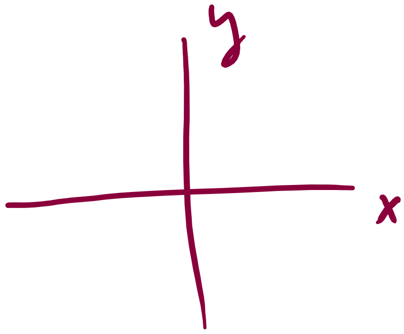
$$\varepsilon(\omega) = \varepsilon_0 + \frac{\varepsilon_s - \varepsilon_\infty}{1 + j\omega\tau}$$

KRAMERS-KRONIG RELATIONS

$$f(z) = u + jv \qquad z = x + jy$$



$$z^2 = (x+jy)^2 = \underbrace{x^2 - y^2}_{u} + j\,\underbrace{2xy}_{v}$$

$$\frac{\partial u}{\partial x} = 2x \qquad \frac{\partial u}{\partial y} = -2y$$

$$\frac{\partial v}{\partial x} = 2y \qquad \frac{\partial v}{\partial y} = 2x$$

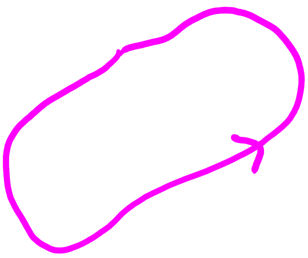
Cauchy-Riemann

$$\begin{cases} \dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y} \\[2ex] \dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x} \end{cases}$$

LAPLACE : $\nabla^2 \phi = 0$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi(x,y)$$

$$\nabla^2 u(x,y) = \frac{\partial}{\partial x}\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}\frac{\partial}{\partial y}u = \frac{\partial}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial}{\partial y}\frac{\partial v}{\partial x} = 0$$

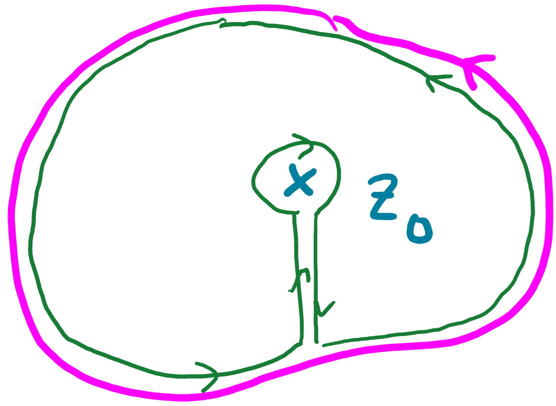


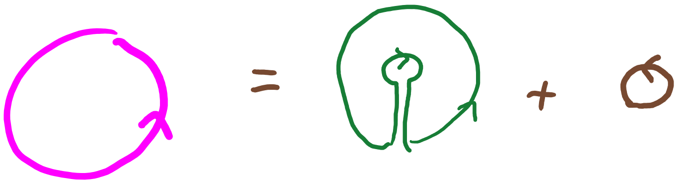
z

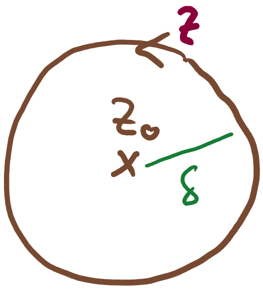
$$\oint f(z)\,dz = 0$$

$\uparrow$ ANALYTIC

$$\oint_C \frac{f(z)}{z-z_0}\,dz = ?$$







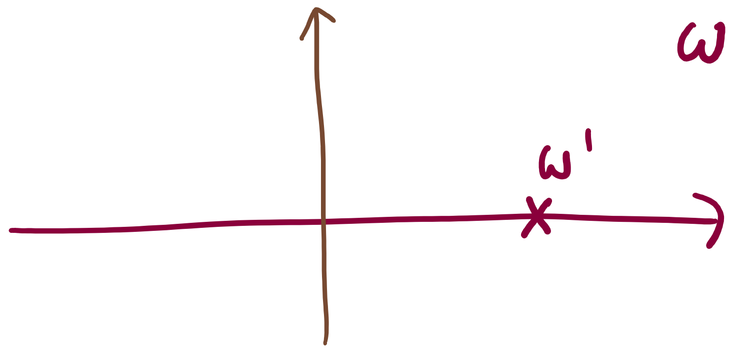
$$z = z_0 + \delta e^{j\varphi}$$

$$dz = j\delta e^{j\varphi}\,d\varphi$$

$$\oint \frac{f(z)}{z-z_0}\,dz = \int_0^{2\pi} \frac{f(z)}{\cancel{\delta e^{j\varphi}}}\, j\cancel{\delta e^{j\varphi}}\,d\varphi$$

$$= j f(z_0) \int_0^{2\pi} d\varphi = j2\pi f(z_0)$$

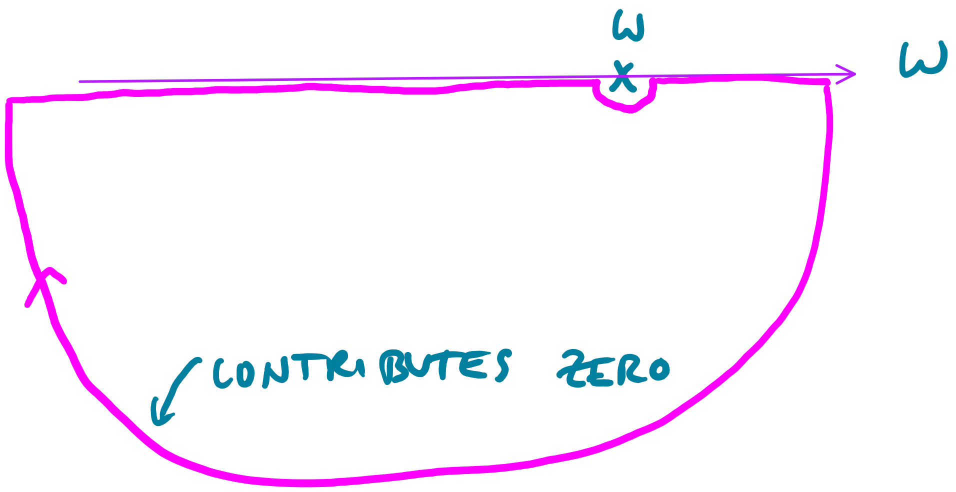
$$\int_{-\infty}^{\infty} \frac{X(\omega')}{\omega' - \omega} \, d\omega' = ?$$

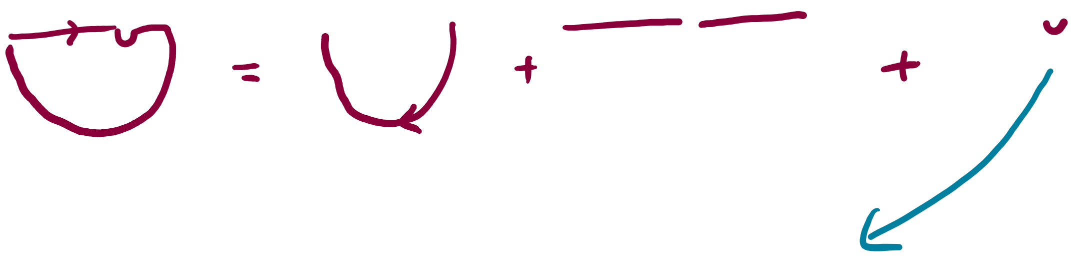


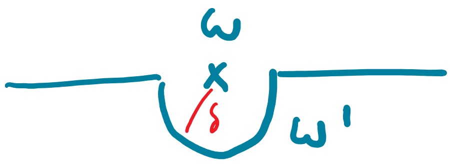
$$X(\omega) = \int x(t) \, e^{-j\omega t} \, dt$$

$e^{-j\omega t} \to 0$ as $\mathrm{Im}\{\omega\} \to -\infty$

$$\int_{-\infty}^{\infty} \frac{X(\omega')}{\omega' - \omega} \, d\omega'$$







$\omega' = \omega + \delta e^{j\varphi}$

$d\omega' = j\delta e^{j\varphi} \, d\varphi$

$$\int \frac{X(\omega) \, j\delta e^{j\varphi}}{\delta e^{j\varphi}} \, d\varphi = j X(\omega) \int^{2\pi} d\varphi = j\pi X(\omega)$$

$$\int_{\cup} \frac{\chi(\omega)\, j\delta e^{i\varphi}}{\omega + \delta e^{i\varphi} - \omega}\, d\varphi = j\chi(\omega) \int_{\pi} d\varphi = j\pi\chi(\omega)$$

$$\underbrace{\int_{-\infty}^{\omega-\delta} + \int_{\omega+\delta}^{\infty} \frac{\chi(\omega')}{\omega'-\omega}\, d\omega'}_{PV \int_{-\infty}^{\infty} \frac{\chi(\omega')}{\omega'-\omega} d\omega'} + j\pi\chi(\omega) = 0$$

$$\chi(\omega) = \frac{j}{\pi}\, PV \int_{-\infty}^{\infty} \frac{\chi(\omega')}{\omega'-\omega}\, d\omega'$$

$$\chi = \chi' - j\chi''$$

$$\Rightarrow \chi'(\omega) = \frac{1}{\pi}\, PV \int_{-\infty}^{\infty} \frac{\chi''(\omega)}{\omega'-\omega}\, d\omega'$$

$$\chi''(\omega) = -\frac{1}{\pi}\, PV \int_{-\infty}^{\infty} \frac{\chi'(\omega)}{\omega'-\omega}\, d\omega'$$

BIANISOTROPIC MEDIA

$$\bar{D} = \bar{\bar{\varepsilon}} \cdot \bar{E} + \bar{\bar{\xi}} \cdot \bar{H}$$

$$\bar{B} = \bar{\bar{\zeta}} \cdot \bar{E} + \bar{\bar{\mu}} \cdot \bar{H}$$

LOSSLESS:

$$\bar{\bar{\varepsilon}}^{*} = \bar{\bar{\varepsilon}}^{T}, \quad \bar{\bar{\mu}}^{*} = \bar{\bar{\mu}}^{T}$$

$$\bar{\bar{\xi}}^{*} = \bar{\bar{\zeta}}^{T}$$

$$\bar{\bar{\zeta}} = \bar{\bar{\chi}} + j\bar{\bar{\kappa}} \qquad \bar{\bar{\xi}} = \bar{\bar{\chi}}^{T} - j\bar{\bar{\kappa}}^{T}$$

$$\bar{\bar{\xi}}^{*} = \bar{\bar{\chi}}^{T*} + j\,\bar{\bar{\kappa}}^{T*}$$

$$\bar{\bar{\zeta}}^{T} = \bar{\bar{\chi}}^{T} + j\,\bar{\bar{\kappa}}^{T}$$

LOSSLESS:

$\bar{\bar{\chi}}, \bar{\bar{\kappa}}$ real

RECIPROCITY

$$\bar{\bar{\varepsilon}} = \bar{\bar{\varepsilon}}^{T}, \quad \bar{\bar{\mu}} = \bar{\bar{\mu}}^{T}, \quad \bar{\bar{\xi}} = -\bar{\bar{\zeta}}^{T}$$

$$\bar{\bar{\xi}} = \bar{\bar{\chi}}^{T} - j\bar{\bar{\kappa}}^{T}$$

$$-\bar{\bar{\zeta}}^{T} = -\left(\bar{\bar{\chi}}^{T} + j\bar{\bar{\kappa}}^{T}\right) = -\bar{\bar{\chi}}^{T} - j\bar{\bar{\kappa}}^{T}$$

$$\Rightarrow \bar{\bar{\chi}} = 0 \quad (\bar{\bar{\kappa}} \text{ anything})$$