Chapter 5

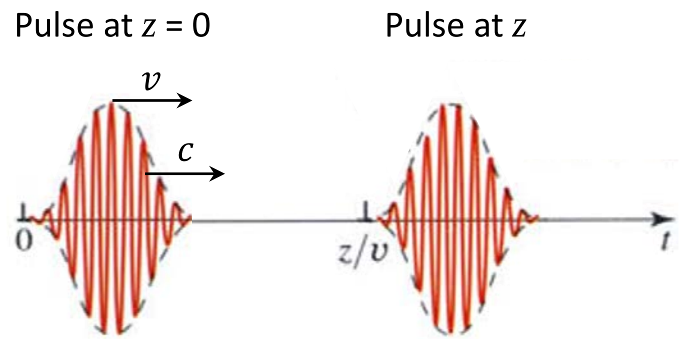
ELECTROMAGNETIC OPTICS II

Pulse propagation in dispersive media

Phase velocity: $c = \frac{\omega}{k}$

Group velocity: $v = \frac{d\omega}{dk} = \frac{c_0}{N}$

Group index: $N = n - \lambda_0 \frac{dn}{d\lambda_0}$



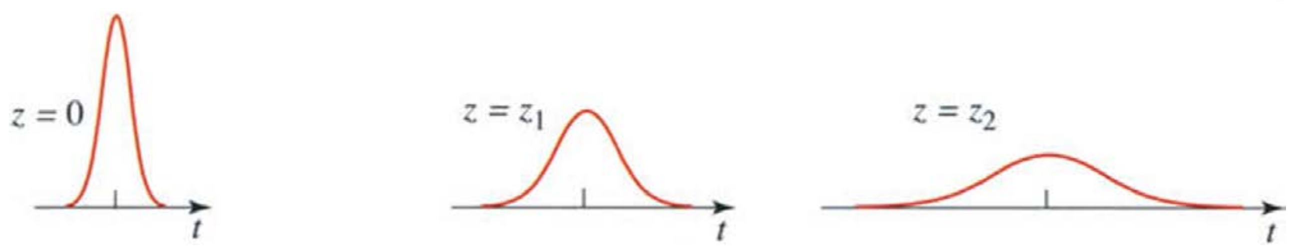
$$U(z,t) = \mathcal{A}(t - z/v)\exp[j\omega_0(t - z/c)]$$

Group velocity dispersion (GVD): v is frequency dependent $\Rightarrow$ Different freque-ncy components travel to the same z in different times. The delay due to $\delta\nu$ is

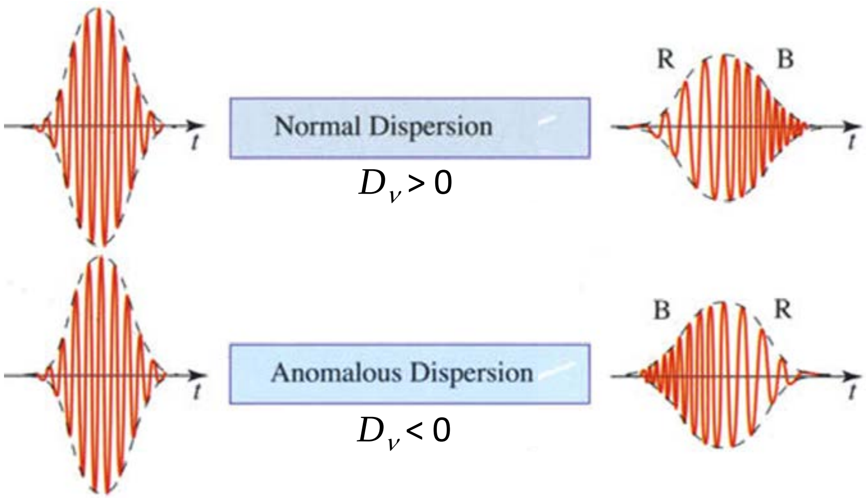
$$\delta\tau = \frac{d\tau_d}{d\nu}\,\delta\nu = \frac{d}{d\nu}\left(\frac{z}{v}\right)\delta\nu = D_\nu\, z\, \delta\nu$$

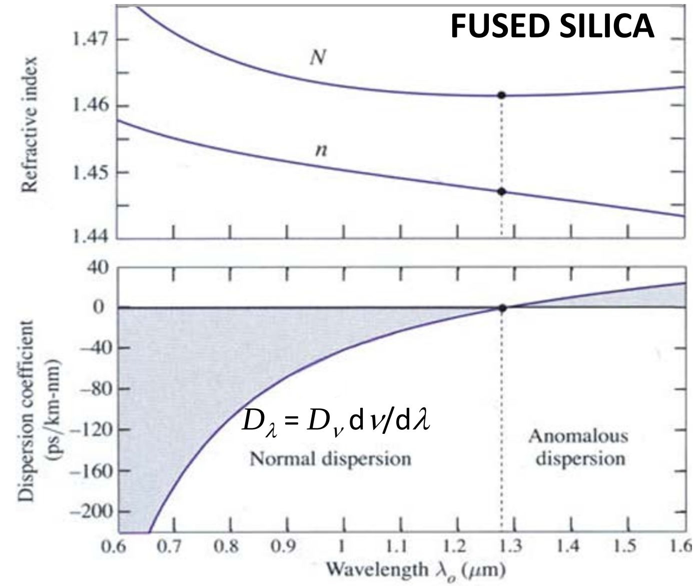
$$D_\nu = \frac{d}{d\nu}\left(\frac{1}{v}\right) = \frac{\lambda_o^3}{c_o^2}\frac{d^2 n}{d\lambda_o^2}$$ – GVD coefficient

❖ If the spectral width of a pulse is σ_ν, the pulse spread will be $\sigma_\tau = |D_\nu|\sigma_\nu z$.

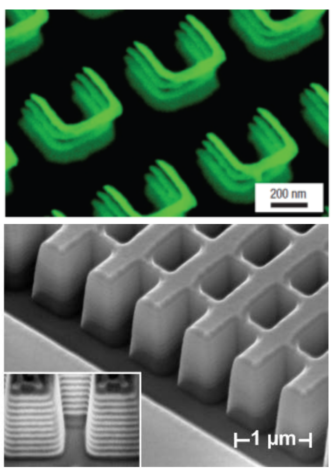


Normal and anomalous dispersion





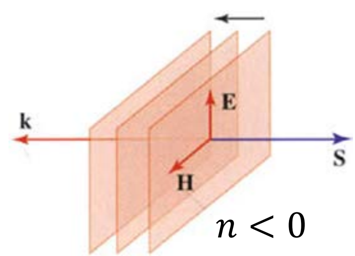
Artificial optical nanomaterials: Metamaterials ($\mu \neq \mu_0$)

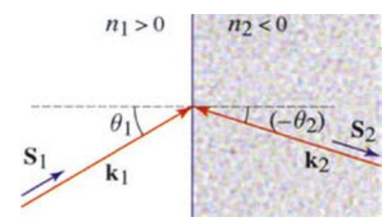


$$\mathbf{k} \times \mathbf{H}_0 = -\omega\,\epsilon\,\mathbf{E}_0$$
$$\mathbf{k} \times \mathbf{E}_0 = \omega\,\mu\,\mathbf{H}_0.$$
$$\eta = \sqrt{\frac{\mu}{\epsilon}},$$
$$n = \sqrt{\frac{\epsilon}{\epsilon_0}}\sqrt{\frac{\mu}{\mu_0}}$$
$$\mathbf{S} = \tfrac{1}{2}\mathbf{E} \times \mathbf{H}^*$$

$$\epsilon = -|\epsilon|, \quad \mu = -|\mu| \;\longrightarrow$$

$$\mathbf{k} \times \mathbf{H}_0 = \omega\,|\epsilon|\,\mathbf{E}_0$$
$$\mathbf{k} \times \mathbf{E}_0 = -\omega\,|\mu|\,\mathbf{H}_0$$
$$\eta > 0, \quad n < 0$$





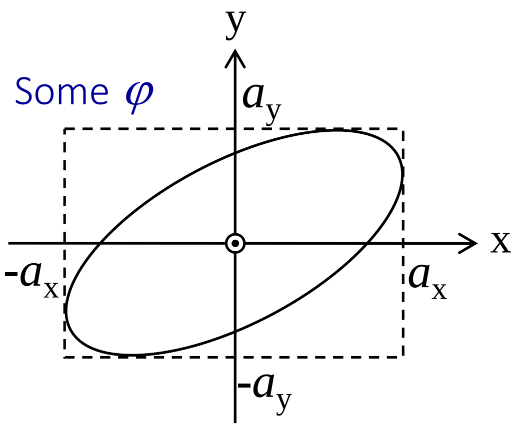
Chapter 6

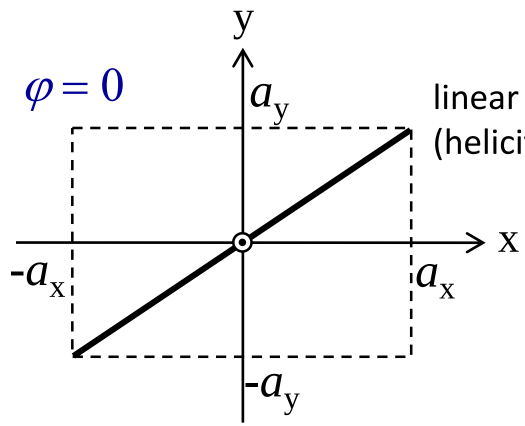
POLARIZATION OPTICS I

Light polarization

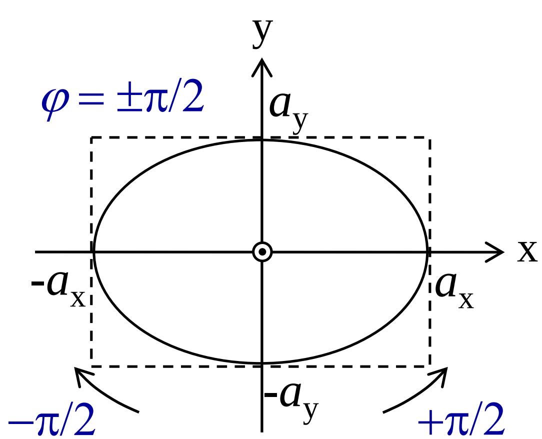
$$\boldsymbol{\mathcal{E}}(z,t) = \mathcal{E}_x \hat{\mathbf{x}} + \mathcal{E}_y \hat{\mathbf{y}}, \qquad \begin{cases} \mathcal{E}_x = \mathfrak{a}_x \cos\left[\omega\left(t - \frac{z}{c}\right) + \varphi_x\right] \\ \mathcal{E}_y = \mathfrak{a}_y \cos\left[\omega\left(t - \frac{z}{c}\right) + \varphi_y\right] \end{cases}$$

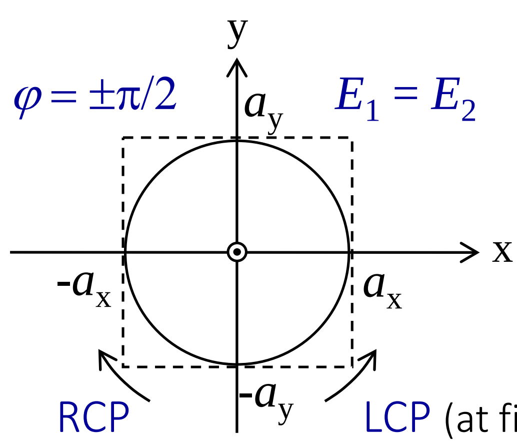
$$\varphi = \varphi_y - \varphi_x$$



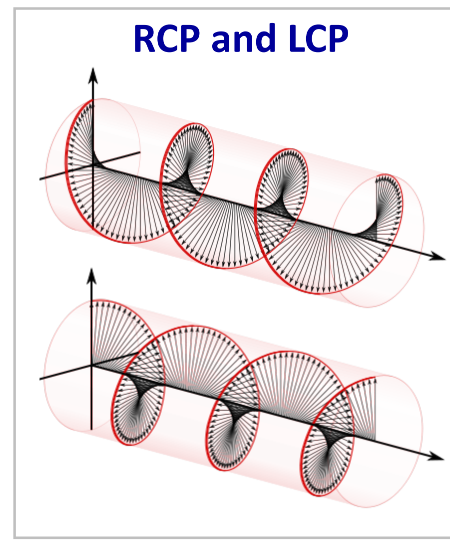


linear polarization
(helicity is not defined)





LCP (at fixed t, looks like a **left screw**)



Poincaré sphere and Stokes parameters

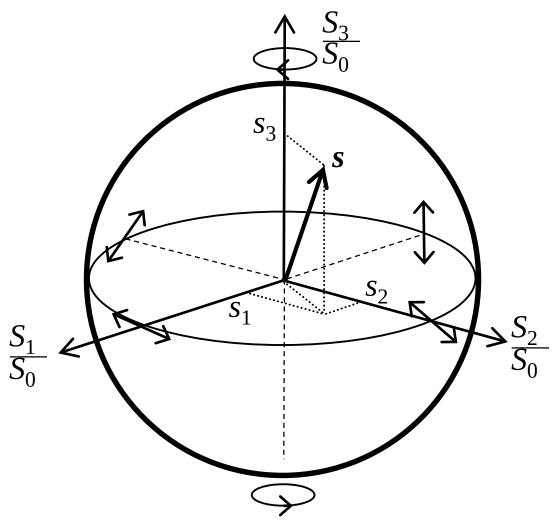
$$\mathcal{E}(z,t) = \mathrm{Re}\left\{\mathbf{A}\exp\left[j\omega\left(t-\frac{z}{c}\right)\right]\right\}$$, where $\mathbf{A} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}}$.

Stokes parameters:

$$\begin{cases} \mathrm{S}_0 = \mathrm{a}_x^2 + \mathrm{a}_y^2 & = |A_x|^2 + |A_y|^2 \quad \text{(intensity)} \\ \mathrm{S}_1 = \mathrm{a}_x^2 - \mathrm{a}_y^2 & = |A_x|^2 - |A_y|^2 \\ \mathrm{S}_2 = 2\mathrm{a}_x\mathrm{a}_y\cos\varphi & = 2\,\mathrm{Re}\{A_x^* A_y\} \\ \mathrm{S}_3 = 2\mathrm{a}_x\mathrm{a}_y\sin\varphi & = 2\,\mathrm{Im}\{A_x^* A_y\}\,. \end{cases}$$

$$\mathrm{S}_1^2 + \mathrm{S}_2^2 + \mathrm{S}_3^2 = \mathrm{S}_0^2$$

Unit-radius ***Poincaré sphere*** is the surface of coordinates $(s_1, s_2, s_3) = \left(\frac{S_1}{S_0}, \frac{S_2}{S_0}, \frac{S_3}{S_0}\right)$.



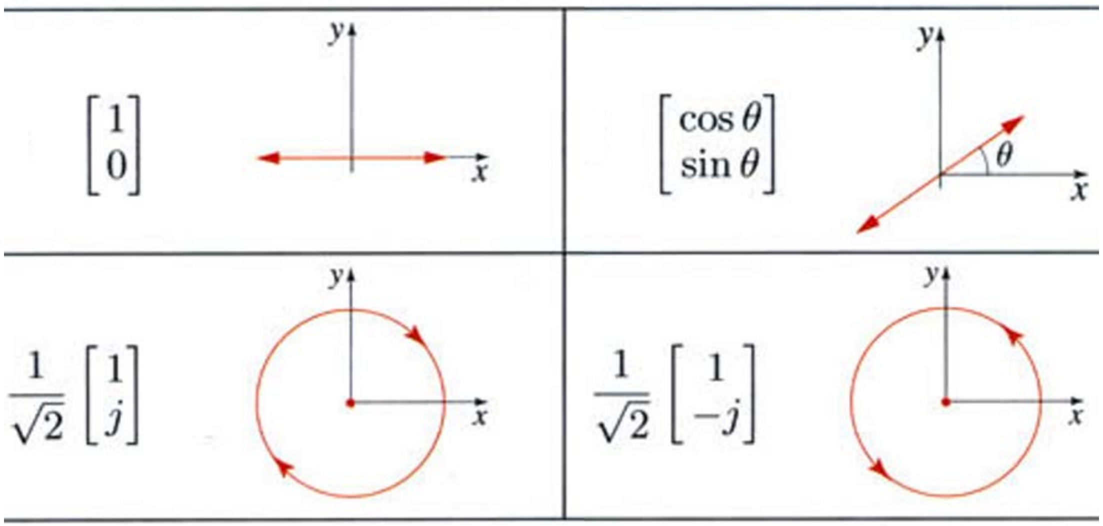
Each point (s_1, s_2, s_3) on the sphere defines a certain polarization state.

Jones vectors

$$\mathcal{E}(z,t) = \mathrm{Re}\left\{\mathbf{A}\exp\left[j\omega\left(t-\frac{z}{c}\right)\right]\right\},$$ where $\mathbf{A} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}}$.

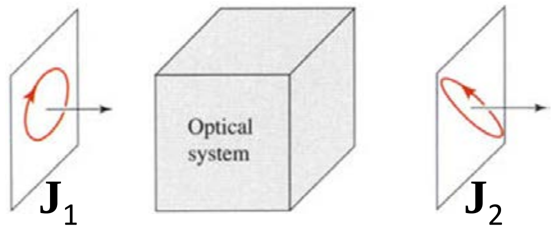
Jones vector: $\mathbf{J} = \begin{bmatrix} A_x \\ A_y \end{bmatrix}$

The Jones vector can be normalized by requiring $|A_x|^2 + |A_y|^2 = 1$. Then,



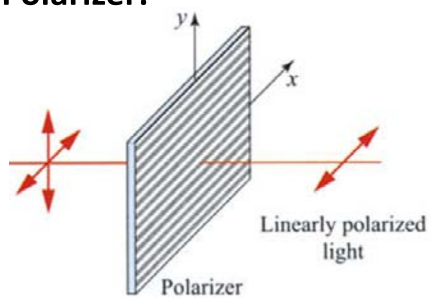
For orthogonal polarizations, $(\mathbf{J}_1 \cdot \mathbf{J}_2) = A_{1x}A_{2x}^* + A_{1y}A_{2y}^* = 0$. Any polarization can then be expanded as $\mathbf{J} = \alpha_1\mathbf{J}_1 + \alpha_2\mathbf{J}_2 = (\mathbf{J}\cdot\mathbf{J}_1)\mathbf{J}_1 + (\mathbf{J}\cdot\mathbf{J}_2)\mathbf{J}_2$.

Jones matrices



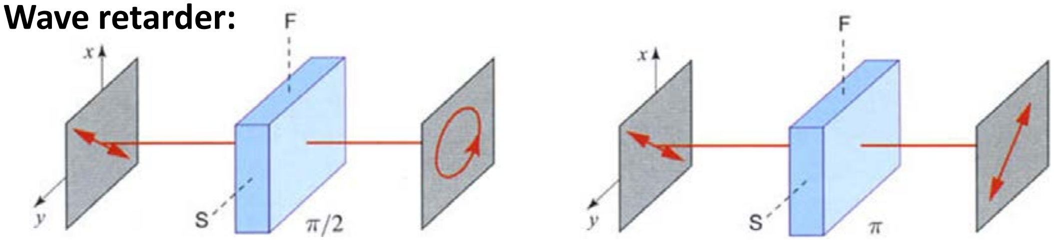
$$\begin{bmatrix} A_{2x} \\ A_{2y} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} A_{1x} \\ A_{1y} \end{bmatrix} \quad \Rightarrow \quad \mathbf{J}_2 = \mathbf{T}\mathbf{J}_1$$

Polarizer:



$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

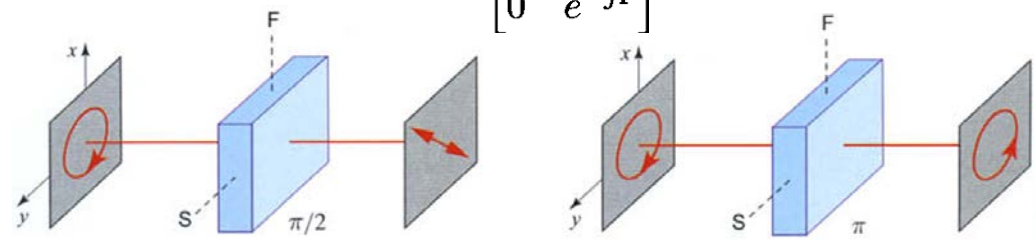
Wave retarder:



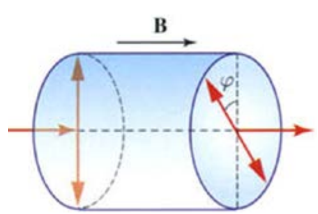
λ/4

$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & e^{-j\Gamma} \end{bmatrix}$$

λ/2



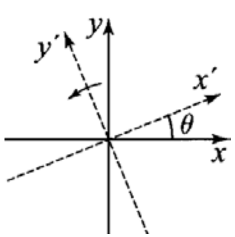
Polarization rotator:



$$\mathbf{T} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

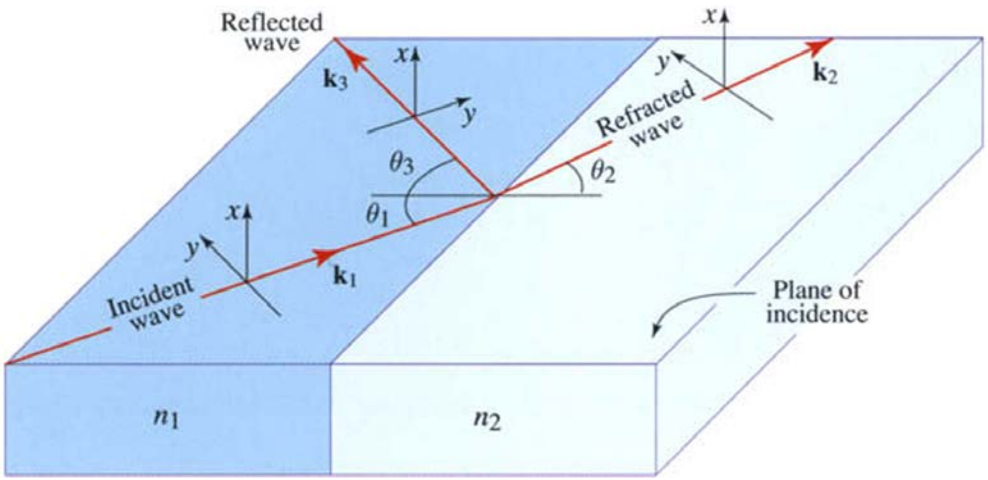
Coordinate transformation:

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$



$$\mathbf{J}' = \mathbf{R}(\theta)\,\mathbf{J}$$

Reflection and refraction



$$\mathbf{t} = \begin{bmatrix} \mathfrak{t}_x & 0 \\ 0 & \mathfrak{t}_y \end{bmatrix}, \qquad \mathbf{r} = \begin{bmatrix} \mathfrak{r}_x & 0 \\ 0 & \mathfrak{r}_y \end{bmatrix}$$

$$E_{2x} = \mathfrak{t}_x E_{1x}, \qquad E_{2y} = \mathfrak{t}_y E_{1y}$$
$$E_{3x} = \mathfrak{r}_x E_{1x}, \qquad E_{3y} = \mathfrak{r}_y E_{1y}.$$

The electromagnetic boundary conditions yield the solutions:

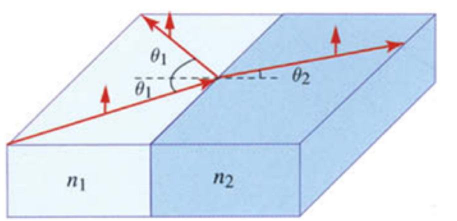
$$\mathfrak{r}_x = \frac{\eta_2 \sec\theta_2 - \eta_1 \sec\theta_1}{\eta_2 \sec\theta_2 + \eta_1 \sec\theta_1}, \quad \mathfrak{t}_x = 1 + \mathfrak{r}_x,$$
$$\mathfrak{r}_y = \frac{\eta_2 \cos\theta_2 - \eta_1 \cos\theta_1}{\eta_2 \cos\theta_2 + \eta_1 \cos\theta_1}, \quad \mathfrak{t}_y = (1 + \mathfrak{r}_y)\frac{\cos\theta_1}{\cos\theta_2}$$

For nonmagnetic transparent dielectrics, one obtains the Fresnel equations:

$$\mathfrak{r}_x = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}, \quad \mathfrak{t}_x = 1 + \mathfrak{r}_x,$$
$$\mathfrak{r}_y = \frac{n_1 \sec\theta_1 - n_2 \sec\theta_2}{n_1 \sec\theta_1 + n_2 \sec\theta_2}, \quad \mathfrak{t}_y = (1 + \mathfrak{r}_y)\frac{\cos\theta_1}{\cos\theta_2}$$

$$\cos\theta_2 = \sqrt{1 - \sin^2\theta_2} = \sqrt{1 - (n_1/n_2)^2 \sin^2\theta_1}$$

Reflection at a boundary



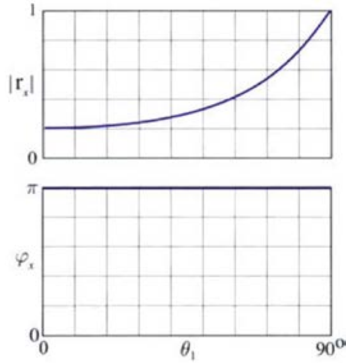
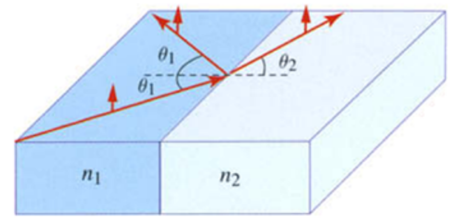


Figure 6.2-2 Magnitude and phase of the reflection coefficient as a function of the angle of incidence for *external reflection* of the *TE-polarized* wave ($n_2/n_1 = 1.5$).



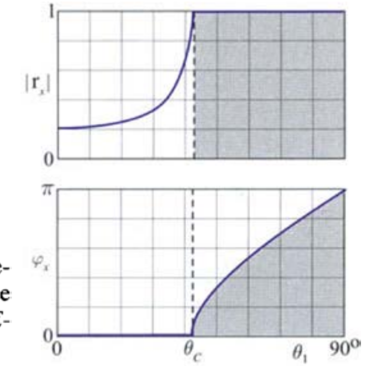
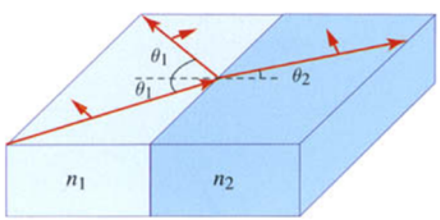


Figure 6.2-3 Magnitude and phase of the reflection coefficient as a function of the angle of incidence for *internal reflection* of the *TE-polarized* wave ($n_1/n_2 = 1.5$).

TE and TM polarizations show different magnitudes and phases of both *r* and *t*.

Total internal reflection at

$$\sin\theta_c = \frac{n_2}{n_1}\sin 90^\circ = \frac{n_2}{n_1}$$



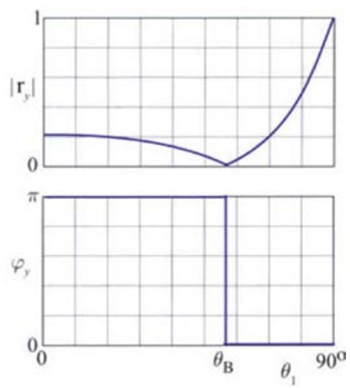
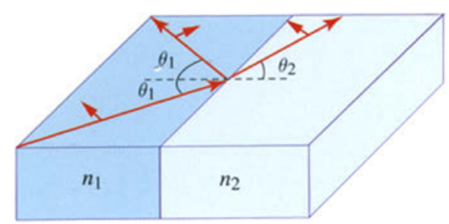


Figure 6.2-4 Magnitude and phase of the reflection coefficient as a function of the angle of incidence for *external reflection* of the *TM-polarized* wave ($n_2/n_1 = 1.5$).



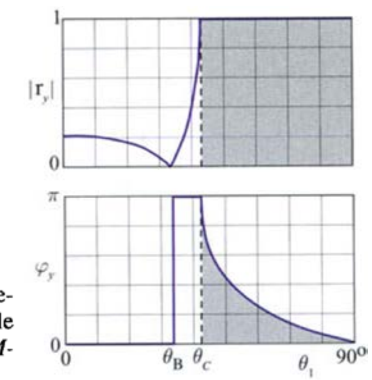


Figure 6.2-5 Magnitude and phase of the reflection coefficient as a function of the angle of incidence for *internal reflection* of the *TM-polarized wave* ($n_1/n_2 = 1.5$).

We have $r_{TM} = 0$ at the Brewster angle: $\tan\theta_B = n_2/n_1$.

Power reflection and transmission: $R = |r|^2$ and $T = 1 - R \neq |t|^2$.