COEFFICIENT OF PARTIAL DETERMINATION AND PARTIAL CORRELATION

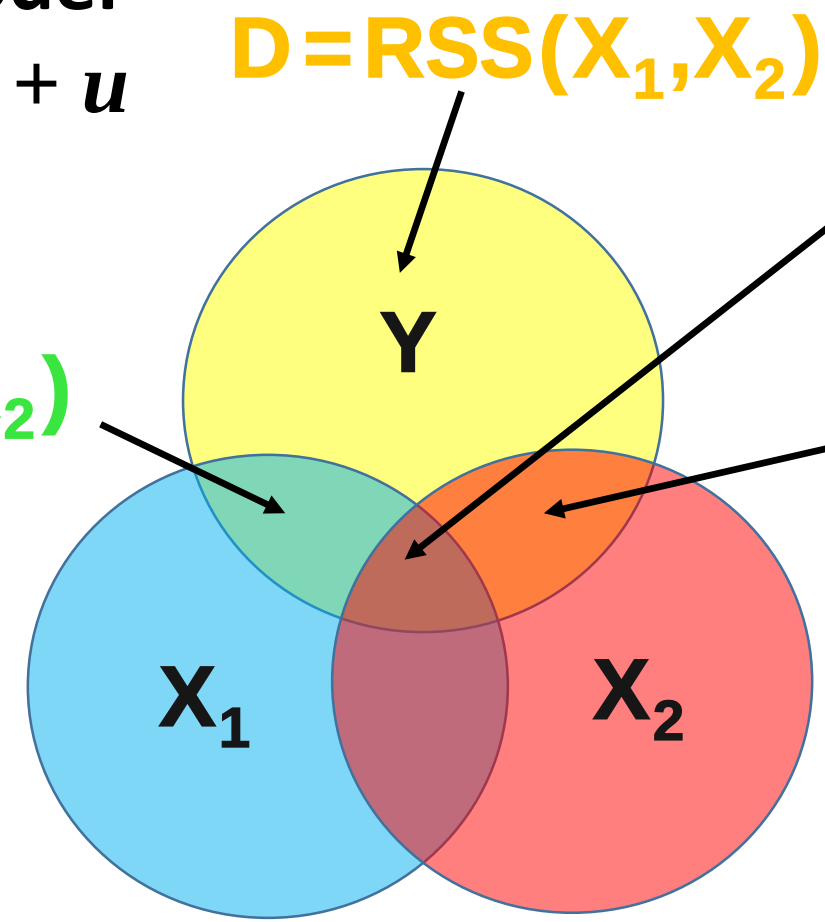
Tomi Seppälä

Aalto School of Business

2018

Linear regression model

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + u$$



TSS(Y)=
A+B+C+D

COEFFICIENT OF DETERMINATION: $R^2(X_1,X_2) = \frac{A+B+C}{A+B+C+D}$

COEFFICIENT OF partial DETERMINATION between Y and X2, when given X1:

$$r^2_{YX2|X1} = \frac{B}{B+D}$$

Tells how many percentages of the remaining variation in Y can be explained by X_2 after the effect of X_1 on Y has been taken into account (effect of X_1 has been controlled for).

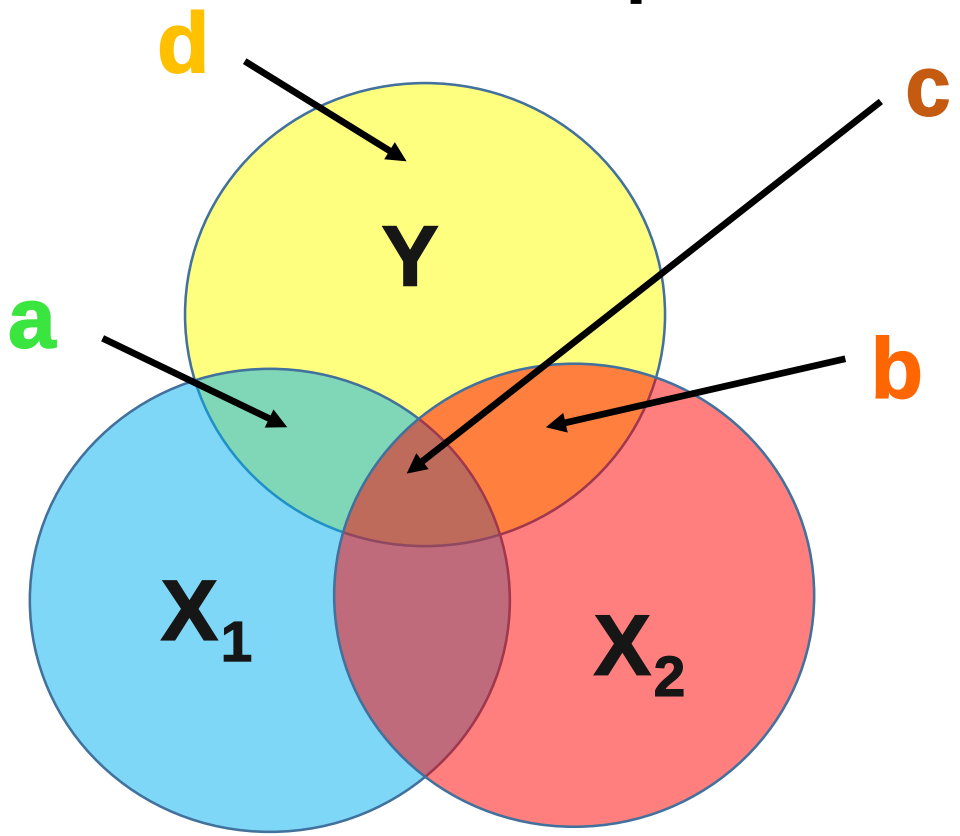
COEFFICIENT OF partial CORRELATION between Y and X_2, when given X_1:

$$r_{YX2|X1} = \sqrt{\frac{B}{B+D}}$$

Tells the correlation between Y and X_2 after the effect of X_1 on Y has been taken into account (effect of X_1 has been controlled for).

Formula to calculate partial correlation

Here all variations (ball sizes) are scaled to 1 (or 100%):
a+b+c+d=1



$$r_{YX2|X1} = \frac{r_{YX2} - r_{YX1} \cdot r_{X1X2}}{\sqrt{(1 - r^2_{YX1})(1 - r^2_{X1X2})}}$$

Example: consider the following correlation matrix

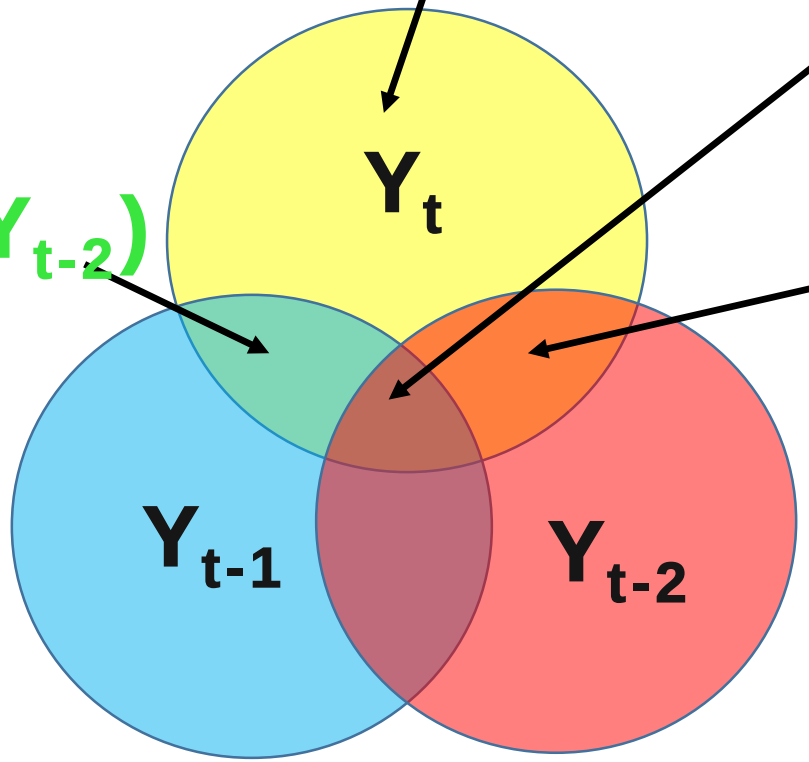
	Y	X1	X2
Y	1	0.6	0.5
X1	0.6	1	0.7
X2	0.5	0.7	1

Partial correlation between Y and X_2, when given X_1:

$$r_{YX2|X1} = \frac{0.5 - 0.6 \cdot 0.7}{\sqrt{(1 - 0.6^2)(1 - 0.7^2)}} = 0.35$$

Time Series model, eg. AR(2)

$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + u_t$



$D = RSS(X_1, X_2)$

C = 'COMMON VARIATION'

$A = ESS(Y_{t-1} \mid Y_{t-2})$

$B = ESS(Y_{t-2} \mid Y_{t-1})$

$TSS(Y_t) = A + B + C + D$

COEFFICIENT OF DETERMINATION: $R^2(Y_{t-1}, Y_{t-2}) = \frac{A+B+C}{A+B+C+D}$

COEFFICIENT OF partial DETERMINATION of order 2 is the coefficient of determination between Y_t and Y_{t-2}, when given Y_{t-1}:

$$\tau^2_{22} = \frac{B}{B+D}$$

Tells how many percentages of the remaining variation in Y_t can be explained by Y_{t-2} after the effect of Y_{t-1} on Y_t has been taken into account (effect of Y_{t-1} has been controlled for).

COEFFICIENT OF partial AUTOCORRELATION of order 2 is the partial correlation between Y_t and Y_{t-2}, when given Y_{t-1}:

$$\tau_{22} = \sqrt{\frac{B}{B+D}}$$

Tells the autocorrelation between Y_t and Y_{t-2} after the effect of Y_{t-1} on Y_t has been taken into account (effect of Y_{t-1} has been controlled for).

τ_{22} Partial autocorrelation of order 2

Same formula as for cross sectional variables!
But simplifies in this case if the process is stationary!
This formula works for any stationary process

$$\tau_{22} = \frac{\tau_2 - \tau^2_1}{1 - \tau_1^2}$$