$$d \wedge \Phi = 0$$

$$a_1 \wedge a_2 = -a_2 \wedge a_1$$

$$d \wedge \Psi = \gamma$$

$$d = d_E + d\tau\, \partial_\tau$$

$$\Psi = D - H \wedge d\tau$$

$$\Phi = B + E \wedge d\tau$$

$$\gamma = \rho - J \wedge d\tau$$

$$d \wedge \Phi = (d_E + d\tau\, \partial_\tau) \wedge (B + E \wedge d\tau)$$

$$= d_E \wedge B + d_E \wedge E \wedge d\tau + d\tau \wedge \partial_\tau B$$

$$= d_E \wedge B + (d_E \wedge E + \partial_\tau B) \wedge d\tau = 0$$

$$d \wedge \Psi = \gamma$$

$$\nabla\times\bar{E} = -\frac{\partial}{\partial t}\bar{B} \qquad (\bar{r},t)$$

$$\nabla\times\bar{H} = \frac{\partial}{\partial t}\bar{D} + \bar{J}$$

$$\nabla\cdot\bar{D} = \rho \qquad \times\,\nabla\cdot \;\Rightarrow\; \nabla\cdot\bar{J} = -\frac{\partial}{\partial t}\overbrace{\nabla\cdot\bar{D}}^{\rho}$$

$$\nabla\cdot\bar{B} = 0$$

$$\bar{D} = \varepsilon\bar{E}, \quad \bar{B} = \mu\bar{H} \qquad \text{(ISOTROPY)}$$

$$\nabla\times(\nabla\times\bar{E}) = -\frac{\partial}{\partial t}\mu\,\nabla\times\bar{H} = -\mu\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}\varepsilon\bar{E} + \bar{J}\right)$$

$$\underbrace{\nabla\times(\nabla\times\bar{E})}_{\nabla(\nabla\cdot E)-\nabla^2\bar{E}} + \mu\varepsilon\frac{\partial^2}{\partial t^2}\bar{E} = -\mu\frac{\partial}{\partial t}\bar{J}$$

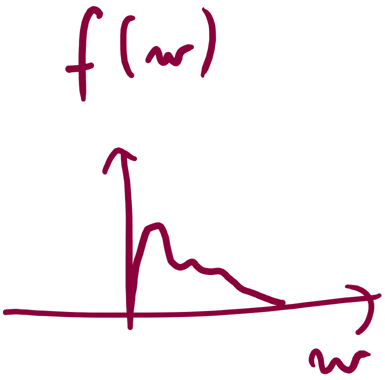
$$\underline{\rho, \bar{J} = 0}$$

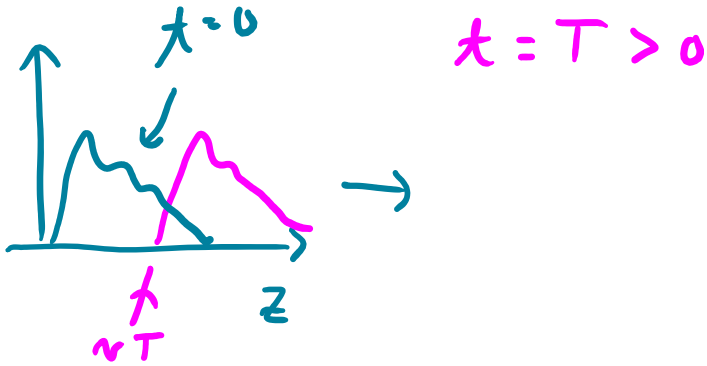
$$\nabla^2\bar{E} - \mu\varepsilon\frac{\partial^2}{\partial t^2}\bar{E} = 0 \qquad \text{WAVE EQUATION}$$

$$f(z-vt) \;\Rightarrow\; \frac{\partial^2}{\partial z^2}f(z-vt) = f''(z-vt)$$

$$\frac{\partial^2}{\partial t^2}f(z-vt) = +v^2 f''(z-vt)$$

$$\frac{\partial^2}{\partial z^2}f(z-vt) - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}f(z-vt) = 0$$





$f(z + vt)$

$\longleftarrow$

$e^{j\omega t}$: $\frac{\partial}{\partial t} = j\omega$

temporal Fourier transform

$$f(\omega) = \int_{-\infty}^{\infty} F(t)\, e^{-j\omega t}\, dt$$

$$F(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\omega)\, e^{j\omega t}\, d\omega$$

$f(-\omega) = f^*(\omega)$

ISOTROPIC $\quad \bar{D} = \varepsilon \bar{E}$, $\quad \bar{D}(\omega) = \varepsilon(\omega)\,\bar{E}(\omega)$

$$\varepsilon(\omega) \rightarrow \varepsilon_r \varepsilon_0, \qquad \varepsilon_r \rightarrow 1 + \chi, \qquad \chi \rightarrow \text{SUSCEPTIBILITY}$$

$$\bar{D}(\omega) = \varepsilon_0 \bar{E} + \bar{P}(\omega)$$

$$\bar{P}(\omega) \rightarrow \varepsilon_0\, \chi(\omega)\, \bar{E}(\omega)$$

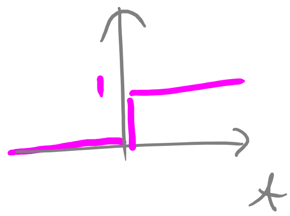
$$\bar{P}(t) = \frac{1}{2\pi}\int_{\omega} \bar{P}(\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{\varepsilon_0}{2\pi}\int_{\omega} \chi(\omega)\, \underbrace{\bar{E}(\omega)}_{\int_{t'} \bar{E}(t')\, e^{-j\omega t'} dt'}\, e^{j\omega t}\, d\omega$$

$$= \varepsilon_0 \int_{t'} \underbrace{\frac{1}{2\pi}\int_{\omega} \chi(\omega)\, e^{j\omega(t-t')}\, d\omega}_{\chi(t-t')}\; \bar{E}(t')\, dt'$$

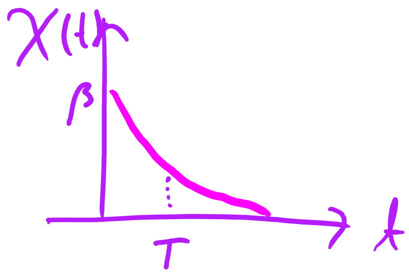
$$\bar{P}(t) = \varepsilon_0 \int_{t'=-\infty}^{t\,\cancel{+\infty}} \chi(t-t')\, \bar{E}(t')\, dt'$$

HEAVISIDE FUNCTION $\quad H(t)$



WATER: SUSCEPTIBILITY KERNEL

$$\chi(t) = H(t)\,\beta\, e^{-t/T}$$



RELAXATION TIME

$$\chi(\omega) = \int_{-\infty}^{\infty} H(t)\,\beta\, e^{-t/T} e^{-j\omega t}\, dt$$

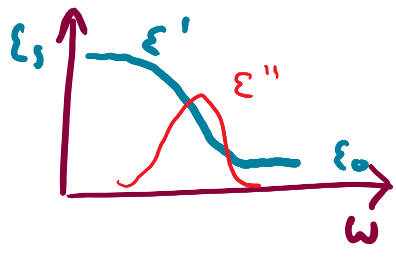
$$= \beta \int_{0}^{\infty} e^{-\left(j\omega + \frac{1}{T}\right)t}\, dt$$

$$= \beta \left. \frac{e^{-\left(j\omega + \frac{1}{T}\right)t}}{-\left(j\omega + \frac{1}{T}\right)} \right|_{0}^{\infty} = \beta\left(0 + \frac{1}{j\omega + 1/T}\right) = \frac{\beta T}{1 + j\omega T}$$

$$\varepsilon(\omega) = \varepsilon_0\left(1 + \chi(\omega)\right) = \varepsilon_0 + \varepsilon_0 \frac{\beta T}{1 + j\omega T}$$

$$\varepsilon(0) = \varepsilon_0 + \varepsilon_0 \beta T = \varepsilon_s \Rightarrow \beta T = \frac{\varepsilon_s - \varepsilon_0}{\varepsilon_0}$$

$$\varepsilon(\omega) = \varepsilon_0 + \frac{\varepsilon_s - \varepsilon_0}{1 + j\omega T}$$



$$\nabla \times \bar{E} = -j\omega \bar{B} - \bar{J}_m$$

$$\nabla \times \bar{H} = +j\omega \bar{D} + \bar{J}$$

$$\nabla \cdot \bar{D} = \rho \quad \frac{As}{m^3}$$

$$\nabla \cdot \bar{B} = \rho_m \quad \frac{Vs}{m^3}$$

$$\frac{1}{m} \qquad \frac{Vs}{m^2}$$

$$\nabla \cdot \bar{J}_m = -j\omega \rho_m$$

CONTINUITY EQN

$$j\omega \nabla \cdot \bar{D} + \nabla \cdot \bar{J} = 0$$

$$\nabla \cdot \bar{J} = -j\omega \rho$$

FOURIER:

$$\bar{r} \leftrightarrow \bar{k}$$

$$f(\bar{k}) = \int F(\bar{r})\, e^{+j\bar{k}\cdot\bar{r}}\, dV_r$$

$$F(\bar{r}) = \frac{1}{(2\pi)^3} \int f(\bar{k})\, e^{-j\bar{k}\cdot\bar{r}}\, dV_k$$