$$
\begin{array}{rlr}
d \wedge \Phi=0 & a_{1} \wedge a_{2}=-a_{2} \wedge a_{1} \\
d \wedge \Psi=\gamma & \\
d=d_{E}+d \tau \partial_{\tau} & \Psi=D-H \wedge d \tau \\
\Phi=B+E \wedge d \tau & \nu=\rho-J \wedge d \tau \\
d \wedge \Phi & =\left(d_{E}+d \tau \partial_{\tau}\right) \wedge(B+E \wedge d \tau) \\
& =d_{E} \wedge B+d_{E} \wedge E \wedge d \tau+d \tau \wedge \partial_{\tau} B \\
& =d_{E} \wedge B+\left(d_{E} \wedge E+\partial_{\tau} B\right) \wedge d \tau=0 \\
d \wedge \Psi & =\gamma &
\end{array}
$$

$$
\begin{aligned}
& \nabla \times \bar{E}=-\frac{\partial}{\partial t} \bar{B} \\
& (\bar{r}, t) \\
& \nabla \times \bar{H}=\frac{2}{\partial t} \bar{D}+\bar{j} \\
& \begin{array}{l}
\nabla \cdot \bar{D}=\rho \\
\nabla \cdot \bar{B}=0
\end{array} \quad \dot{\sim} \cdot \Rightarrow \nabla \cdot \bar{j}=-\frac{\partial}{\partial t} \stackrel{\rho}{\overbrace{\nabla} \cdot \bar{D}} \\
& \bar{D}=\varepsilon \bar{E}, \bar{B}=\mu \bar{H} \quad \text { (ISOTROPY) } \\
& \nabla \times(\nabla \times \bar{E})=-\frac{\partial}{\partial t} \mu \nabla \times \bar{H}=-\mu \frac{\partial}{\partial t}\left(\frac{\partial}{\partial t} \varepsilon \bar{E}+\bar{j}\right) \\
& \underbrace{\nabla \times(\nabla \times \bar{E})}+\mu \varepsilon \frac{\partial^{2}}{\partial t^{2}} \bar{E}=-\mu \frac{\partial}{\partial t} \bar{\jmath} \\
& \nabla(\nabla \cdot E)-\nabla^{2} \bar{E} \\
& \rho_{1} j=0 \\
& \nabla^{2} \bar{E}-\mu \varepsilon \frac{\partial^{2}}{\partial t^{2}} \bar{E}=0 \\
& \text { WAVE } \\
& \text { EqUATION } \\
& f(z-v t) \quad \Rightarrow \quad \frac{\partial^{2}}{\partial z^{2}} f(z-v t)=f^{\prime \prime}(z-v t) \\
& \frac{\partial^{2}}{\partial t^{2}} f(z-v t)=+v^{2} f^{\prime \prime}(z \cdot v t) \\
& \frac{\partial^{2}}{\partial z^{2}} f(z-v t)-\frac{1}{v^{2}} \frac{\partial^{2}}{\partial t^{2}} f(z-v t)=0
\end{aligned}
$$




$$
\begin{gathered}
f(z+w t) \\
\leftarrow
\end{gathered}
$$

$$
e^{j \omega t}: \quad \frac{\partial}{\partial t}=j \omega
$$

temporal Fourier transform

$$
\begin{array}{ll}
f(\omega)=\int_{-\infty}^{\infty} F(t) e^{-j \omega t} d t & \quad f(-\omega)=f^{*}(\omega) \\
F(t)=\frac{1}{2 \pi} \int_{-\infty}^{\infty} f(\omega) e^{j \omega t} d \omega &
\end{array}
$$

ISOTROPIC $\bar{D}=\varepsilon \bar{E}, \quad \bar{D}(\omega)=\varepsilon(\omega) \bar{E}(\omega)$
SUSCEPTIBILITY
heaviside function $H(t)$


$$
\begin{aligned}
& \bar{D}(\omega)=\varepsilon_{0} \bar{E}+\bar{P}(\omega) \\
& \varepsilon_{0} X(\omega) E(\omega) \\
& \varepsilon_{r} \varepsilon_{0} \\
& t_{1+x} \\
& \bar{P}(t)=\frac{1}{2 \pi} \int_{\omega} \bar{P}(\omega) e^{j \omega t} d \omega \\
& \text { x } \\
& =\frac{\varepsilon_{0}}{2 \pi} \int_{\omega} X(\omega) \underbrace{E(\omega)} e^{j \omega t} d \omega \\
& \int_{A^{\prime}} E\left(t^{\prime}\right) e^{-j \omega t^{\prime}} d t^{\prime} \\
& =\varepsilon_{0} \int_{t^{\prime}} \underbrace{\frac{1}{2 \pi} \int_{\omega} X(\omega) e^{j \omega\left(t-t^{\prime}\right)} d \omega \bar{E}\left(t^{\prime}\right) d t^{\prime}}_{\chi\left(t-t^{\prime}\right)} \\
& \bar{P}(t)=\varepsilon_{0} \int_{t^{\prime}=-\infty}^{++x} x\left(t-t^{\prime}\right) E\left(t^{\prime}\right) d t^{\prime}
\end{aligned}
$$

Water: susceptibility kernel

$$
\begin{aligned}
& X(t)=H(t) \beta e^{-t / T} \\
& X(\omega)=\int_{-\infty}^{\infty} H(t) \beta e^{-t / T} e^{-j \omega t} d t \\
& =\beta \int_{0}^{\infty} e^{-\left(j \omega+\frac{1}{T}\right) t} d t \\
& \xrightarrow[\substack{\text { RELAXATIoN } \\
\text { TIME }}]{\substack{\chi(1)}} \neq \\
& =\beta \int_{0}^{\infty} \frac{e^{-\left(j \omega+\frac{1}{T}\right) t}}{-\left(j \omega+\frac{1}{T}\right)}=\beta\left(0+\frac{1}{j \omega+1 / r}\right)=\frac{\beta T}{1+j \omega T} \\
& \varepsilon(\omega)=\varepsilon_{0}(1+\chi(\omega))=\varepsilon_{0}+\varepsilon_{0} \frac{\beta T}{1+j \omega T} \\
& \varepsilon(0)=\varepsilon_{0}+\varepsilon_{0} \rho T=\varepsilon_{s} \Rightarrow \beta T=\frac{\varepsilon_{s}-\varepsilon_{0}}{\varepsilon_{0}} \\
& \varepsilon(\omega)=\varepsilon_{0}+\frac{\varepsilon_{s}-\varepsilon_{0}}{1+j \omega T}
\end{aligned}
$$

$$
\begin{aligned}
& \nabla \times \bar{E}=-j \omega \bar{B}-\bar{j}_{m} \\
& \nabla \times \bar{H}=+j \omega \bar{D}+\bar{j} \\
& \nabla \cdot \bar{D}=\rho \times \frac{A s}{m^{3}} \\
& \nabla \cdot \bar{B}=\rho_{m} \\
& k \frac{k}{m} \frac{v_{s}}{m^{2}}
\end{aligned}
$$

$$
\nabla \cdot \bar{j}_{m}=-j \omega \rho_{m}
$$

CONTINUITY EaR

$$
\begin{gathered}
j \omega \cdot \nabla \cdot \bar{b}+\nabla \cdot \bar{j}=0 \\
\nabla \cdot \bar{j}=-j \omega \rho
\end{gathered}
$$

FOURIER: $\quad f(\bar{k})=\int F(\bar{r}) e^{+j \bar{L} \cdot \bar{r}} d V_{r}$

$$
\bar{r} \leftrightarrow \bar{k}
$$

$$
\begin{aligned}
& f(\bar{k})=\int F(\bar{r}) e^{+j \bar{L} \cdot \bar{r}} d V_{r} \\
& F(\bar{r})=\frac{1}{(2 \pi)^{3}} \int f(\bar{k}) e^{-j \bar{k} \cdot \bar{r}} d V_{k}
\end{aligned}
$$

