate different curve pieces
 - Always pick the best
- Obtain trajectory estimate for ICP!

3D pointclouds (4)



- “Each point is a landmark”
 - Bend the trajectory with curve-pieces
 - Bend again with different characteristic length
 - Apply ICP
 - Done

Lehtola, Ville V., et al. "Localization of a mobile laser scanner via dimensional reduction." ISPRS Journal of Photogrammetry and Remote Sensing 121 (2016): 48-59.

More information

- A Tutorial on Graph-Based SLAM
<http://www2.informatik.uni-freiburg.de/~stachnis/pdf/grisetti10titsmag.pdf>
- Book: “Probabilistic robotics”, Thrun, Burgard, and Fox

Available methods

- g2o: A General Framework for Graph Optimization
<https://openslam.org/g2o.html>
- C++ code, LGPL v3, Open source
- Others: TreeMap, TORO, sqrt(SAM), iSAM, Sparse Pose Adjustment, iSAM2

Graph SLAM – learning summary

- GraphSLAM solves the full SLAM problem **offline**
 - **Measurement** constraints integrate the measurement model
 - Landmarks
 - **Motion** constraints integrate the motion model
 - Data from controls not necessarily needed (use IMU or smoothing)
- Ability to build large scale global maps
 - Sparse graph, motion constraints build linearly in time
 - Eased loop closure
 - Amount of landmarks may be very large, > 1000

$$J_{graphSLAM} = \underbrace{x_0^T \Omega_0 x_0^T}_{\text{Initial or anchor constraint}} + \sum_t [x_t - g(u_t, x_{t-1})] R^{-1} [x_t - g(u_t, x_{t-1})] + \sum_t [z_t - h(m_c, x_t)] Q^{-1} [z_t - h(m_c, x_t)]$$

Initial or anchor constraint